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SCHOOL OF MECHANICAL ENGINEERING
DEPARTMENT OF AERONAUTICAL ENGINEERING

UNIT – I- AIRCRAFT STRUCTURES – SAE1303

UNIT – I

STATICALLY DETERMINATE STRUCTURES

Analysis of 2 D, 3 D trusses

Frames

Composite beams,

Propped cantilever

Fixed-fixed beams-

Clapeyron's Three Moment Equation

Moment Distribution Method, Super position method (brief).

Statically determinate structure.

If the structure can be analyzed and the reactions at the support can be determined by using the equations of static equilibrium such åFx = 0 and SFy = 0 and SM = 0, then it is called as a statically determine structure. Example: Simply supported beam, pin jointed truss or frame.

Truss and Frame

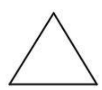
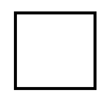
Truss	Frame
Truss is defined as number of members riveted together to carry the horizontal, vertical and inclined loads in equilibrium.	Frame is defined as number of members together to carry the horizontal. Vertical loads in equilibrium.

Types of Frames

Frames are classified into two types.

1. Perfect
2. Imperfect
   (i) Deficient frame
   (ii) Redundant frame

Sl.No	Perfect frame	Imperfect frame
1.	Perfect frames have sufficient or enough members to carry the load.	Imperfect frames have less or more members to carry the load than the required numbers.
2	It satisfies the formula n = 2j-3	It does not satisfy the formula n = 2j-3

Sl. No	Deficient frame	Redundant frame
1.	If the number of members are less than the required number of members n < 2j-3	If the number of members are more than the required number of members n > 2j-3
2	Eg. Triangular frame n = 3, j = 3 n = 2j-3 3 = 2×3 -3, 3 = 3 	Eg: Square frame n = 4, j = 4 n = 2j-3 4 = 2×4 -3, 4 ≠ 5 

Where, n = number of members, j = number of joints.

conditions of equilibrium used in the method of joints

The conditions of equilibrium used in the method of joints are, å Fx = 0, å Fy = 0. One of the assumption is all the joints are pin jointed, there is no moment. The equilibrium condition åMx = 0 is not used.

Pin-joined plane frame.

Pin-joined plane frames (also known as trusses) are commonly used in structures to span large distances where constructing beams is uneconomical. They are common as roof structures in industrial buildings, and large assembly building and bridges.

Assumptions made in the analysis of a pin-jointed plane frame.

The structural action of a frame is derived from the following assumptions to get an ideal frame.

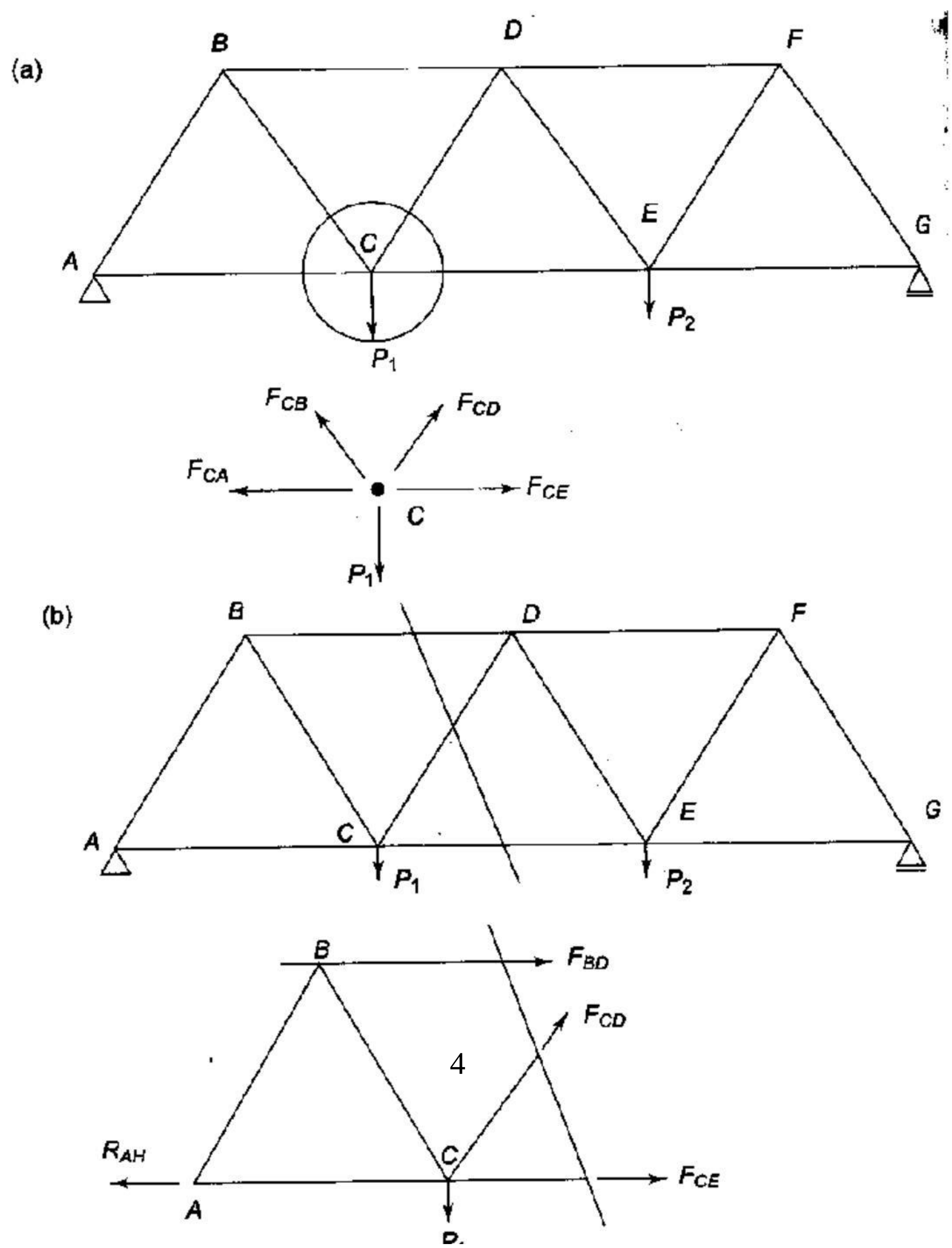
The frame has perfect hinge joints. For practical purposes, this assumption gives reasonable results and makes the actual frame more stable.

The frame is loaded only at the joints and not in between the joints. The weights of members acting over their length are calculated and transferred to the joints.

The centroidal lines of the members meet at the joint. By careful fabrication and design, this can be reasonably achieved. If th e lines are not concurrent, some moment due to eccentricity is developed.

Two methods employed for the analysis of a pin -jointed frame and principle involved in each case:

The basic approach to the analysis of a frame is the section method. We take a se ction cutting a number of bars, and consider the equilibrium of either of the two parts so obtained. On solution, the equilibrium equations so formulated can give us values of unknown bar forces and reactions. Depending upon the method of taking a section, there are basically two methods of analysing a frame, as shown in figs.



Section around a joint or method of joints:

In this case, as in fig, a section is taken around a joint, isolating the joint completely. The important point to note is that the isolated joint is in equilibrium under the action of a set of concurrent forces. Thus, there are two equilibrium equations for each joint, □H=0 and □V=0, where □H and □V re the summations o components along two mutually perpendicular directions.

In a stable, determinate frame, there are 2j equations available and the number of members is only (2j-3). The three extra equations available can be used to calculate the three unknown reactions or for checking.

Ritter's method of section:

Here, the section is taken as shown in fig. the truss is separated into two parts by such a section and each part is in equilibrium under the action of a general coplanar fore system. There are three equilibrium equations. □H=0, □V=0, and □M=0, available for such a force system and three unknown forces can, therefore, be determined.

If the retains are calculated from the conditions of equilibrium for the frame as a whole, then the advantage of Ritter's method of section is that it enables us to determine the force in any member by taking a section cutting that member. In the method of joints, it is necessary to go from free ends as in a cantilever truss.

Before we discuss these methods in detail, we need to look at some simple procedures to enable us to find forces in some members through visual inspection or to check the results.

Two methods of building a frame work:

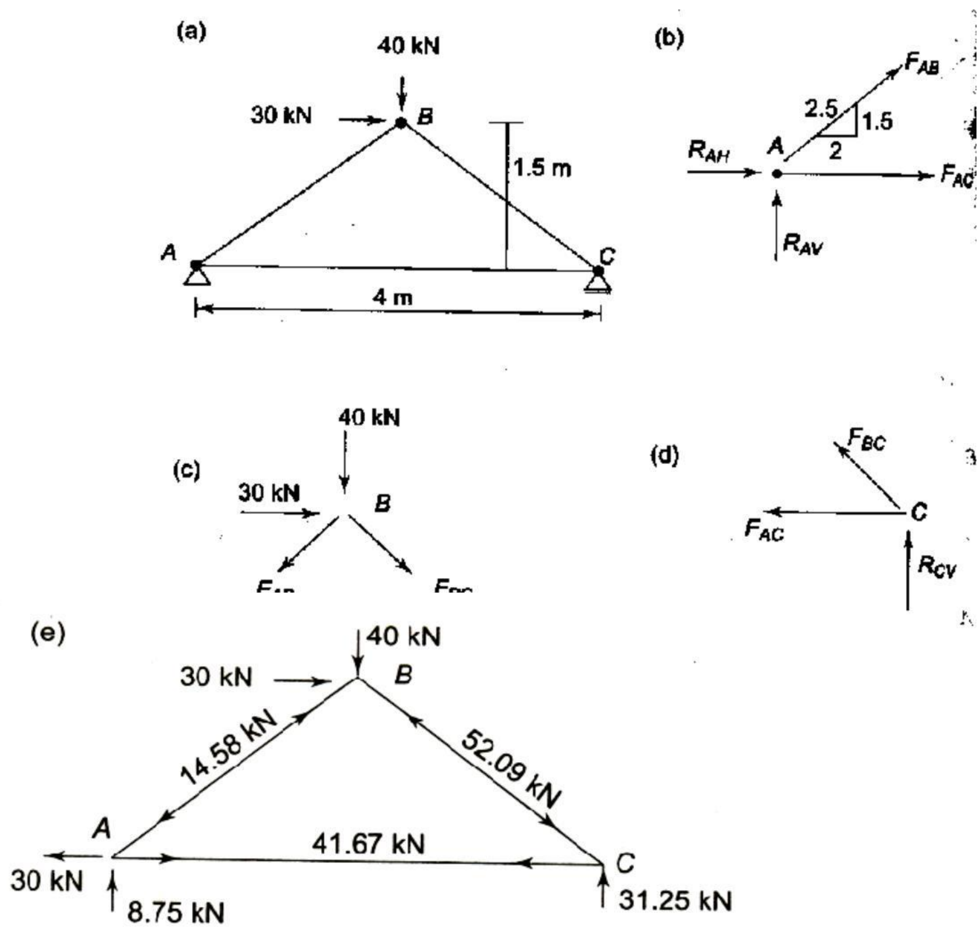
Staring with a triangle of three members and three joints, the frame can be built up to any extent by adding two members for every additional joint. This gives an internally stable frame work, which can be supported suitably for external stability.

Starting from a firm foundation, two members can be made to form a joint. The frame can be built up further as described. Note that the frame work is dependent upon its attachment to the foundation for internal stability.

Analyse the frame shown in fig. and find the forces in all the members.

Solution :

There are three members forces and three reactive components – R_{AH}, R_{AV}, and $R_{CV,}$ we formulate two equations for each of the joints a, B and C and determine the six unknowns.



These equations can be solved to evaluate the six unknowns.

$F_{AB} = -14.58$ kN

(The member is under compression and not tension as assumed)

$F_{BC} = -52.09$ kN

(The member is in compression)

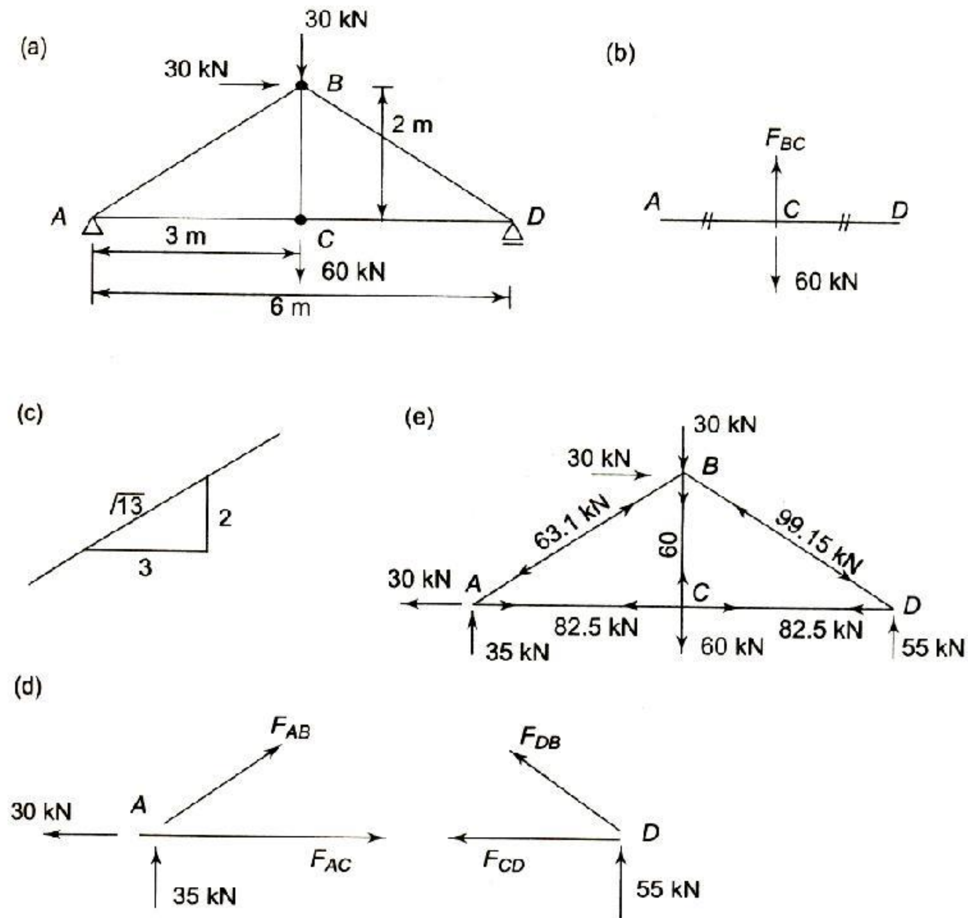
$F_{AC} = 41.67$ kN

(The member is in tension as assumed)

$R_{CV} = 31.25$ kN $\quad$ $R_{AV} = 8.75$ kN

and $\quad$ $R_{AH} = -30$ kN

Analyse the frame loaded as shown in fig. and determine the forces in all the members.



solution :

The reactions are determined from the equilibrium conditions of the frame as a whole. Let R_{AH} and R_{AV} be the vertical reaction at B.

$$R_{AH} = -\ 30\text{kN}$$

Acting towards the left.

$$R_{BV} = 55\text{kN}$$

$$R_{AV} = 35\text{kN}$$

From visual inspection, considering joint C shown in fig. F_{BC} = 60 kN (tensile) and F_{AC} = F_{CD}, both tensile or both compressive. As shown in fig, the ratio of the length, horizontal and vertical projections are 3 and 2 for AB and BD.

