

Composite Materials for Aircraft Structures: A Brief Review of Practical Application

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ME 480 Introduction to Aerospace,
Spring 2010



Introduction

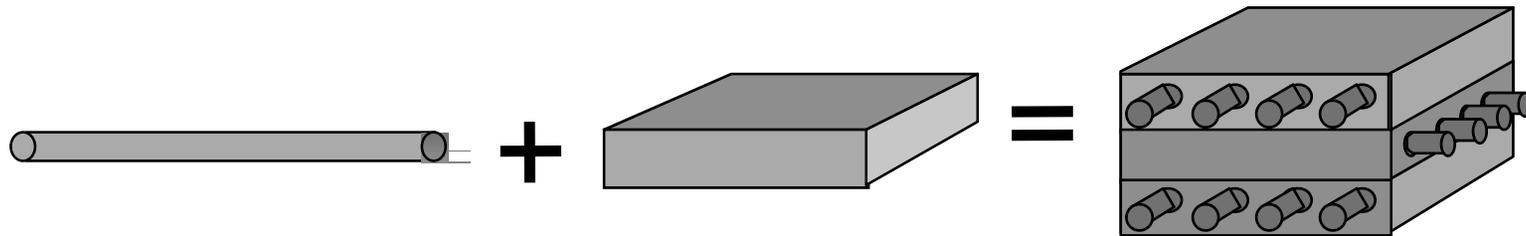
- **Composite materials are used more and more for primary structures in commercial, industrial, aerospace, marine, and recreational structures**



From Last Time

- **Composite parts used for aircraft applications are defined by**
 - Material, process, and manufacturing specifications.
 - Material allowable (engineering definition).
- **All of these have a basis in regulatory requirements.**
- **Most efficient use of advanced composites in aircraft structure is in applications with**
 - Highly loaded parts with thick gages.
 - High fatigue loads (fuselage and wing structure, etc).
 - Areas susceptible to corrosion (fuselage, etc).
 - Critical weight reduction (empennage, wings, fuselage, etc).
- **Use must be justified by weighing benefits against costs.**

Composition of Composites



Fiber/Filament Reinforcement

- High strength
- High stiffness
- Low density
- Carbon, Glass, Aramid, etc

Matrix

- Good shear properties
- Low density
- Thermoset & Thermoplastic
- Epoxy, Polyester, PP, Nylon, Ceramics, etc.

Composite

- High strength
- High stiffness
- Good shear properties
- Low density
- **Anisotropic!**

Overview

- **Micromechanics**
 - Study of mechanical behavior of a composite material in terms of its constituent materials
- **Ply Mechanics**
 - Study of mechanical behavior of individual material plies based on variations from global coordinate system
- **Macromechanics**
 - Study of mechanical behavior utilizing ply mechanics of a homogenized composite material
- **Failure Theories**

CADEC: Introduction



Back

Computer Aided Design Environment For Composites

Complement to "Introduction to Composite Materials Design", Taylor&Francis (1999) by Ever J. Barbero

Compliment to text: Barbero, EJ. **Introduction to Composite Materials Design**; Taylor & Francis, 1999.

Software free online—search keywords CADEC & Barbero

~~Chapter 6 - Macromechanics~~

Chapter 7 - Failure Theory

Chapter 8 - Thin Walled Beams

Field Types:

Hotword	These will take the user to a chapter or page when clicked.
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Micromechanics: Assumptions

- **Lamina:**

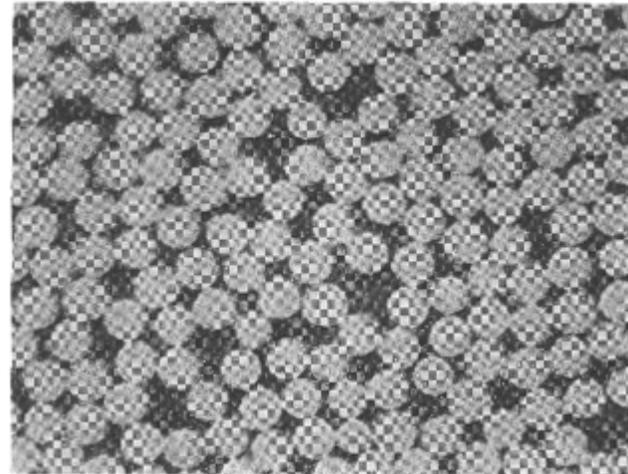
- Macroscopically homogeneous
- Linearly elastic
- Macroscopically Orthotropic
- Initially stress free