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Plane truss and Space truss

A plane truss is a two dimension truss structure composed of number of bars hinged together to form a rigid framework, all the members are lie in one plane. Eg: Roof truss in industries.

A space truss is a three dimension truss structure composed of number of bars hinged together to form a rigid framework, the members are lie in different plane. Eg: Transmission line towers, crane parts.

Methods used to analyze the plane & space frames

- Analytical method.
  1. Method of joints
  2. Method of sections (Method of moments)
  3. Tension coefficient method.
- Graphical method.

Assumptions made in the analyze of a truss

1. In a frame or truss all the joints will be pin jointed.
2. All the loads will be acting at the joints only.
3. The self-weight of the members of the truss is neglected. Only the live load is considered.
4. The frame is a perfect one.

Cantilever truss

If anyone of the member of the truss is fixed and the other end is free, it is called a cantilever truss. There is no reaction force at the fixed end.

Simply supported truss

If the members of the truss are supported by simple supports.

Hints to be followed while analyzing a cantilever truss using method of joints

- There is no need to find the support reactions.
- The analysis is to be started from the free end where there is a maximum of two unknown forces, using the condition of equilibrium å Fx = 0, and å Fy = 0 .
- All the members are assumed to be tensile.
- Consider tensile forces as positive and compressive as negative.
- The force convention is, upward force assigns positive sign and downward force assigns negative sign.

Hints to be followed while analyzing a simply supported truss using method of joints

- The support reactions are determined first.
- The analysis is to started from the free end where there is a maximum of two unknown forces, using the condition of equilibrium å Fx = 0, and å Fy = 0
.
- All the members are assumed to be tensile.
- Consider tensile forces as positive and compressive as negative.
- The force convention is, upward force assigns positive sign and downward force assigns negative sign.

Relation between the numbers of members and joints in a truss

$n = 2j - 3$, Where, n = number of members, j = number of joints. This relation is used to find the type of the frames. Perfect frame is only solved by method of joints.

Primary and secondary stresses in the analysis of a truss

If the stresses are produced due to direct loads like tension, compression and torsion then the stresses are called primary stress. If the stresses are produced due to expansion, compression and temperature variation then the stresses are called secondary stress.

Statically indeterminate structure:

The simple equations are not sufficient to solve some problems. Such problems are called statically indeterminate structures.

For solving statically indeterminate problems, the deformation characteristics of the structure are also taken into account along with the statical equilibrium equations. Such equation, which contain the deformation characteristics, are called compatibility equations.

Statically indeterminate structures.

If the forces on the members of a structure cannot be determined by using conditions of equilibrium (åFx = 0 and SFy = 0 and SM = 0), it is called statically indeterminate
structures.

STATICALLY INDETERMINATE STRUCTURES

Types of statically indeterminate structures:

1. Simple statically indeterminate structures
2. Indeterminate structures of equal lengths
3. Composite structures of equal length

Continuous beam:

A beam, which is supported on more than two supports, is called a continuous beam. Such a beam, when loaded will deflect with convexity upwards, over the intermediate supports and with concavity upwards over the mid of the spans. The intermediate supports of a continuous beam are always subjected to some bending moment. The end supports, if simply supported will not be subjected to any bending moment. But the end supports, if fixed, will be subjected to fixing moments and the slope of the beam, at the fixed ends will be zero.

1. Beams of unsymmetrical sections
2. Beams of uniform strength
3. Flitched beams

Flexural Rigidity of Beams.

The product of young's modulus (E) and moment of inertia (I) is called Flexural Rigidity (EI) of Beams. The unit is N mm^2.

Constant strength beam.

If the flexural Rigidity (EI) is constant over the uniform section, it is called Constant strength beam.

Composite beam.

A structural member composed of two or more dissimilar materials jointed together to acts as a unit. The resulting system is stronger than the sum of its parts. The composite action can better utilize the properties of each c constituent material.

Example : Steel – Concrete composite beam, Steel-Wood beam.

Application of the theorem of three moments to a fixed beam:

Sometimes, a continuous beam is fixed at its one or both ends. If the beams is fixed at the left end A, then an imaginary zero span is taken to the left of A and the three moments theorem is applied as usual. Similarly, if the both beam is fixed at the right end, then an imaginary zero span is taken after the right end support and the three moments theorem is applied as usual.

Carry over factor:

Consider a beam Av fixed at A and simply supported at B, let a clockwise moment be applied at the support B of the beam as shown in fig below.

Stiffness factor.

It is the moment required to rotate the end while acting on it through a unit rotation, without translation of the far end being

(i) Simply supported is given by k = 3 EI / L
(ii) Fixed is given by k = 4 EI / L

Where, E = Young’s modulus of the beam material.
I = Moment of inertia of the beam
L = Beam’s span length.

Sl. No	Statically determinate structures	Statically indeterminate structures
1.	Conditions of equilibrium are sufficient to analyze the structure	Conditions of equilibrium are insufficient to analyze the structure.
2.	Bending moment and shear force is independent of material and cross sectional area.	Bending moment and shear force is dependent of material and independent of cross sectional area.
3.	No stresses are caused due to temperature change and lack of fit.	Stresses are caused due to temperature change and lack of fit.

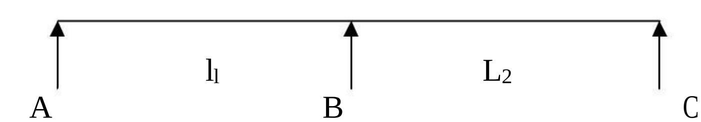
Continuous beam:

A Continuous beam is one, which is supported on more than two supports. For usual loading on the beam hogging (- ive) moments causing convexity upwards at the supports and sagging (+ ive) moments causing concavity upwards occur at mid span.

Advantages of Continuous beam over simply supported beam:

1. The maximum bending moment in case of continuous beam is much less than in case of simply supported beam of same span carrying same loads.
2. In case of continuous beam, the averaging bending moment is lesser and hence lighter materials of construction can be used to resist the bending moment.

General form of Clapeyron's three moment equations for the continuous beam:



$$M_a l_1 + 2 M_b l_2 + M_c l_2 = -\left(\frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2}\right)$$

Where,

M_a = Hogging bending moment at A
M_b = Hogging bending moment at B
M_c = Hogging bending moment C
l_1 = length of span between supports A, B
l_2 = length of span between supports B, C
$\bar{x}_1$ = CG of bending moment diagram from support A
$\bar{x}_2$ = CG of bending moment diagram from support C
A_1 = Area of bending moment diagram between supports A, B
A_2 = Area of bending moment diagram between supports B, C

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UNIT – II- AIRCRAFT STRUCTURES – SAE1303

UNIT – 2
ENERGY METHODS

1. Strain Energy:

The strain energy of a member will be defined as the increase in energy associated with the deformation of the member. The strain energy is equal to the work done by a slowly increasing load applied to the member.

2. Define Strain energy density.

The strain-energy density of a material will be defined as the strain energy per unit volume.

3. Define Modulus of toughness.

The area under the entire stress-strain diagram was defined as the modulus of toughness and is a measure of the total energy that can be acquired by the material.

4. Define Modulus of resilience.

The area under the stress-strain curve from zero strain to the strain ϵ_y at yield is referred to as the modulus of resilience of the material and represents the energy per unit volume that the material can absorb without yielding.

5. Write the expression for strain energy under axial load.

If the rod is of uniform cross section of area A, the strain energy is

$$U=P^2L/2AE$$

6. Write the expression for strain energy due to bending.

For a beam subjected to transverse loads the strain energy associated with the normal stresses is

$$U=M^2L/2EI$$

where M is the bending moment and EI the flexural rigidity of the beam.

7. Write the expression for strain energy due to torsion.

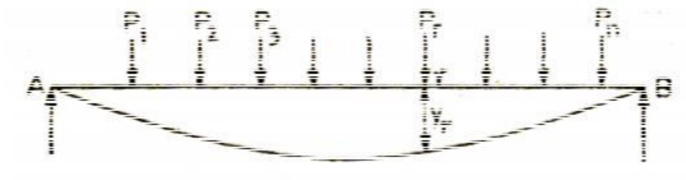
For a shaft of length Land uniform cross section subjected at its ends to couples of magnitude T the strain energy was found to be

$$U=T^2L/2GJ$$

Where J is t~~he po~~lar moment of inertia of the cross-sectional area of the shaft.

8. Define Castigliano's theorem.

In any beam or truss subjected to any load system, the deflection at any point r is given by the partial differential coefficient of the total strain energy stored with respect to a force P_r acting at the point r in the direction in which the deflection is desired.



Figure

Figure shows a structure AB carrying a load system P_1, P_2, P_3 ….P_r, ….P_n.

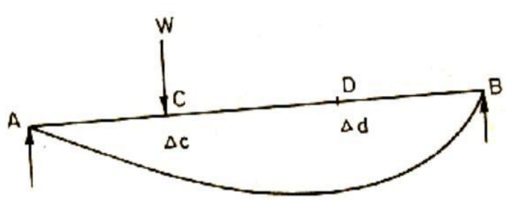
Let the deflection at the point r be y_r.
Let W_e = External work done be the given load system
W_i = Corresponding strain energy stored.
$W_e = W_i$

9. Define Maxwell's reciprocal theorem.

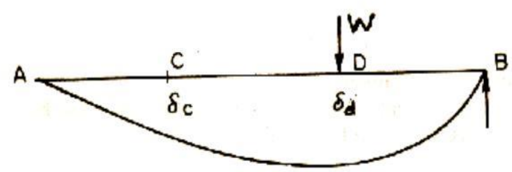
In any beam of truss the deflection at any point D due to a load W at any other point C is the same as the deflection at C due to the same load W applied at D.

Figure (i) shows a structure AB carrying a load W applied at any point C. Let the deflection at C be □c. Let the deflection at another point D be □d.

Figure (ii) shows the same structure AB carrying the same load W at D. Let the deflections at C and D be □$_c$ and □$_d$ respectively.



(i)



(ii)

10. Give the relation between number of joints and the number of members in a perfect frame.

Let there be n members and j joints in a perfect frame, Fig. (a)

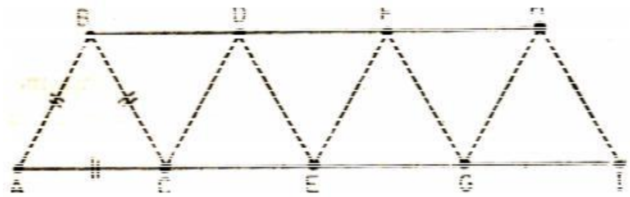


Fig. (a)

Suppose we remove three members AB, BC and CA and the three joints A, B and C. We are now left with (n – 3) members and (j – 3) joints.

Studying this remaining part of the frame (Fig. (b)), we find that the number of members in such that, for each joint, there are two members.

Hence for the (j – 3) joints we have 2(j – 3) members.

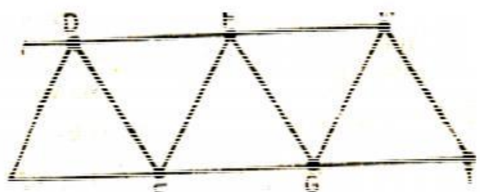


Fig. (b)

$$\Rightarrow n - 3 = 2(j - 3)$$
$$\Rightarrow n = 2j - 3$$

Hence for a stable frame the minimum number of members required = twice the number of joints minus three.

11. Derive the expression for Strain Energy under Axial

Loading. Strain Energy under Axial Loading

When a rod is subjected to centric axial loading, the normal stresses σ_x can be assumed uniformly distributed in any given transverse section. Denoting by A the area of the section located at a distance x from the end B of the rod and by P the internal force, we write $\sigma_x = P/A$.

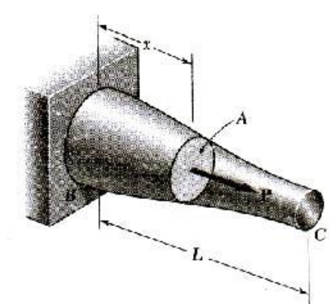


Figure: Axial loading

$$U=(P^2/2AE)dx$$

In the case of a rod of uniform cross section subjected at its ends to equal and opposite forces of magnitude P.

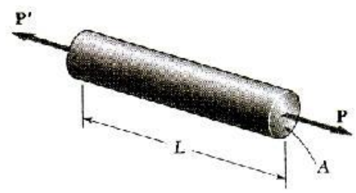
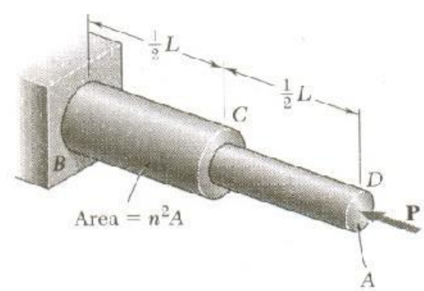


Figure: Axial loading

$$U=P^2L/2AE$$

12. **A rod consists of two portions BC and CD of the same material and same length, but of different cross sections. Determine the strain energy of the rod when it is subjected to a centric axial load P, expressing the result in terms of P, L, E, the cross-sectional area A of portion CD, and the ratio n of the two diameters.**



$$U = \frac{1+n^2}{2n^2}\frac{P^2L}{2AE}$$

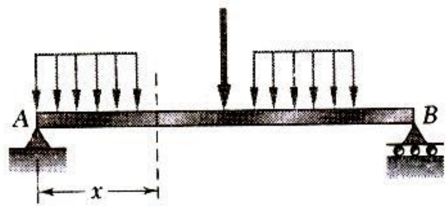
We check that, for n = 1, we have

$$U = \frac{P^2L}{2AE}$$

which is the expression given in equation for a rod of length L and uniform cross section of area A. We also note that, for n > 1, we have $U_n < U_1$; P/A, it follows that, for a given allowable stress, increasing the diameter of portion BC of the rod results in a decrease of the overall energy-absorbing capacity of the rod. Unnecessary changes in cross-sectional area should therefore be avoided in the design of members that may be subjected to loadings, such as impact loadings, where the energy-absorbing capacity of the member is critical.

13. Derive the expression for strain energy in bending. Strain Energy in Bending

Consider a beam AB subjected to a given loading and let M be the bending moment at a distance x from end A. Neglecting for the time being the effect of shear, and taking into account only the normal stresses $s_x = My/I$,



Figure

Setting dV = dA dx, where dA represents an element of the cross-sectional area, and recalling that $M^2/2EI^2$ is a function of x alone, we have

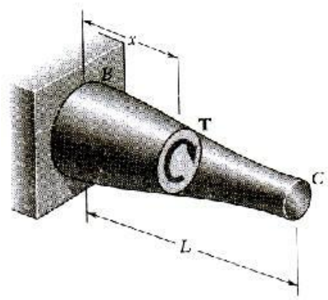
Recalling that the integral within the parentheses represents the moment of inertia I of the cross section about its neutral axis, we write

$$U = M^2L/2EI$$

14. Derive expression for strain energy due to

torsion. Strain Energy in Torsion

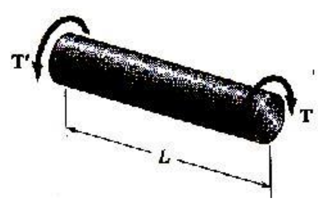
Consider a shaft BC of length L subjected to one or several twisting couples. Denoting by J the polar moment of inertia of the cross section located at a distance x from B and by T the internal torque in that section, we recall that the shearing stresses in the section are □$_{xy}$ = T□/J. Substituting for □$_{xy}$ we have



Setting dV = dA dx, where dA represents an element of the cross-sectional area, and observing that $T^2/2GJ^2$ is a function of x alone, we write

Recalling that the integral within the parentheses represents the polar moment of inertia J of the cross section, we have

$$U=(T^2L/2GJ)dx$$

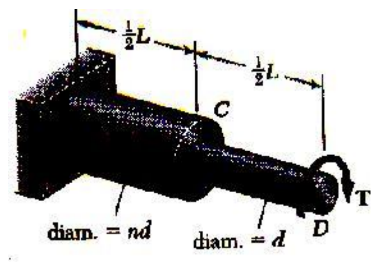


Figure

In the case of a shaft of uniform cross section subjected at its ends to equal and opposite couples of magnitude T yields.

$$U=T^2L/2GJ$$

15. A circular shaft consists of two portions BC and CD of the same material and same length, but of different cross sections. Determine the strain energy of the shaft when it is subjected to a twisting couple T at end D, expressing the result in terms of T, L, G, the polar moment of inertia J of the smaller cross section, and the ratio n of the two diameters.



Figure

$$U = \frac{1+n^4}{2n^4}\,\frac{T^2L}{2GJ}$$

We check that, for n = 1, we have

$$U = \frac{T^2L}{2GJ}$$

Which is the expression given in equation for a shaft of length L and uniform cross section.

Since the maximum shearing stress occurs in the portion CD of the shaft and is proportional to the torque T, we note as we did earlier in the case of the axial loading of a rod that, for a given allowable stress, increasing the diameter of portion BC of the shaft results in a decrease of the overall energy-absorbing capacity of the shaft.

16. Find the deflection at the free end of a cantilever carrying a concentrated load at the free end. Assume uniform flexural rigidity.

Solution:-

Figure shows a cantilever carrying a point load P at the free end A. The bending moment at any section distant x from the free end is given by

$$M = - Px$$

Strain energy stored by the cantilever

$$W_i = \frac{p^2 l^3}{6EI}$$

17. Find the central deflection of a simply supported beam carrying a concentrated load at mid span. Assume uniform flexural rigidity.

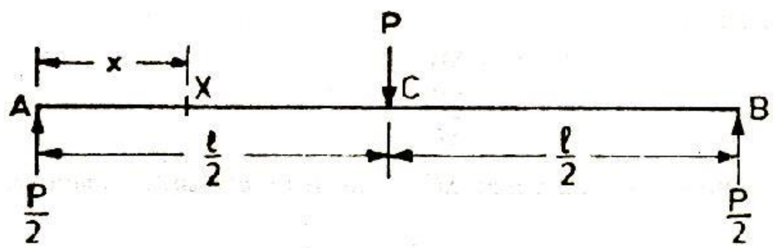
Solution:-

Figure shows a beam AB simply supported at A and B and carrying a central load P.

Each reaction

The bending moment at any section in AC, distant x from the end A is given by,

$$M \square \frac{P}{2} x$$



Figure

□ Strain energy stored by the beam

$$W_i = P^2L^3/96EI$$

□The deflection in the line of action P is given by

$$W_i = PL^3/48EI$$

18. Define: Strain Energy

When an elastic body is under the action of external forces the body deforms and work is done by these forces. If a strained, perfectly elastic body is allowed to recover slowly to its unstrained state. It is capable of giving back all the work done by these external forces. This work done in straining such a body may be regarded as energy stored in a body and is called strain energy or resilience.

19. Define: Proof Resilience.

The maximum energy stored in the body within the elastic limit is called Proof Resilience.

20. Write the formula to calculate the strain energy due to axial loads (tension).

$U=P^2L/2AE$

Where,

P = Applied tensile load.
L = Length of the member
A = Area of the members
E = Young's modulus.

21. Write the formula to calculate the strain energy due to bending.

$U=M^2L/2EI$

Where,

M = Bending moment due to applied loads.
E = Young's modulus
I = Moment of inertia

22. Write the formula to calculate the strain energy due to torsion

$U=T^2L/2GJ$

Where, T = Applied Torsion
G = Shear modulus or Modulus of rigidity
J = Polar moment of inertia

23. Write the down the formula to calculate the strain energy, if the moment value is given

$U=M^2L/2EI$

Where, M = Bending moment
L = Length of the beam
E = Young's modulus
I = Moment of inertia

24. Write the down the formula to calculate the strain energy, if the torsion moment value is given.

$U=T^2L/2GJ$

Where, T = Applied Torsion
L = Length of the beam
G = Shear modulus or Modulus of rigidity
J = Polar moment of inertia

25. Write down the formula to calculate the strain energy, if the applied tension load is given.

$U = P^2L/2AE$

Where, P = Applied tensile load.
L = Length of the member.
A = Area of the member
E = Young's modulus.

26. Write the Castigliano's first theorem.

In any beam or truss subjected to any load system, the deflection at any point is given by the partial differential coefficient of the total strain energy stored with respect to force acting at a point.

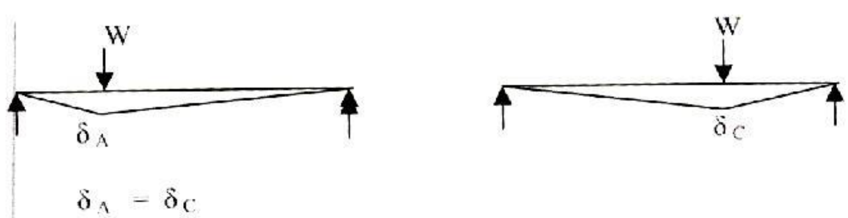
δ = Deflection
U = Strain Energy
stored P = Load

27. What are the uses of Castigliano's first theorem?

a. To determine the deflection of complicated structure.
b. To determine the deflection of curved beams, springs.

28. **Define: Maxwell Reciprocal Theorem.**

In any beam of truss the deflection at any point 'A' due to a load 'W' at any other point 'C' is the same as the deflection at 'C' due to the same load 'W' applied at 'A'.



29. Define: Unit load method.

The external load is removed and the unit load is applied at the point, where the deflection or rotation is to found.

30. Give the procedure for unit load method.

a. Find the forces P1, P2, ………. in all the members due to external loads.
b. Remove the external loads and apply the unit vertical point load at the joint if the vertical deflection is required and find the stress.
c. Apply the equation for vertical and horizontal deflection.

31. Compare the unit load method and Castigliano's first theorem.

In the unit load method, one has to analyze the frame twice to find the load and deflection. While in the latter method, only one analysis is needed.

32. Find the strain energy per unit volume, the shear stress for a material is given as 50 N/mm^2. Take G = 80000 N/mm^2.

$= 50^2 / (2 \times 80000),$
$= 0.015625$ N / mm^2. per unit volume.

33. Find the strain energy per unit volume, the tensile stress for a material is given as 150 N/mm^2. Take E = 2 x 10 N/mm^2.

—

$= (150)^2 / (2 \times (2x\ 10^2)$
$= 0.05625$ N/mm^2 per unit volume.

34. Define: Modulus of resilience.

The proof resilience of a body per unit volume. (ie) The maximum energy stored in the body within the elastic limit per unit volume.

35. Define Trussed Beam?

A beam strengthened by providing ties and struts is known as Trusted Beams.

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UNIT – III- AIRCRAFT STRUCTURES – SAE1303

Unit-III
Unsymmetric Bending of Beams

The learning objectives of this chapter are:

- Understand the theory, its limitations, and its application in design and analysis of unsymmetric bending of beam.

A member may be subjected to a bending moment, which acts on a plane inclined to the principal axis (say). This type of bending does not occur in a plane of symmetry of the cross section, it is called unsymmetrical bending. Since the problem related to flexure in general differs from symmetrical bending, it may be termed as skew bending.

One of the basic assumptions in deriving the flexural formula **f=MY/I**

is that the plane of the load is perpendicular to the neutral axis. Every cross-section has got two mutually perpendicular principal axes of inertia, about one of which the moment of inertia is the maximum and about the other a minimum. It can be shown that a symmetric axis of cross-section is one of the principal axis and one at right angles to the same will be the other principal axis.

For beams having unsymmetrical cross-section such as angle (L) or channel ([) sections, if the plane of loading is not coincident with or parallel to one of the principal axis, the bending is not simple. In that case it is said to be unsymmetrical or non-uniplanar bending.

In the present experiment for a cantilever beam of an angle section, the plane of loading is always kept vertical and the angle iron cantilever beam itself is rotated through angles in steps of 450.

Considering the position of angle iron wherein the plane of loading makes an angle □ with V-V axis of the section, which is one of the principal axes of the section. The components of the vertical load P along V-V and U-U axis are Pcos□ and Psin□ respectively.

The deflection □U and □V along U-U and V-V axis respectively are given by

$$\square U \; \square \; \frac{P \sin\ . L^3}{3EI_{VV}} \qquad (1)$$

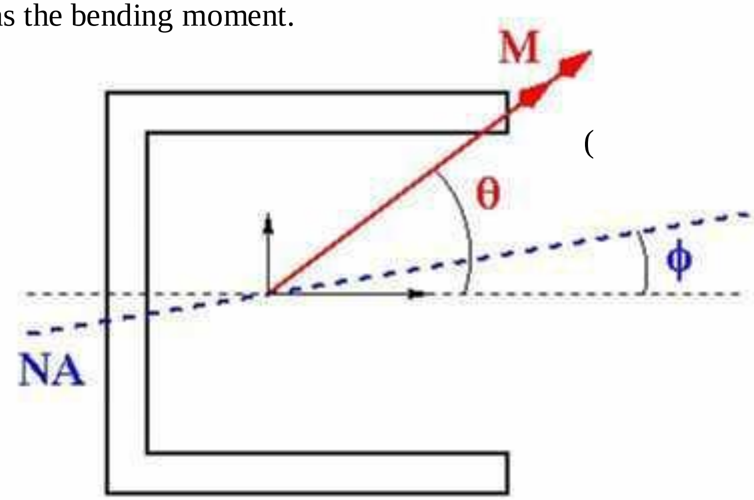
$$\square V \; \square \; \frac{P \cos\ . L^3}{3EI_{UU}} \qquad (2)$$

and the magnitude of resultant deflection $\square_{oo}$, is given by

Where Mx and My are the bending moments about the x and y centroidal axes, respectively. Ix and Iy are the second moments of area (also known as moments of inertia) about the x and y axes, respectively, and Ixy is the product of inertia. Using this equation it would be possible to calculate the bending stress at any point on the beam cross section regardless of moment orientation or cross-sectional shape. Note that Mx, My, Ix, Iy, and Ixy are all unique for a given section along the length of the beam. In other words, they will not change from one point to another on the cross section. However, the x and y variables shown in the equation correspond to the coordinates of a point on the cross section at which the stress.

Neutral Axis:

- When a homogeneous beam is subjected to elastic bending, the neutral axis (NA) will pass through the centroid of its cross section, but the orientation of the NA depends on the orientation of the moment vector and the cross sectional shape of the beam.
- When the loading is unsymmetrical (at an angle) as seen in the figure below, the NA will also be at some angle - NOT necessarily the same angle as the bending moment.



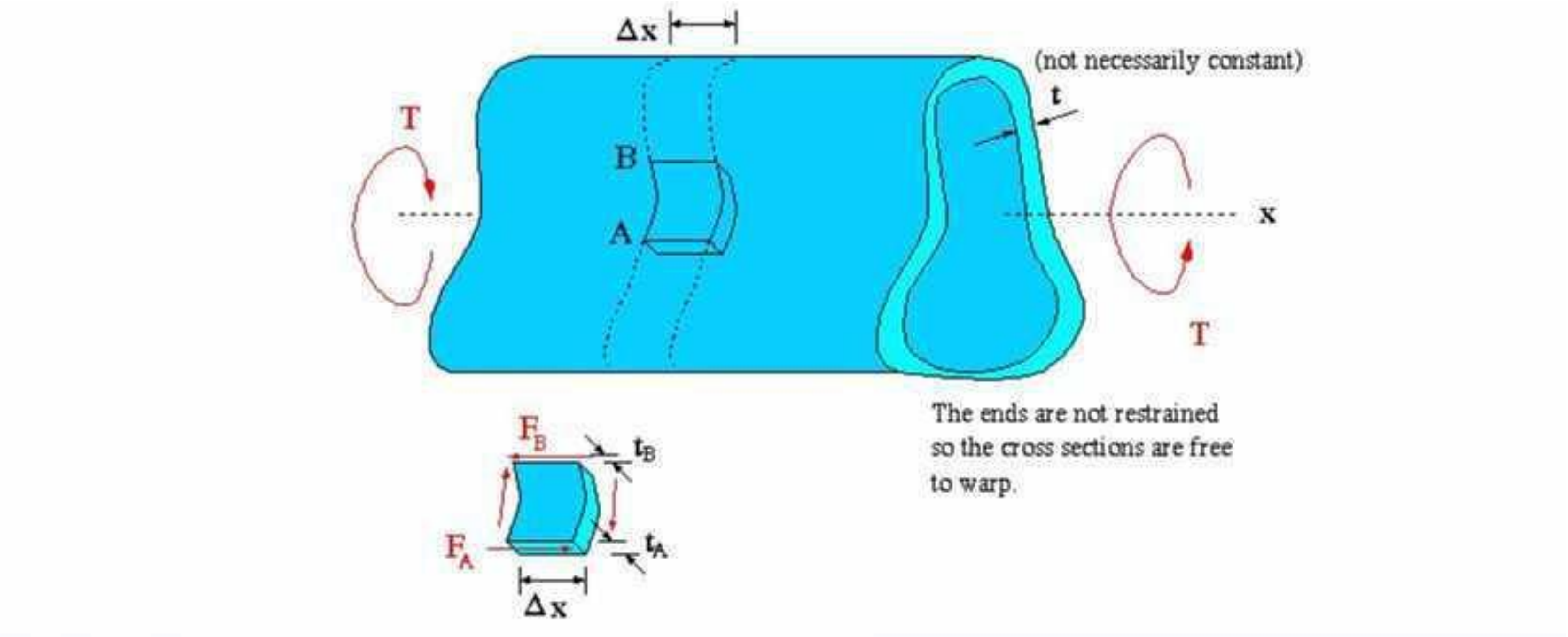
(

- Realizing that at any point on the neutral axis, the bending strain and stress are zero, we can use the general bending stress equation to find its orientation. Setting the stress to zero and solving for the slope y/x gives

Torsion of Thin - Wall Closed Sections

Derivation

Consider a thin-walled member with a closed cross section subjected to pure torsion.