- **Fibers:**

- Homogeneous
- Linearly elastic
- Isotropic/Orthotropic
- Regularly spaced
- Perfectly aligned

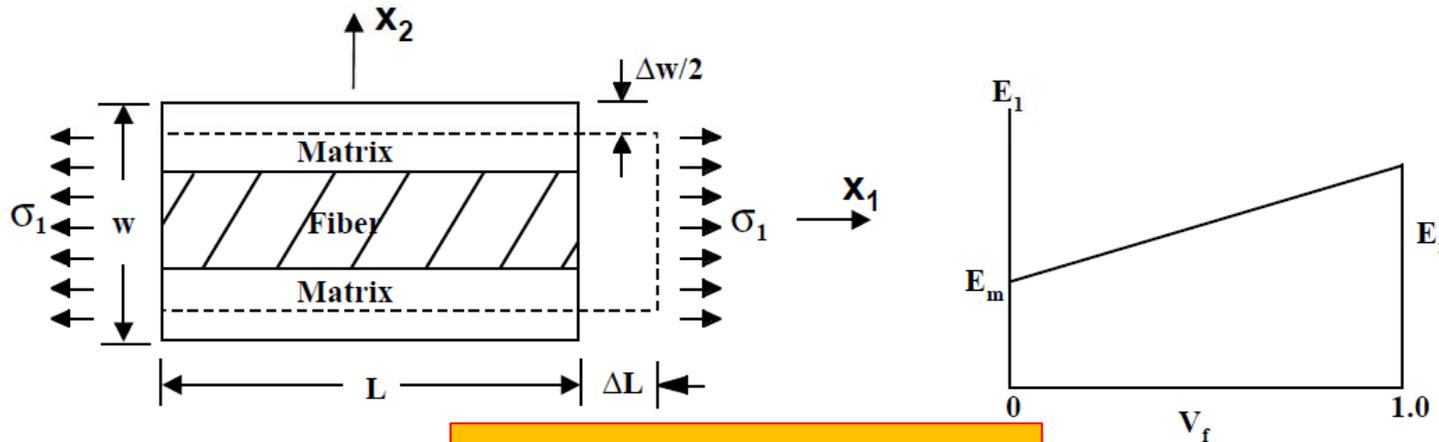
- **Matrix:**

- Homogeneous
- Linearly elastic
- Isotropic
- Assumptions in Micromechanics of Composites



Carbon/epoxy (AS4/3501-6) composite ($V_f=.70$)

Micromechanics: Rule of Mixtures



Assumption: Axial ($V_{f,max}$ **approximately 78%** **fiber and matrix**
Common range = 55-67%

\therefore **Strain in the composite** $\epsilon_1 = \frac{\Delta L}{L} = \epsilon_f = \epsilon_m$

Total force in composite $\sigma_1 A_c = \sigma_f A_f + \sigma_m A_m$

\therefore **Stress in the composite** $\sigma_1 = \sigma_f \frac{A_f}{A_c} + \sigma_m \frac{A_m}{A_c} = \sigma_f V_f + \sigma_m V_m$