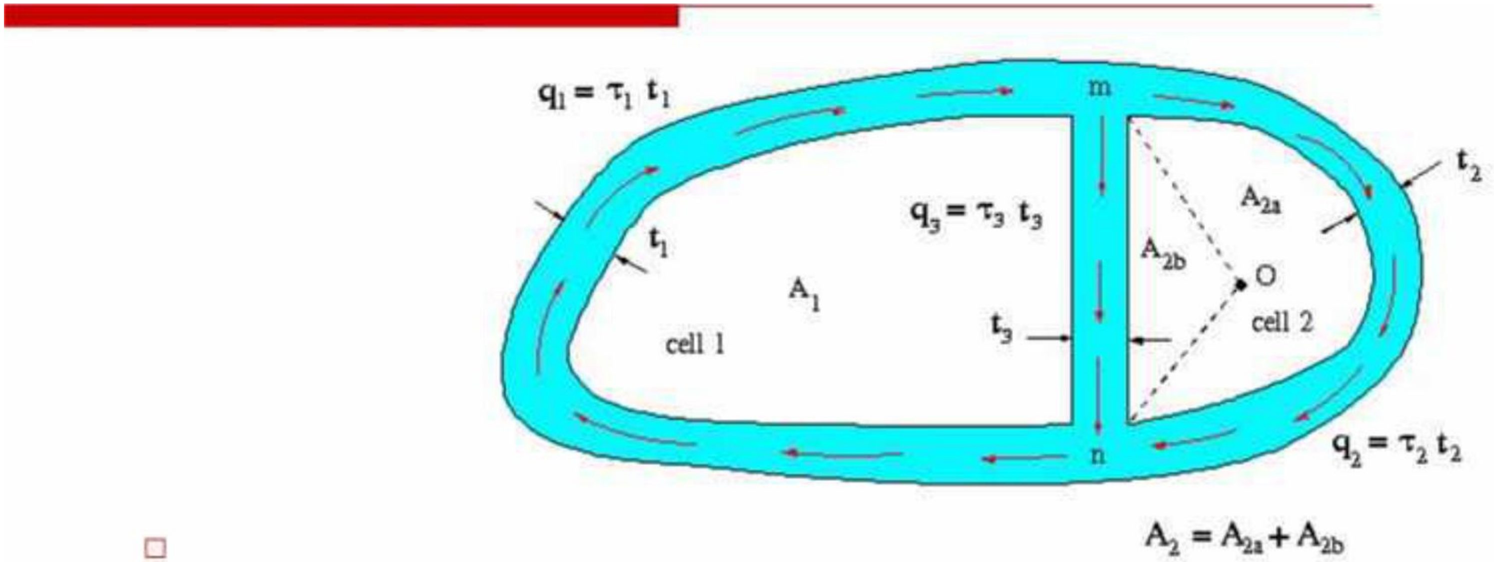
Angle of Twist

By applying strain energy equation due to shear and Castigliano's Theorem the angle of twist for a thin-walled closed section can be shown to be

Since T = 2qA, we have

If the wall thickness is constant along each segment of the cross section, the integral can be replaced by a simple summation

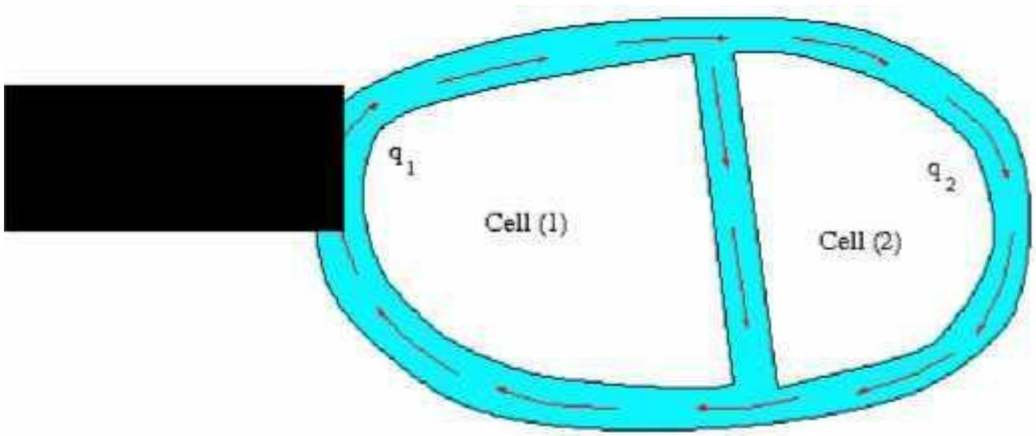
Torsion - Shear Flow Relations in Multiple-Cell Thin- Wall Closed Sections



The torsional moment in terms of the internal shear flow is simply

Shear Stress Distribution and Angle of Twist for Two-Cell Thin-Walled Closed Sections

- The equation relating the shear flow along the exterior wall of each cell to the resultant torque at the section is given as



This is a statically indeterminate problem. In order to find the shear flows q1 and q2, the compatibility relation between the angle of twist in cells 1 and 2 must be used. The compatibility

requirement can be stated as

$$\phi_1 = \phi_2 = \phi$$

$$\phi_1 = \frac{L}{2A_1G}\oint_{cell1}\frac{q}{t}ds$$

$$\phi_2 = \frac{L}{2A_2G}\oint_{cell2}\frac{q}{t}ds$$

where

$$q_1 = \frac{1}{2}\left[\frac{a_{20}A_1 + a_{12}A}{a_{20}A_1^2 + a_{12}A^2 + a_{10}A_2^2}\right]T \qquad q_2 = \frac{1}{2}\left[\frac{a_{10}A_2 + a_{12}A}{a_{20}A_1^2 + a_{12}A^2 + a_{10}A_2^2}\right]T$$

$A = A_1 + A_2$

$$a_{10} = \int \frac{ds}{t} \qquad \text{(along exterior wall of cell 1)}$$

$$a_{20} = \int \frac{ds}{t} \qquad \text{(along exterior wall of cell 2)}$$

$$a_{12} = \int \frac{ds}{t} \qquad \text{(along interior wall between cells 1 \& 2)}$$

- The shear stress at a point of interest is found according to the equation

$$\tau = \frac{q}{t}$$

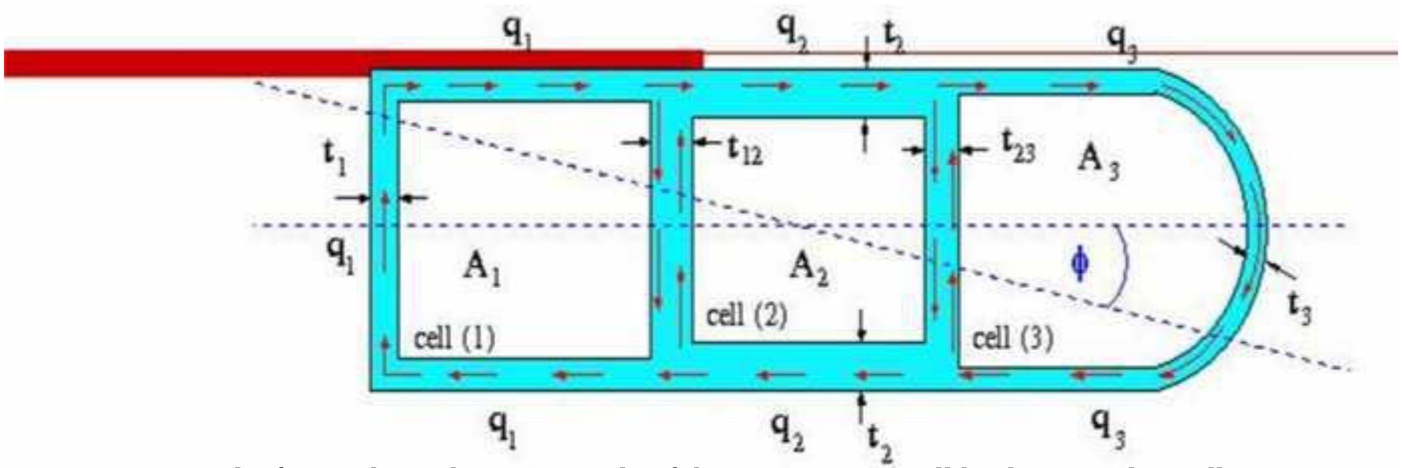
- To find the angle of twist, we could use either of the two twist formulas given above. It is also possible to express the angle of twist equation

  Similar to that for a circular section

$$\phi = \frac{TL}{JG}$$

$$J = 4\left[\frac{a_{20} A_1^2 + a_{12} A^2 + a_{10} A_2^2}{a_{10} a_{12} + a_{12} a_{20} + a_{10} a_{20}}\right]$$

Shear Stress Distribution and Angle of Twist for Multiple-Cell Thin-Wall Closed Sections

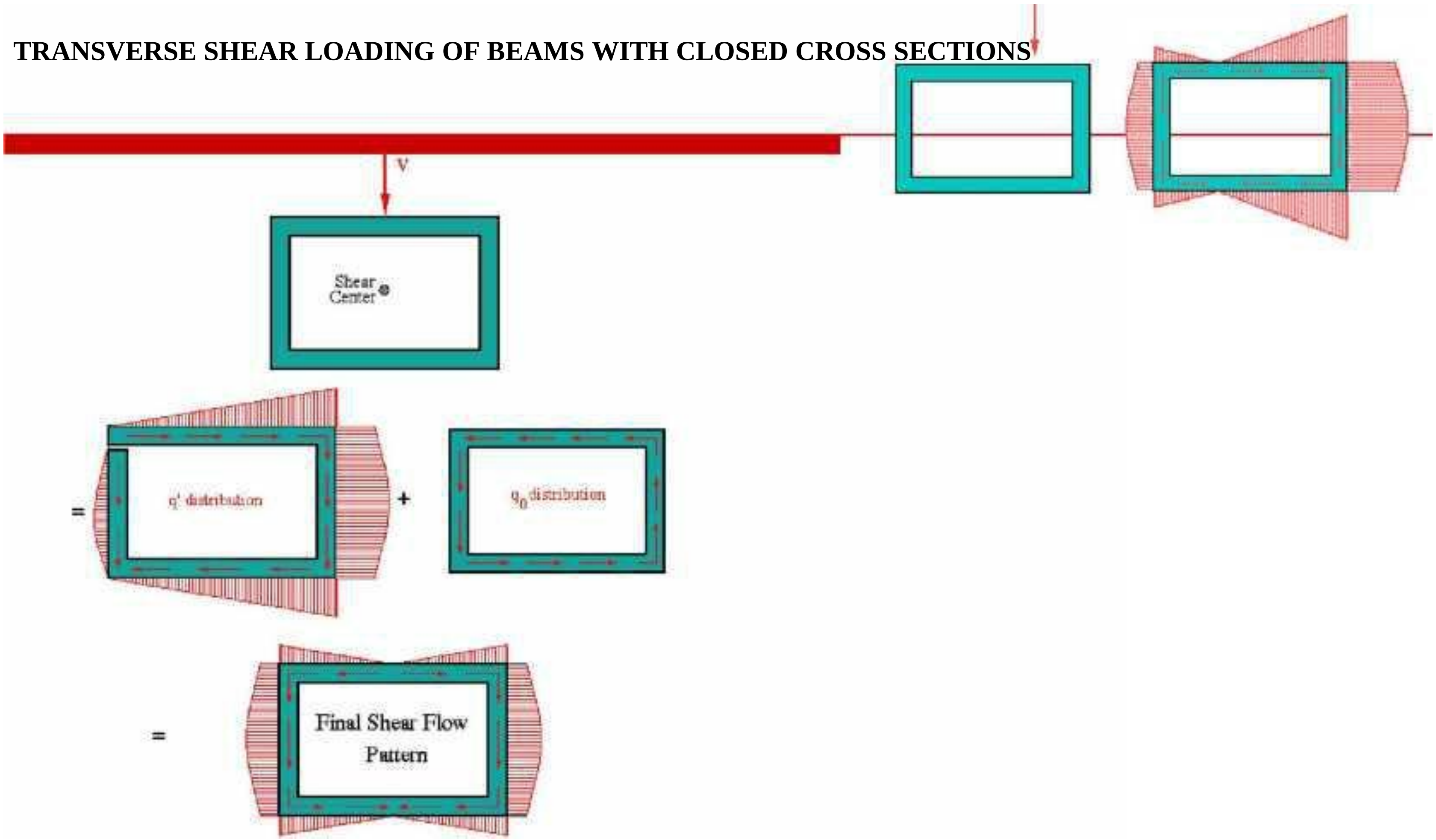


In the figure above the area outside of the cross section will be designated as cell (0). Thus to designate the exterior walls of cell (1), we use the notation 1-0. Similarly for cell (2) we use 2-0 and for cell (3) we use 3-0. The interior walls will be designated by the names of adjacent cells.

The torque of this multi-cell member can be related to the shear flows in exterior walls as follows

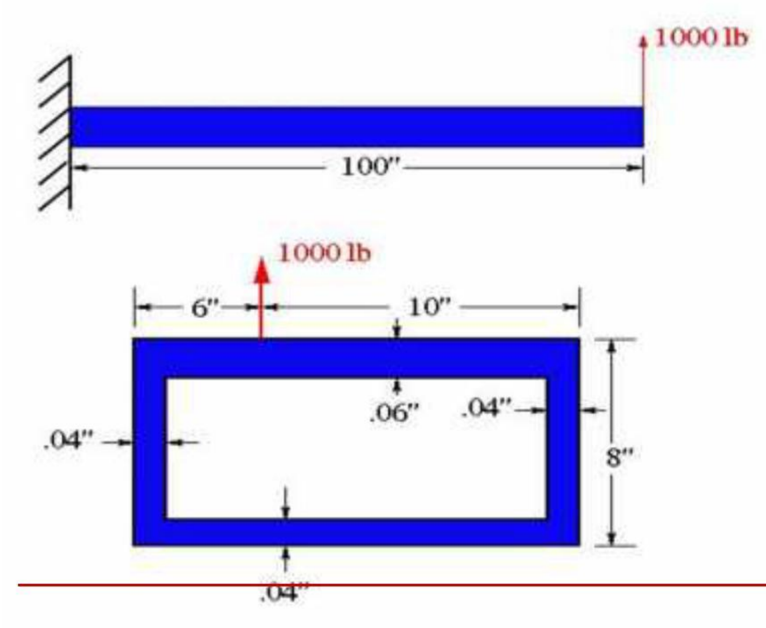
- For elastic continuity, the angles of twist in all cells must be equal

TRANSVERSE SHEAR LOADING OF BEAMS WITH CLOSED CROSS SECTIONS



EXAMPLE

- For the thin-walled single-cell rectangular beam and loading shown, determine
  (a) the shear center location (ex and ey),
  (b) the resisting shear flow distribution at the root section due to the applied load of 1000 lb,
  (c) the location and magnitude of the maximum shear stress



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UNIT – IV- AIRCRAFT STRUCTURES – SAE1303

UNIT-IV
SHEAR FLOW AND SHEAR CENTER

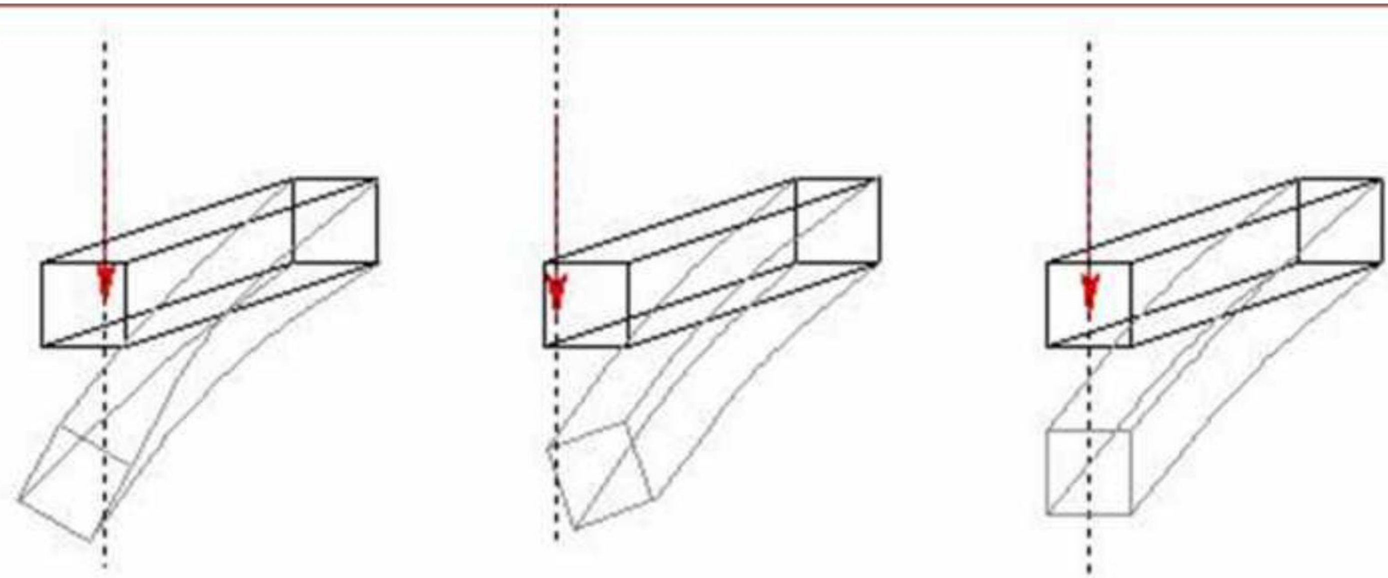
Restrictions:

1. Shear stress at every point in the beam must be less than the elastic limit of the material in shear.
2. Normal stress at every point in the beam must be less than the elastic limit of the material in tension and in compression.
3. Beam's cross section must contain at least one axis of symmetry.
4. The applied transverse (or lateral) force(s) at every point on the beam must pass through the elastic axis of the beam. Recall that elastic axis is a line connecting cross-sectional shear centers of the beam. Since shear center always falls on the cross-sectional axis of symmetry, to assure the previous statement is satisfied, at every point the transverse force is applied along the cross-sectional axis of symmetry.
5. The length of the beam must be much longer than its cross sectional dimensions.
6. The beam's cross section must be uniform along its length.

Shear Center

If the line of action of the force passes through the Shear Center of the beam section, then the beam will only bend without any twist. Otherwise, twist will accompany bending.

The shear center is in fact the *centroid of the internal shear force system.* Depending on the beam's cross-sectional shape along its length, the location of shear center may vary from section to section. A line connecting all the shear centers is called the elastic axis of the beam. When a beam is under the action of a more general lateral load system, then to prevent the beam from twisting, the load must be centered along the elastic axis of the beam.



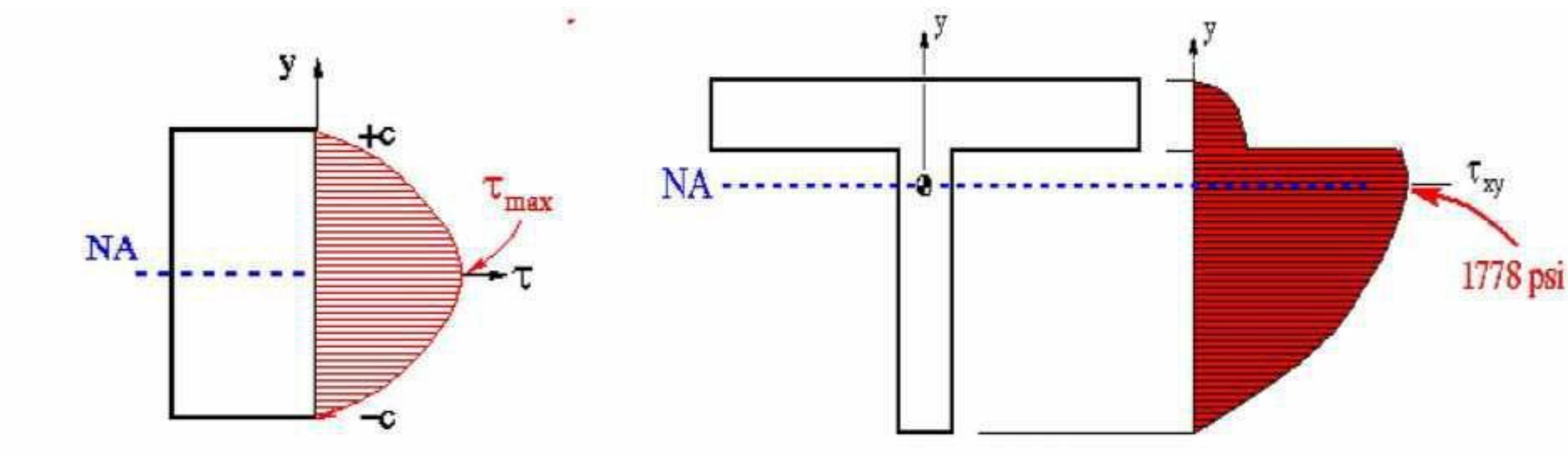
- The two following points facilitate the determination of the shear center location.

1. The shear center always falls on a cross-sectional axis of symmetry.

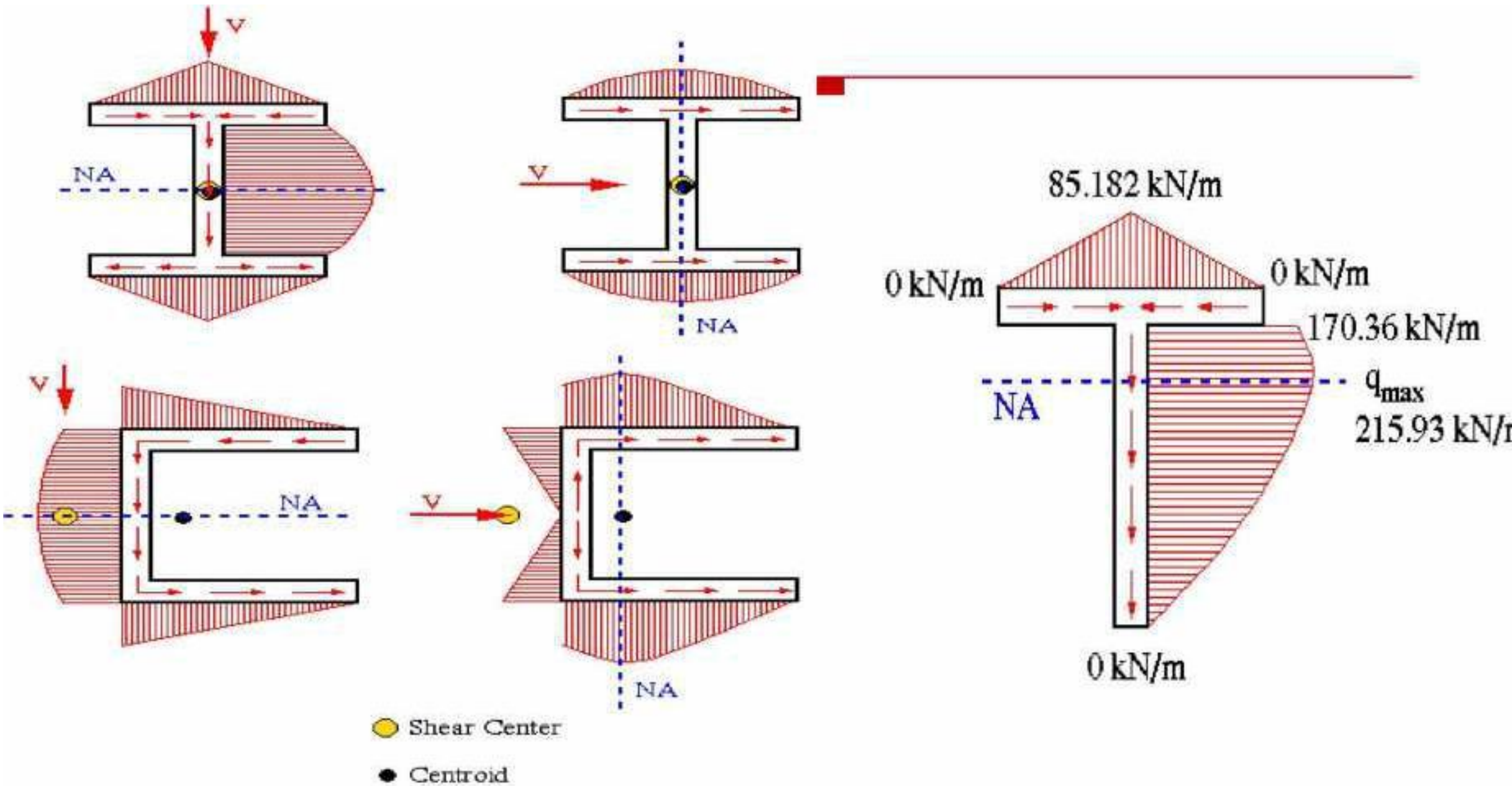
2. If the cross section contains two axes of symmetry, then the shear center is located at their intersection. Notice that this is the only case where shear center and centroid coincide.

SHEAR STRESS DISTRIBUTION

RECTANGLE T-SECTION



SHEAR FLOW DISTRIBUTION



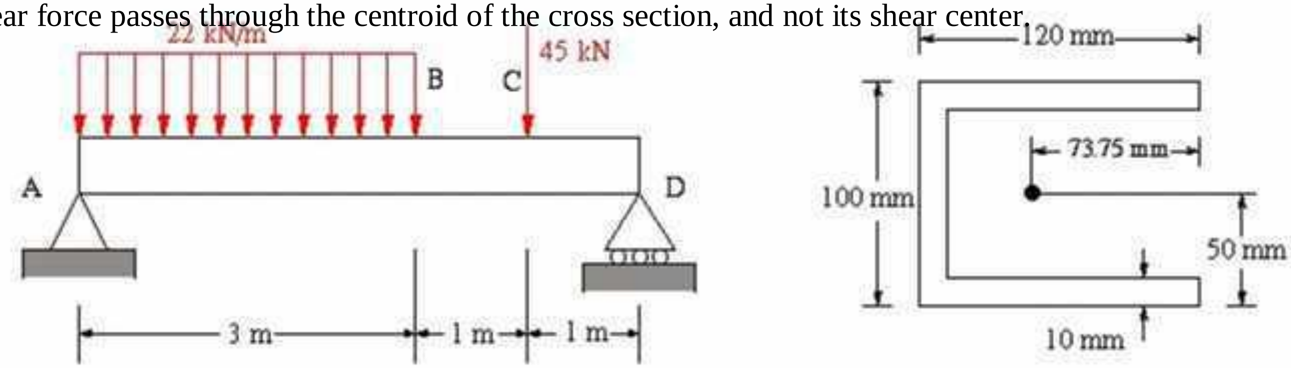
EXAMPLES

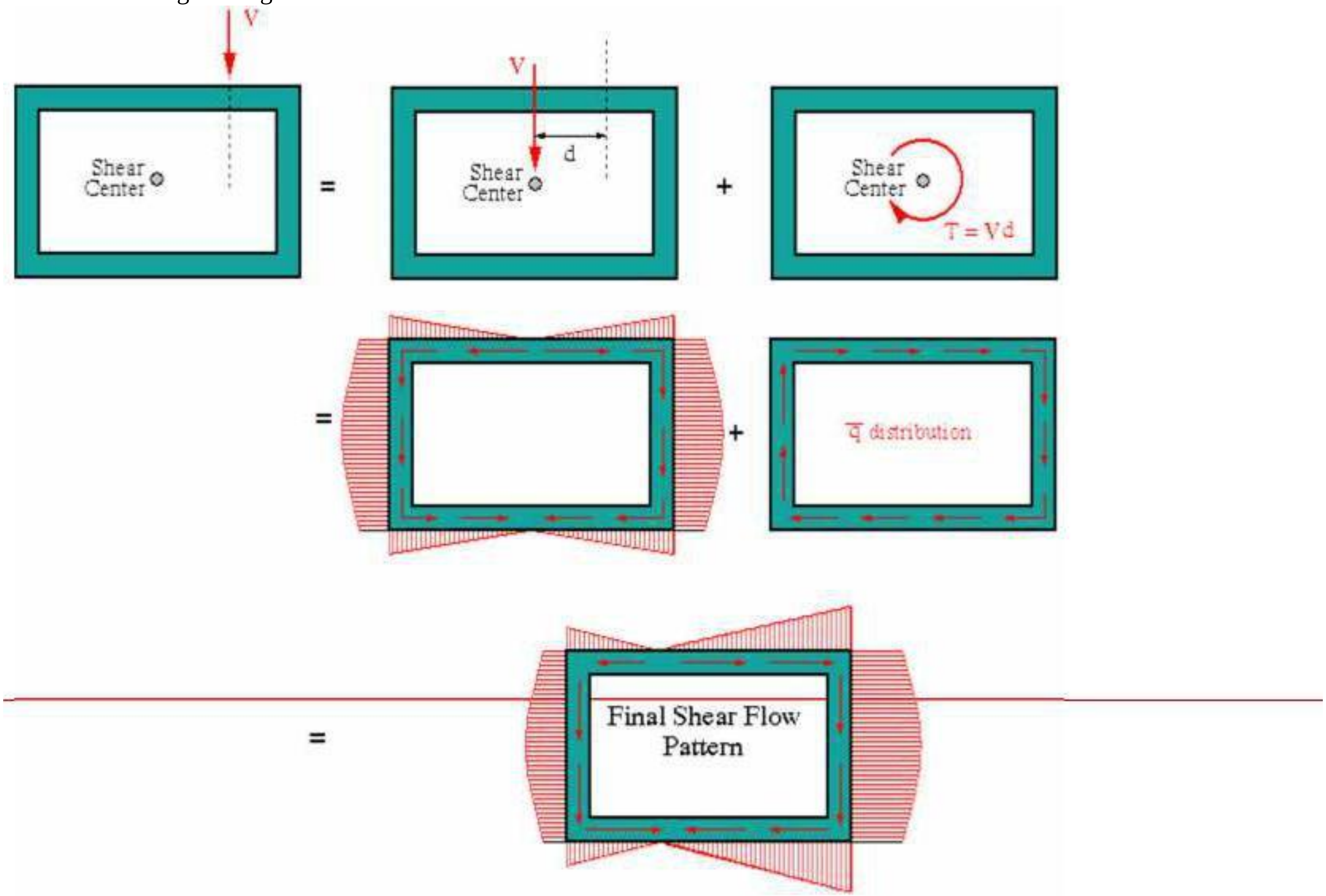
For the beam and loading shown, determine:

(a)the location and magnitude of the maximum transverse shear force 'Vmax', (b)the shear flow 'q' distribution due the 'Vmax',

(c)the 'x' coordinate of the shear center measured from the centroid, (d)the maximum shear stress and its location on the cross section.