$$\epsilon_1 E_1 = \epsilon_1 E_f V_f + \epsilon_1 E_m V_m$$

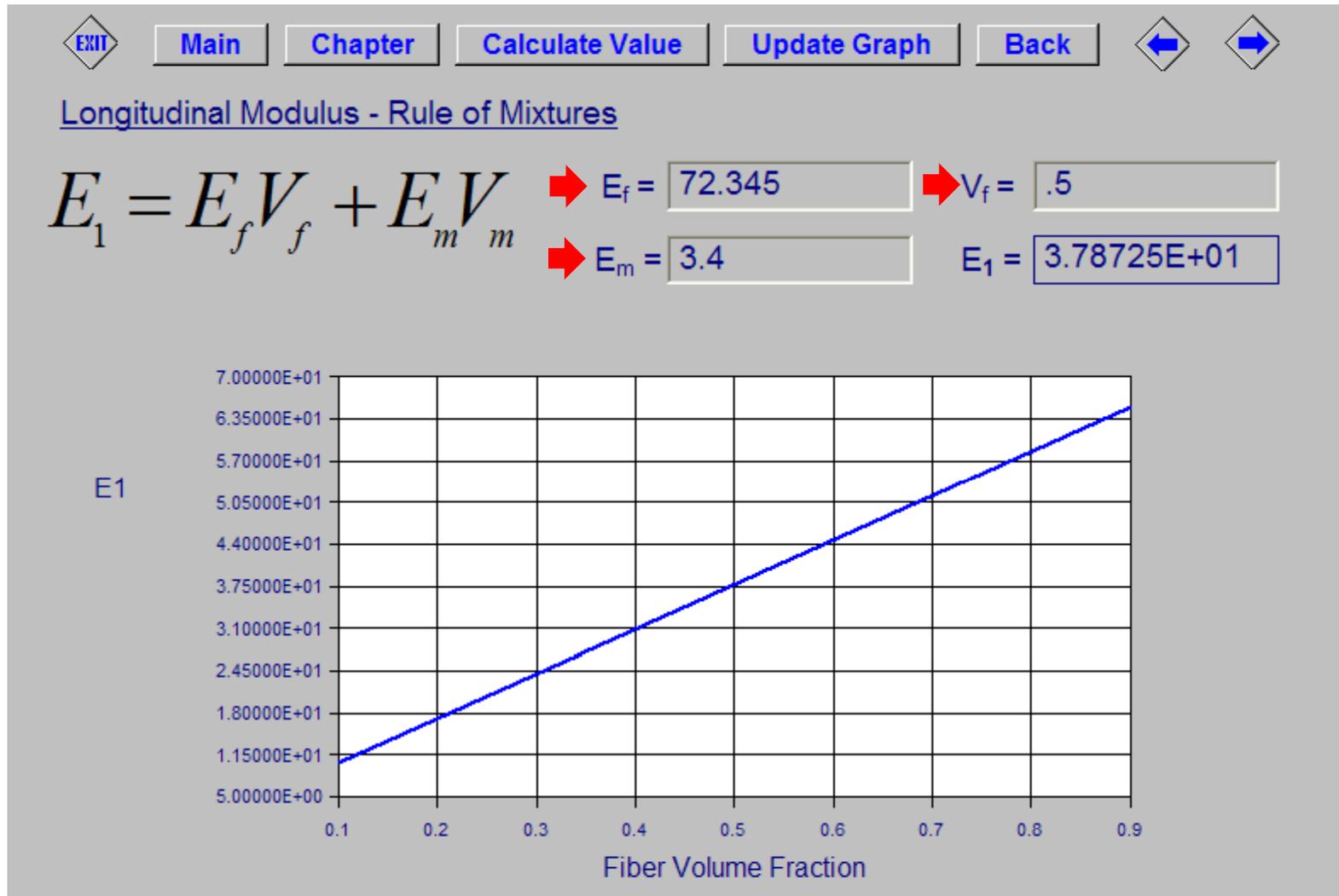
\therefore $E_1 = E_f V_f + E_m V_m$ **Rule of mixtures**

Micromechanics: Determining Properties

EXIT Main Chapter 4 - Micromechanics Back

	E_1 - Rule of Mixtures		V_{12}, V_{23} - Periodic Microstructure
	E_2 - Rule of Mixtures		G_{12}, G_{23} - Periodic Microstructure
	E_2 - Halpin-Tsai		Continuous Strand Mat - E,G,V
	V_{12} - Rule of Mixtures		Continuous Strand Mat - Strengths
	G_{12} - Rule of Mixtures		Continuous Strand Mat - α
	G_{12} - Cylindrical Assemblage		Continuous Strand Mat - β
	G_{23} - Stress Partitioning		Longitudinal Tensile Strength
	Coefficient of Thermal Expansion		Longitudinal Compressive Strength
	Coefficient of Moisture Expansion		Transverse Tensile Strength
	Transport Properties		Transverse Compressive Strength
	E_1, E_2 - Periodic Microstructure		Inplane Shear Strength