Stresses induced by the load do not exceed the elastic limits of the material. NOTE:In this problem the applied transverse shear force passes through the centroid of the cross section, and not its shear center.

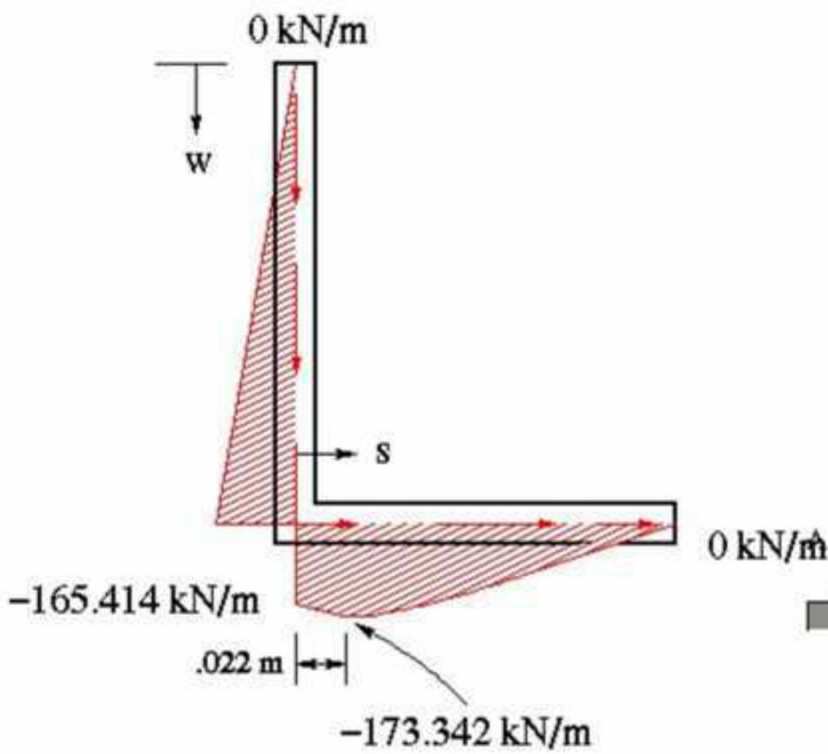


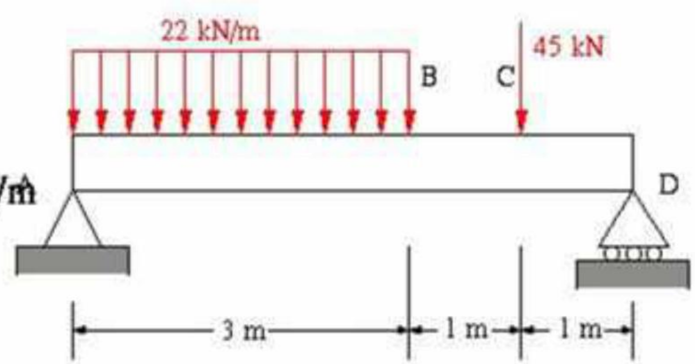


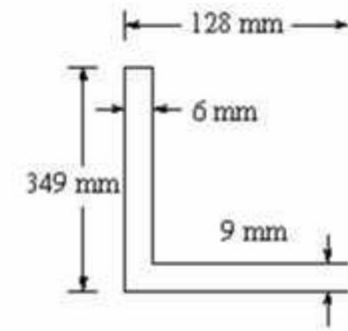
V
Shear Center
=
V
d
Shear Center
+
Shear Center
T = Vd
=
+
q distribution
=
Final Shear Flow Pattern

SHEAR FLOW DISTRIBUTION

- For the beam and loading shown, determine:
- (a) the location and magnitude of the maximum transverse shear force,
- (b) the shear flow 'q' distribution due to 'Vmax',
- (c) the 'x' coordinate of the shear center measured from the centroid of the cross section.
- Stresses induced by the load do not exceed the elastic limits of the material. The transverse shear force is applied through the shear center at every section of the beam. Also, the length of each member is measured to the middle of the adjacent member.





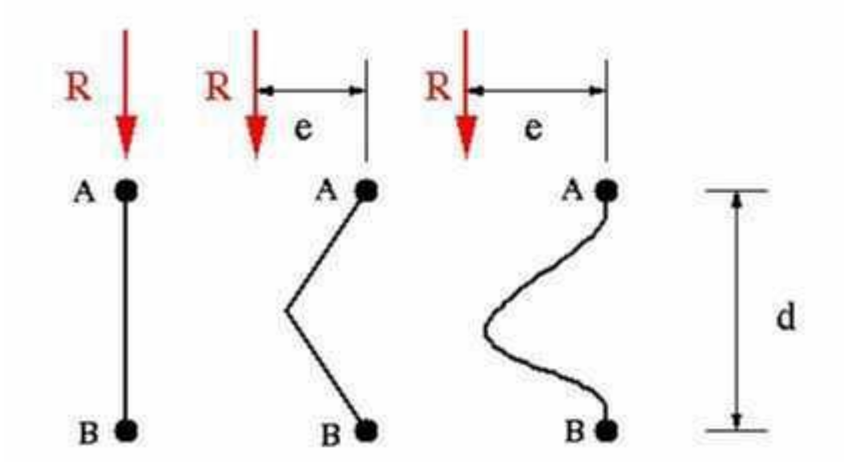


Assumptions:

1. Calculations of centroid, symmetry, moments of area and moments of inertia are based totally on the areas and distribution of beam stiffeners.
2. A web does not change the shear flow between two adjacent stiffeners and as such would be in the state of constant shear flow.
3. The stiffeners carry the entire bending-induced normal stresses, while the web(s) carry the entire shear flow and corresponding shear stresses.

Analysis

- Let's begin with a simplest thin-walled stiffened beam. This means a beam with two stiffeners and a web. Such a beam can only support a transverse force that is parallel to a straight line drawn through the centroids of two stiffeners.
  Examples of such a beam are shown below. In these three beams, the value of shear flow would be equal although the webs have different shapes.

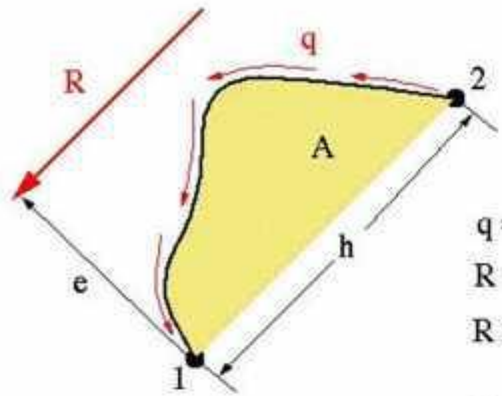


- The reason the shear flows are equal is that the distance between two adjacent stiffeners is shown to be 'd' in all cases, and the applied force is shown to be equal to 'R' in all cases. The shear flow along the web can be determined by the following relationship

Important Features of

Two-Stiffener, Single-Web Beams:

1. Shear flow between two adjacent stiffeners is constant.
2. The magnitude of the resultant shear force is only a function of the straight line between the two adjacent stiffeners, and is absolutely independent of the web shape.
3. The direction of the resultant shear force is parallel to the straight line connecting the adjacent stiffeners.
4. The location of the resultant shear force is a function of the enclosed area (between the web, the stringers at each end and the arbitrary point 'O'), and the straight distance between the adjacent stiffeners. This is the only quantity that depends on the shape of the web connecting the stiffeners.
5. The line of action of the resultant force passes through the shear center of the section.



$q = \text{constant}$

$R = qh$ (magnitude)

R is parallel to h (direction)

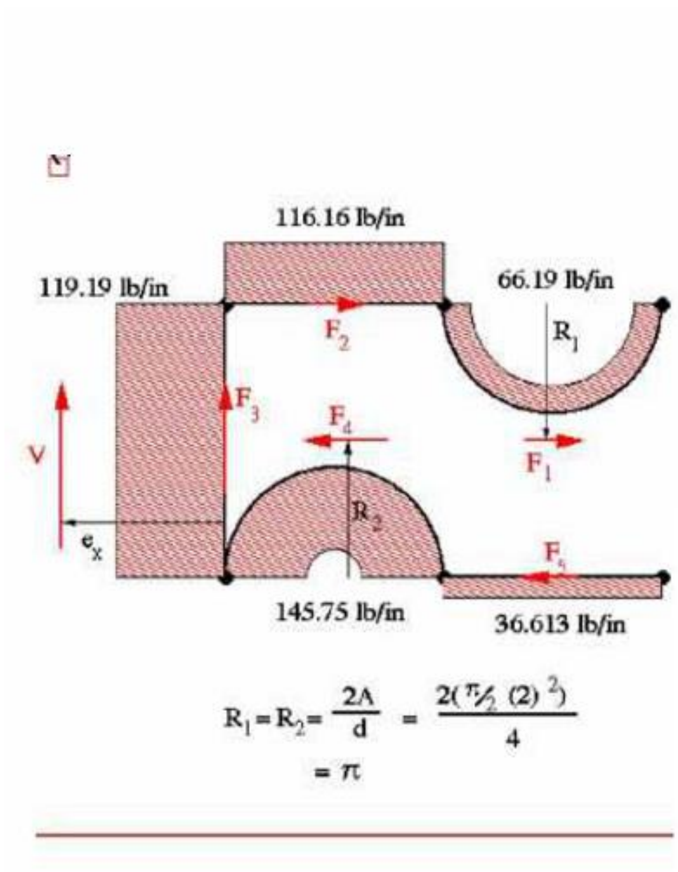
$e = \dfrac{2A}{h}$ (location)

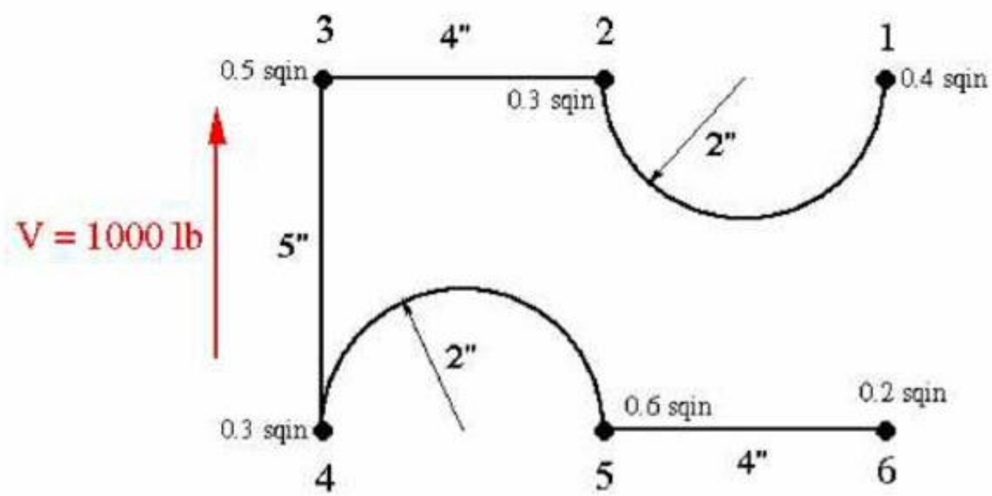
EXAMPLE

For the multi-web, multi-stringer open-section beam shown, determine

(a) the shear flow distribution,

(b) the location of the shear center Answer





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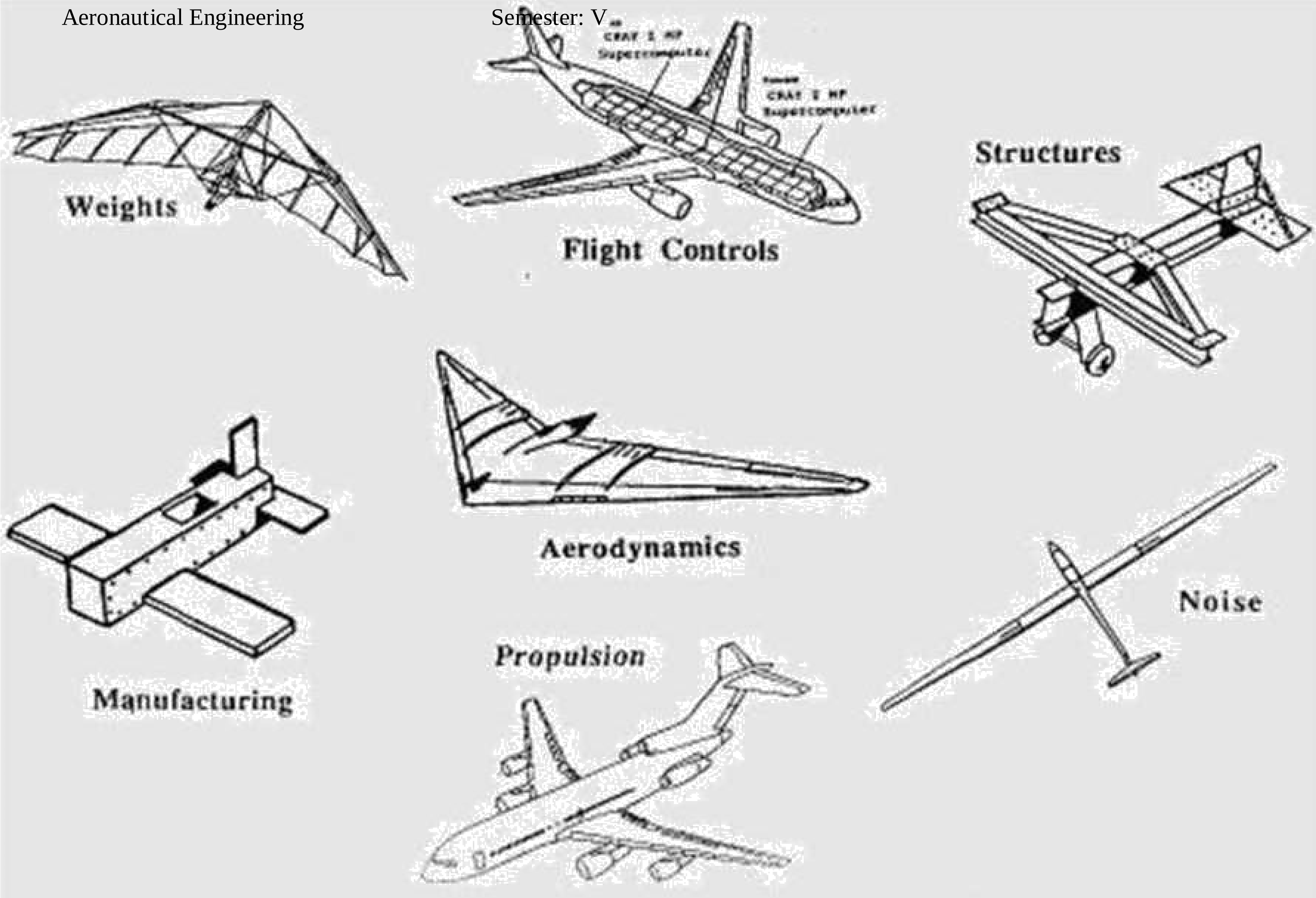
UNIT – V- AIRCRAFT STRUCTURES – SAE1303

Course Objective

- The purpose of the course is to teach the principles of solid and structural mechanics that can be used to design and analyze aerospace structures, in particular aircraft structures.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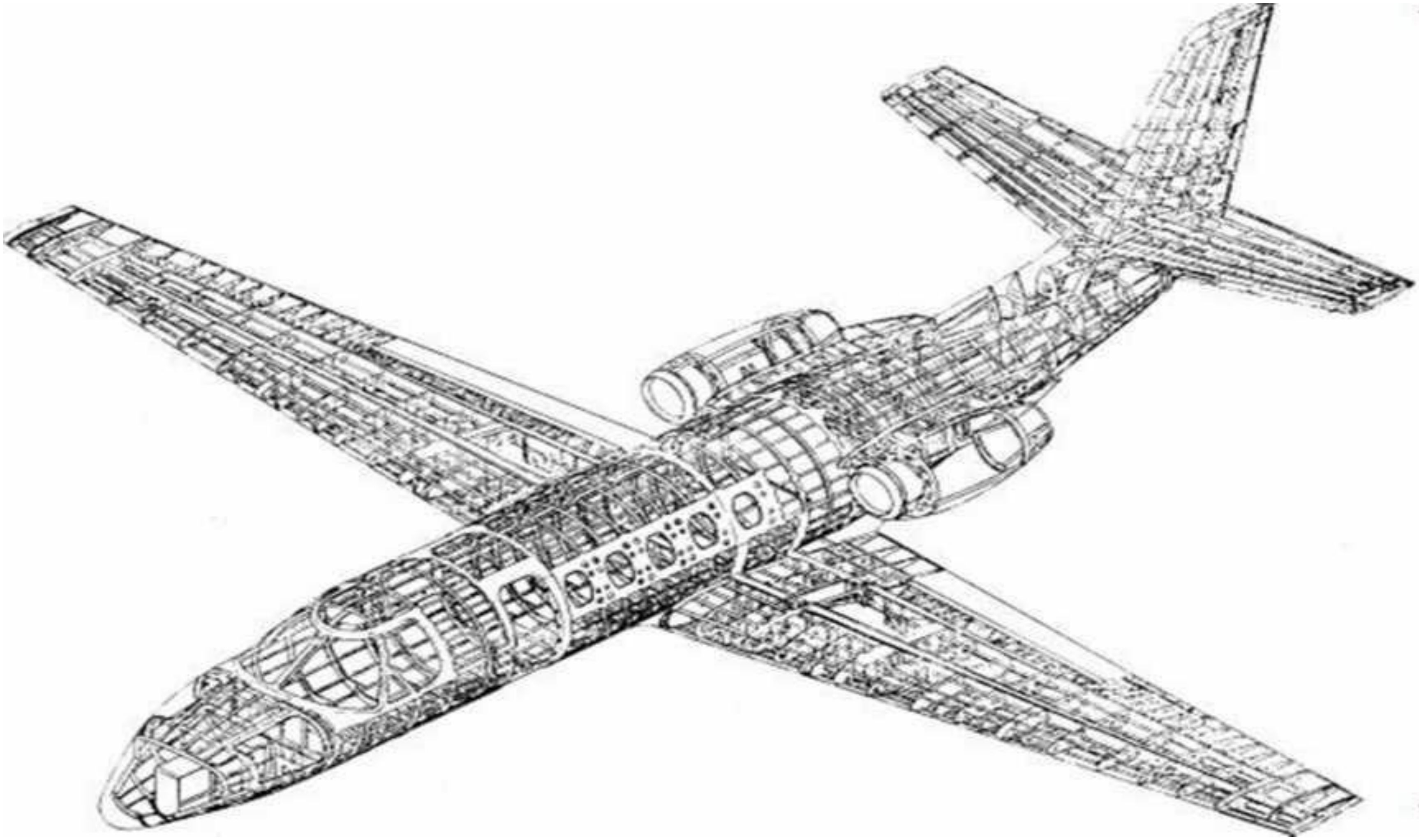
Weights
Flight Controls
Structures
Aerodynamics
Noise
Manufacturing
Propulsion

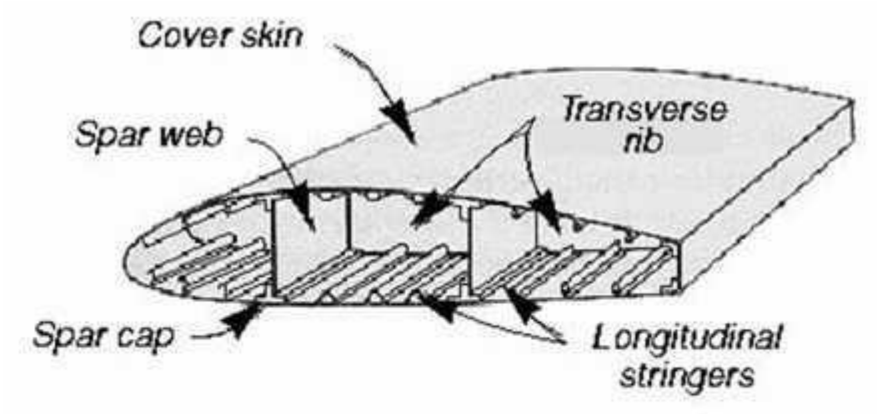
Airframe



Function of Aircraft Structures

General

The structures of most flight vehicles are thin walled structures (shells)



Resists applied loads (Aerodynamic loads acting on the wing structure)

Provides the aerodynamic shape

Protects the contents from the environment

Definitions

Primary structure:

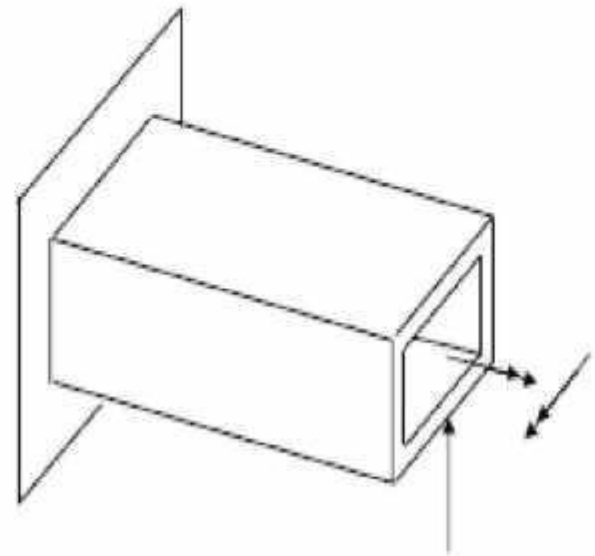
A critical load -bearing structure on an aircraft. If this structure is severely damaged, the aircraft cannot fly.

Secondary structure:

Structural elements mainly to provide enhanced aerodynamics. Fairings, for instance, are found where the wing meets the body or at various locations on the leading or trailing edge of the wing.

Monocoque **structures:**

Unstiffened shells. Must be relatively thick to resist bending, compressive, and torsional loads.

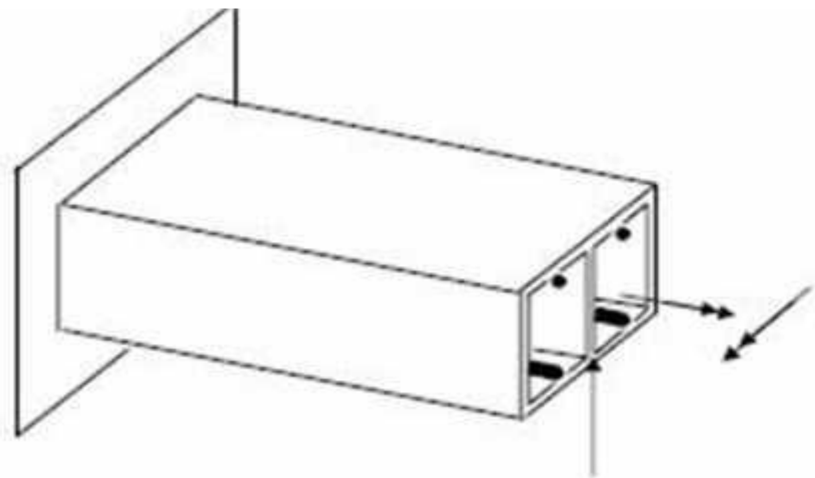


Definitions…

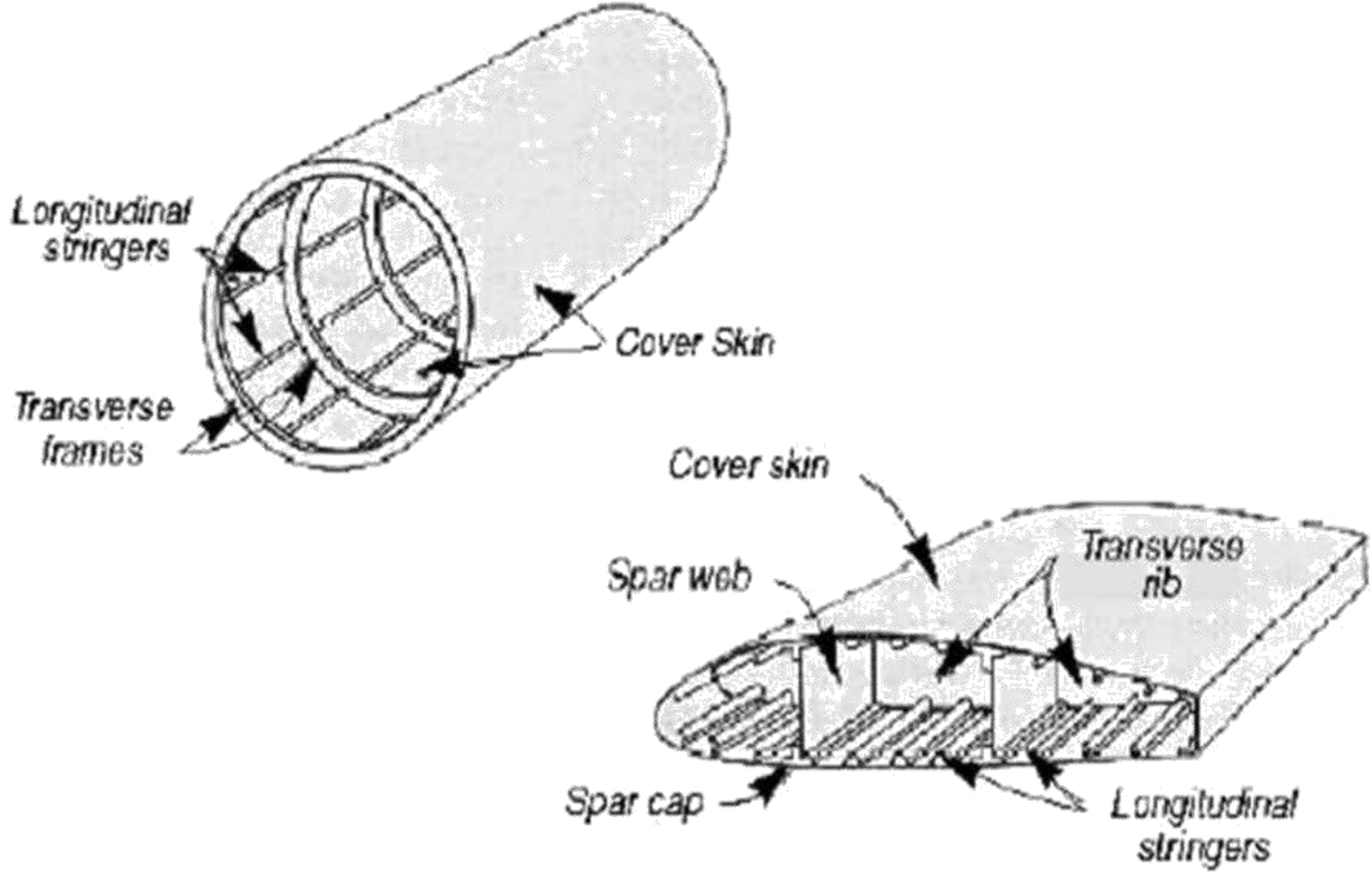
Semi-monocoque Structures:

Constructions with stiffening members that may also be required to diffuse concentrated loads into the cover.