Micromechanics: Rule of Mixtures (E_1)

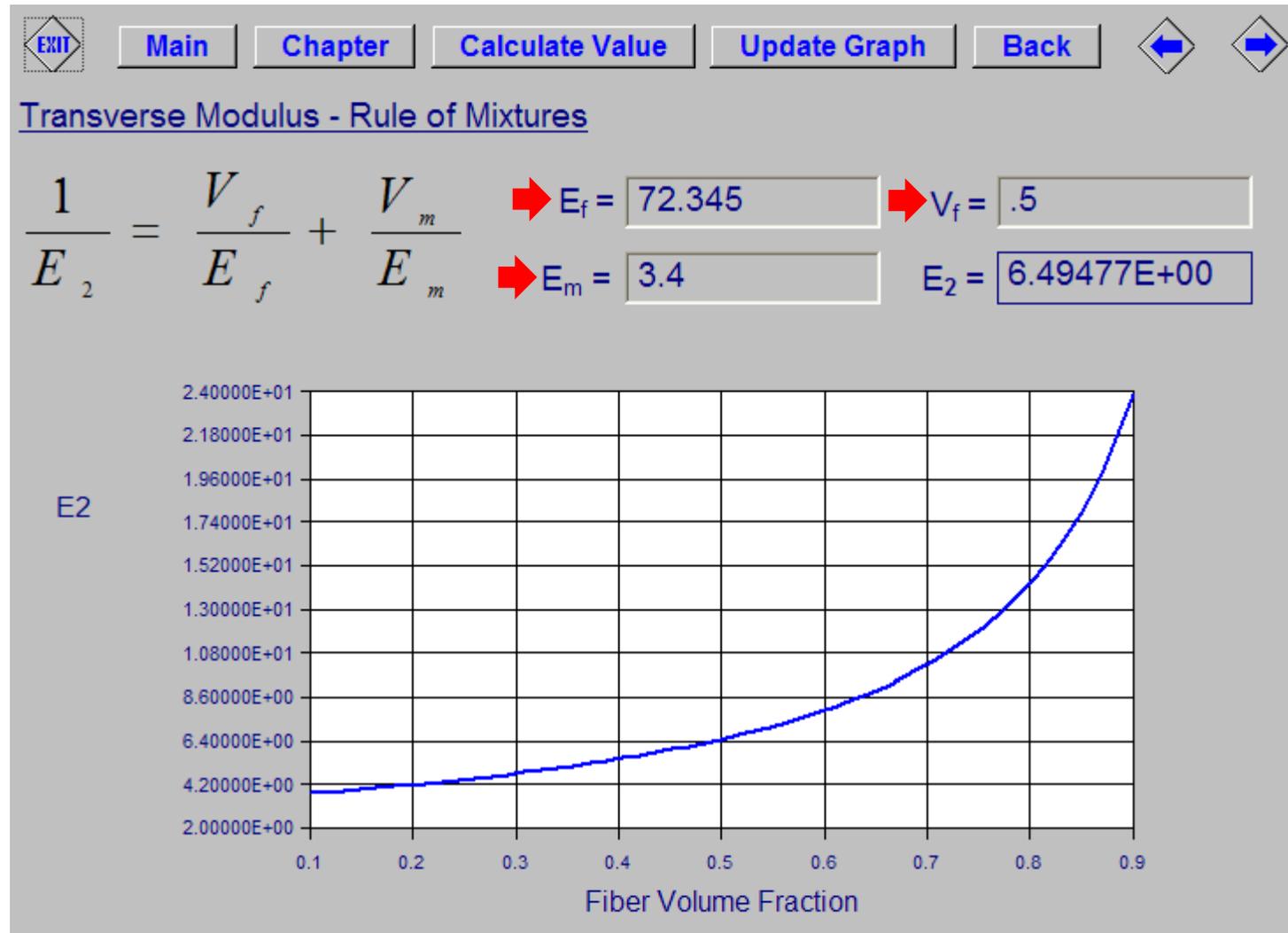


Micromechanics: Determining Properties

EXIT Main Chapter 4 - Micromechanics Back

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Micromechanics: Rule of Mixtures (E_2)

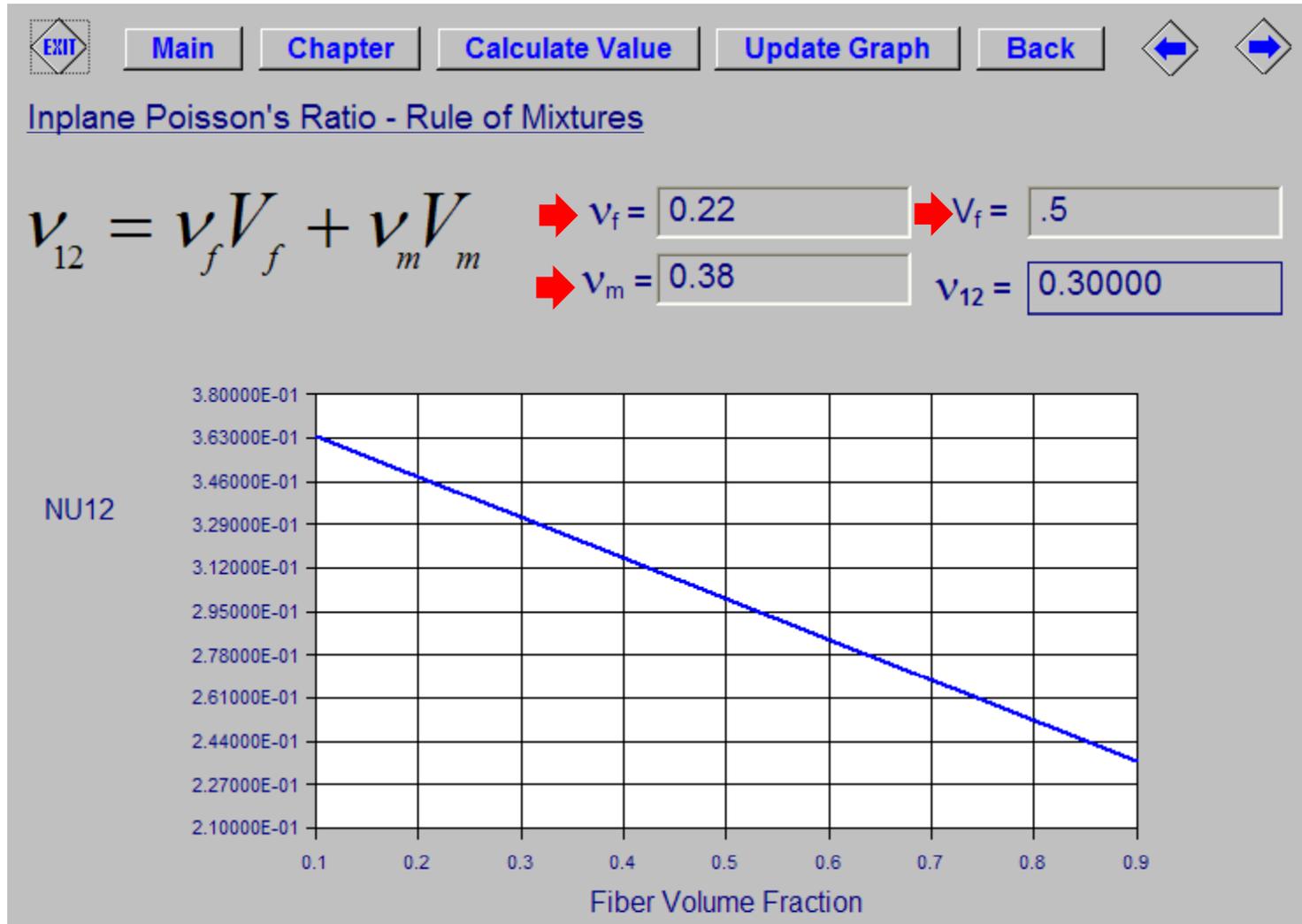


Micromechanics: Determining Properties

EXIT Main Chapter 4 - Micromechanics Back

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Micromechanics: Rule of Mixtures (ν_{12})