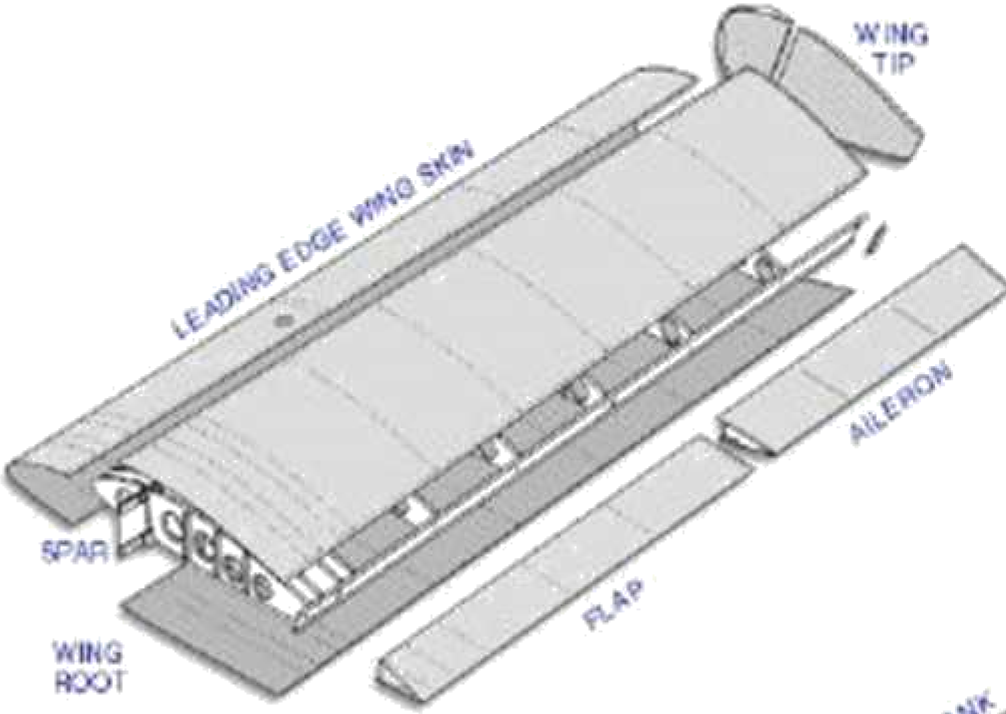
More efficient type of construction that permits much thinner covering shell.

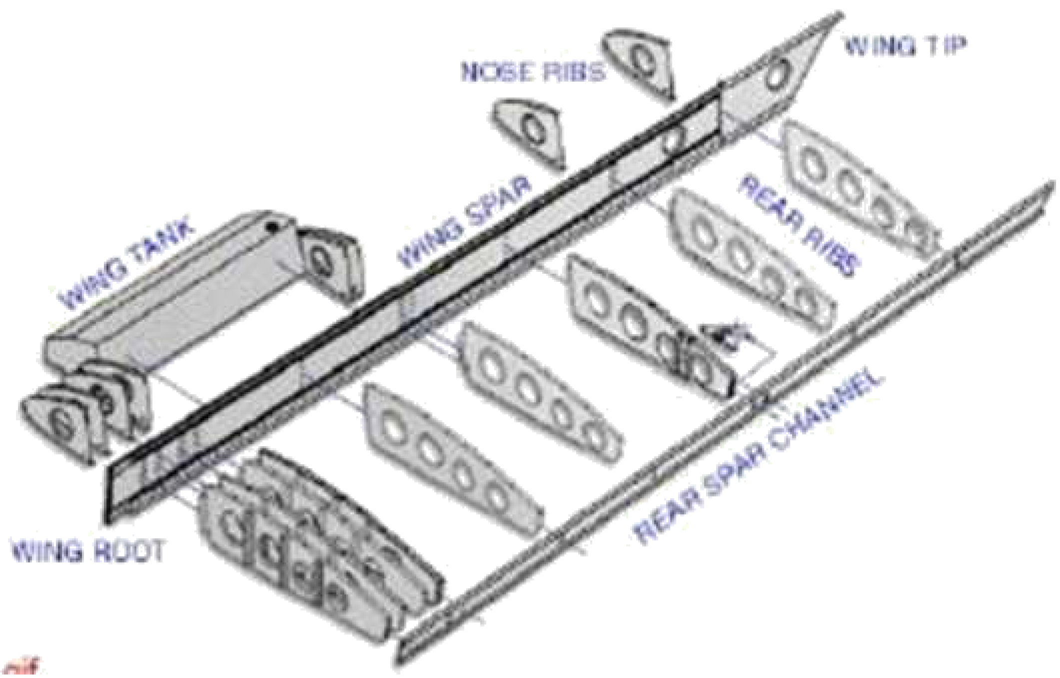


Semi-monocoque structures

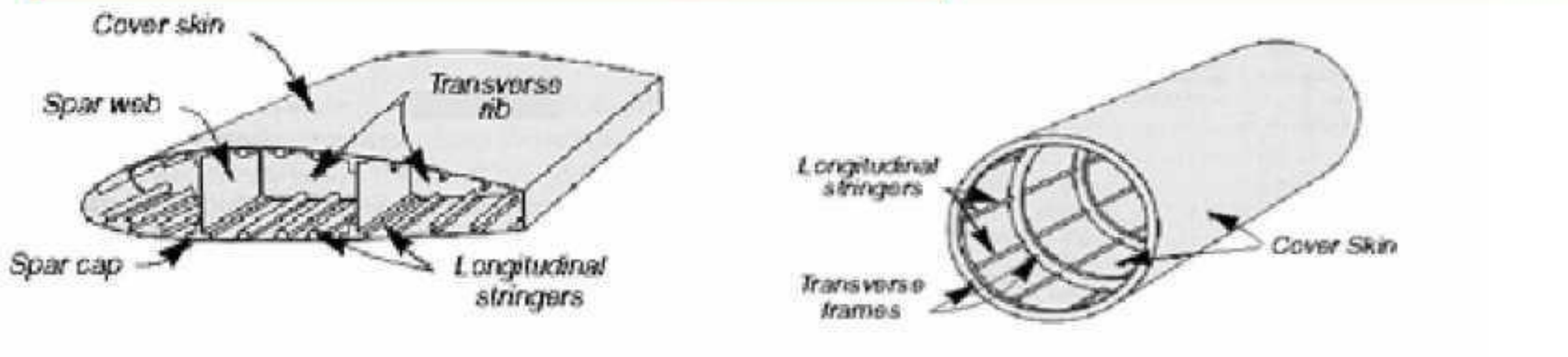


Semi-monocoque structures: Wing layout





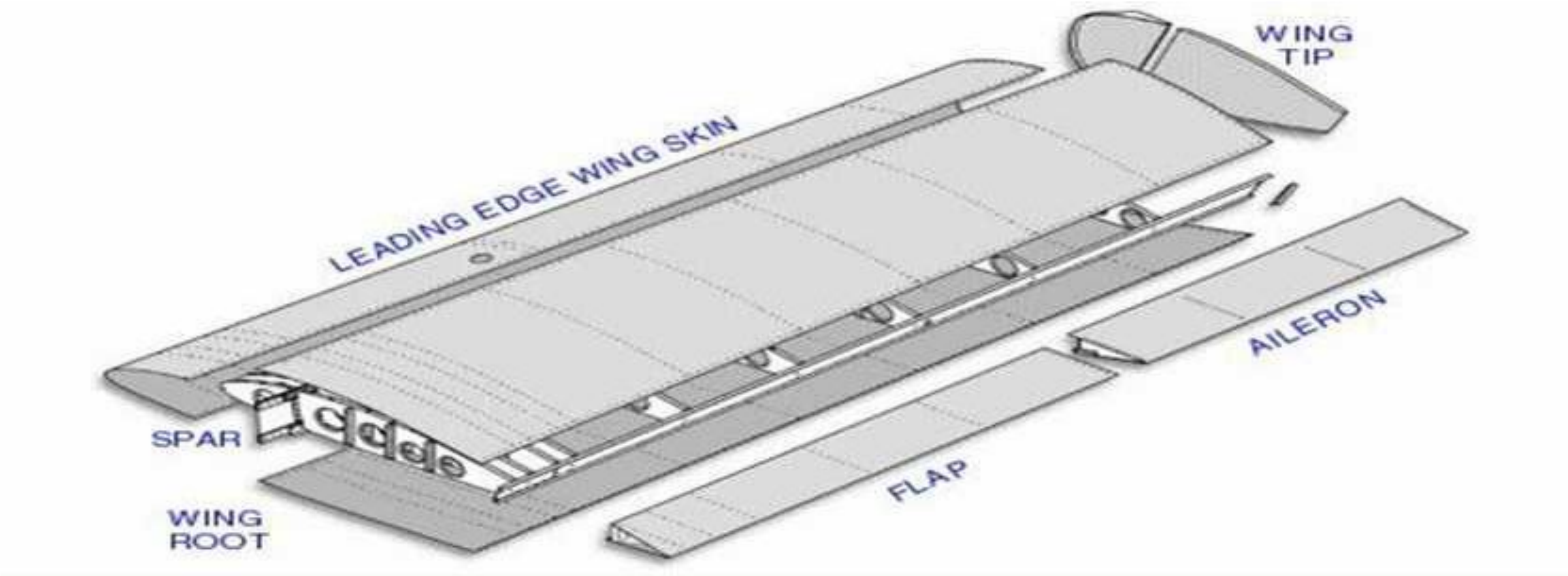
http://www.zenithair.com/zodiac/xl/gif/wing_exploded_tx.gif



Skin reacts the applied torsion and shear forces transmits aerodynamic forces to the longitudinal and transverse supporting membersacts with the longitudinal members in resisting the applied bending and axial loads acts with the transverse members in reacting the hoop, or circumferential, load when the structure is Pressurized.

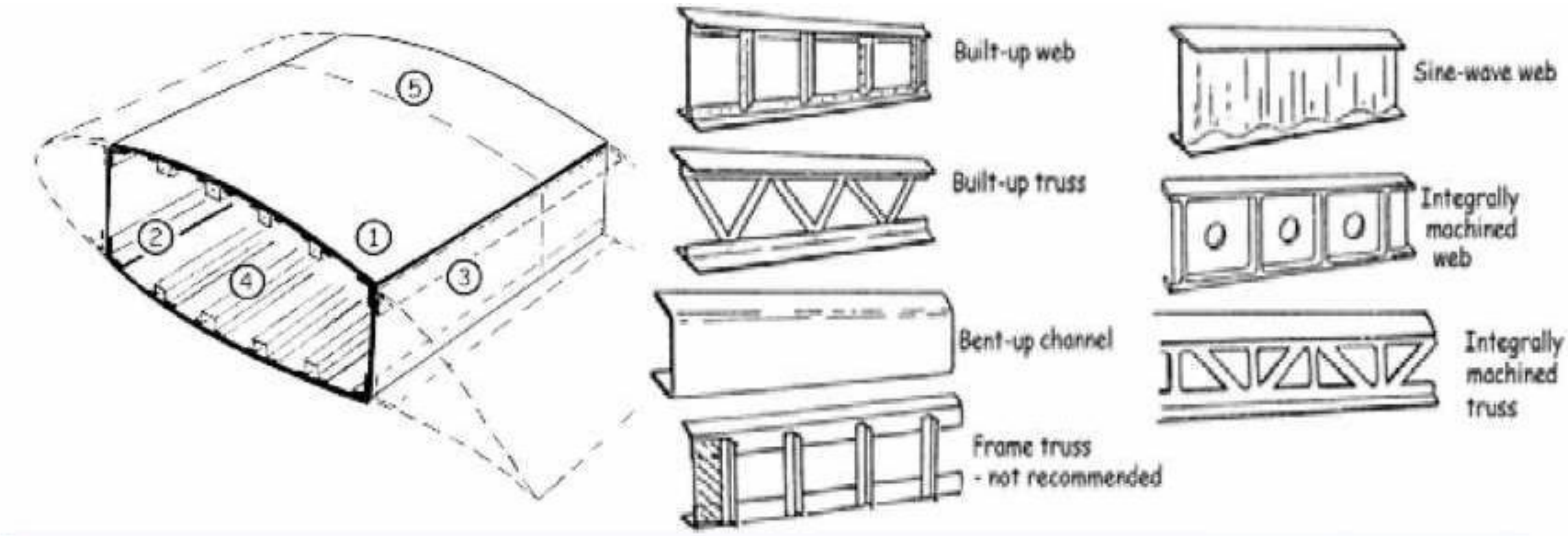
Ribs and Frames

1. Structural integration of the wing and fuselage
2. Keep the wing in its aerodynamic profile



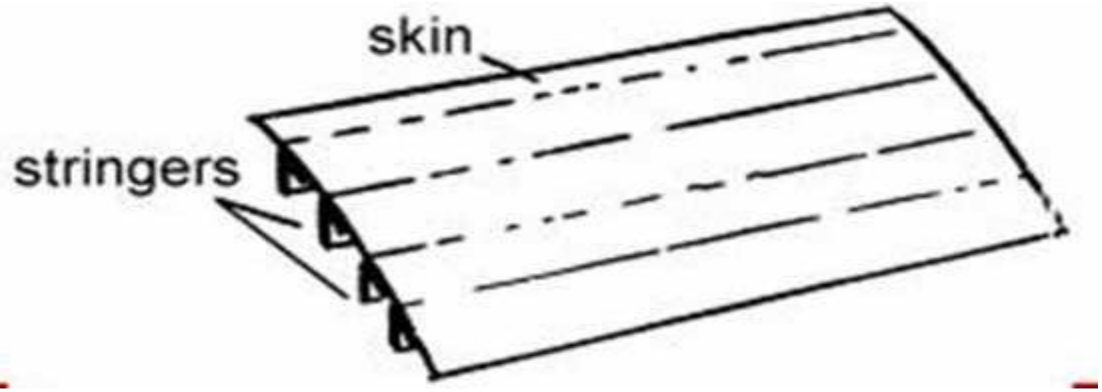
Spar

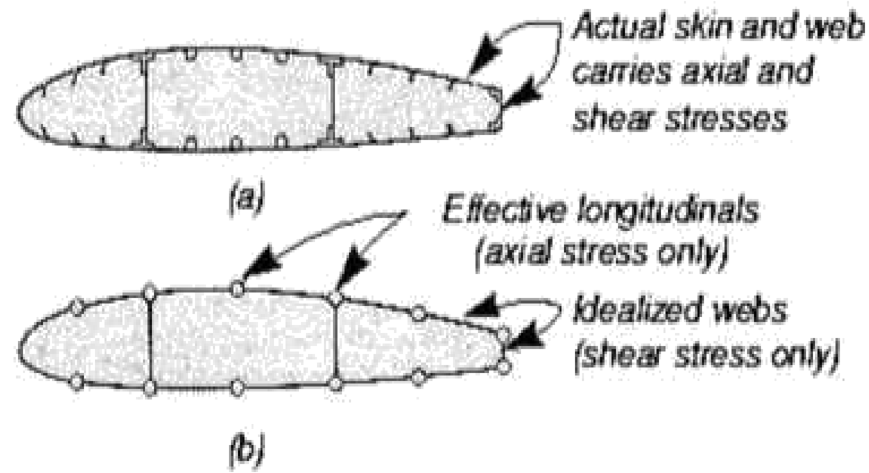
1. Resist bending and axial loads
2. form the wing box for stable torsion resistance



Stiffener or Stringers

i. Resist bending and axial loads along with the skin
ii. divide the skin into small panels and thereby increase its buckling and failing stresses
iii. act with the skin in resisting axial loads caused by pressurization.





The behavior of these structural elements is often idealized to simplify the analysis of the assembled Component Several longitudinal may be lumped into a single effective Longitudinal to shorten computations. The webs (skin and spar webs) carry only shearing Stresses. The longitudinal elements carry only axial stress. The transverse frames and ribs are rigid within their own planes, so that the cross section is maintained unchanged during loading.