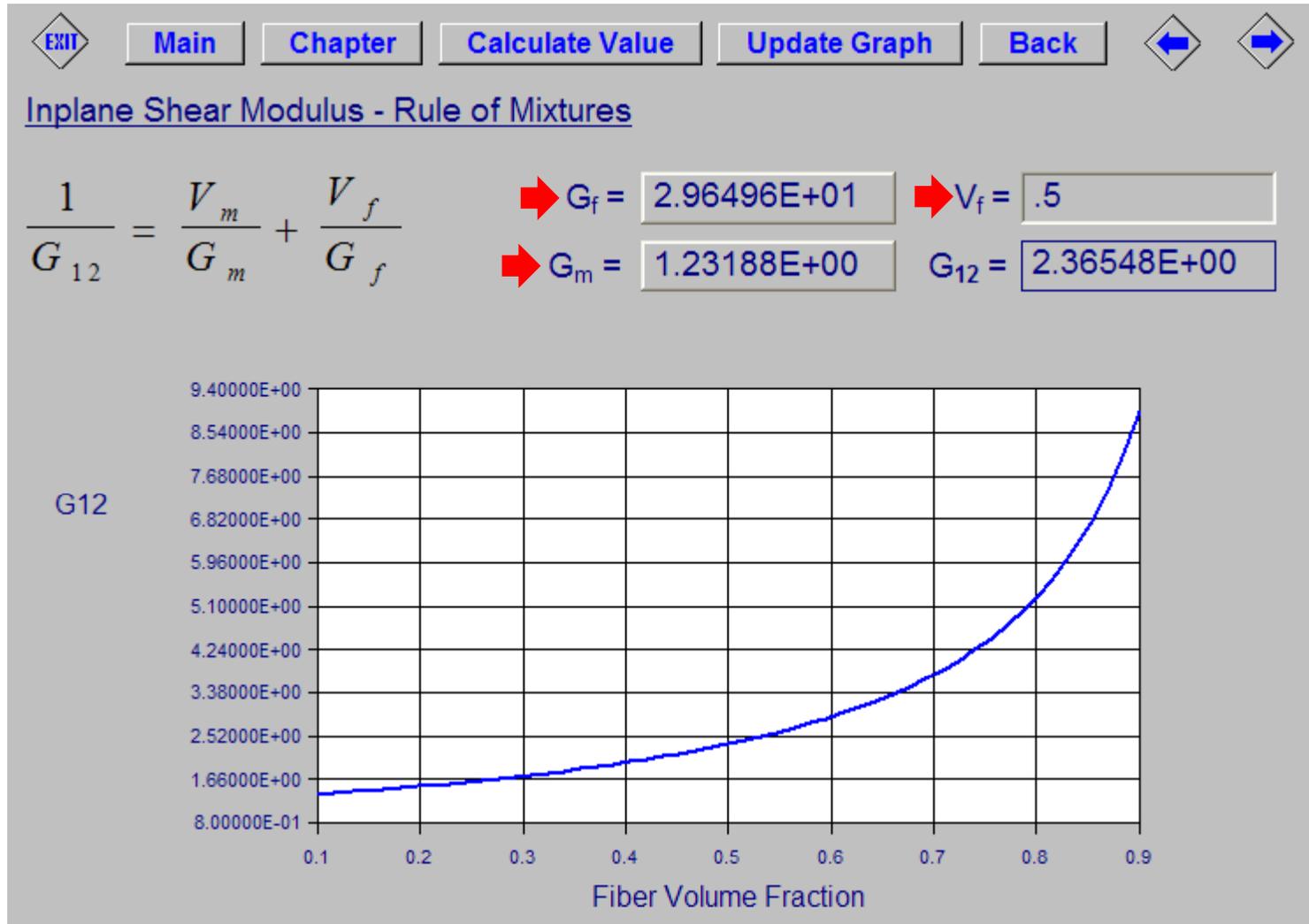


Micromechanics: Determining Properties

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Micromechanics: Rule of Mixtures (G_{12})

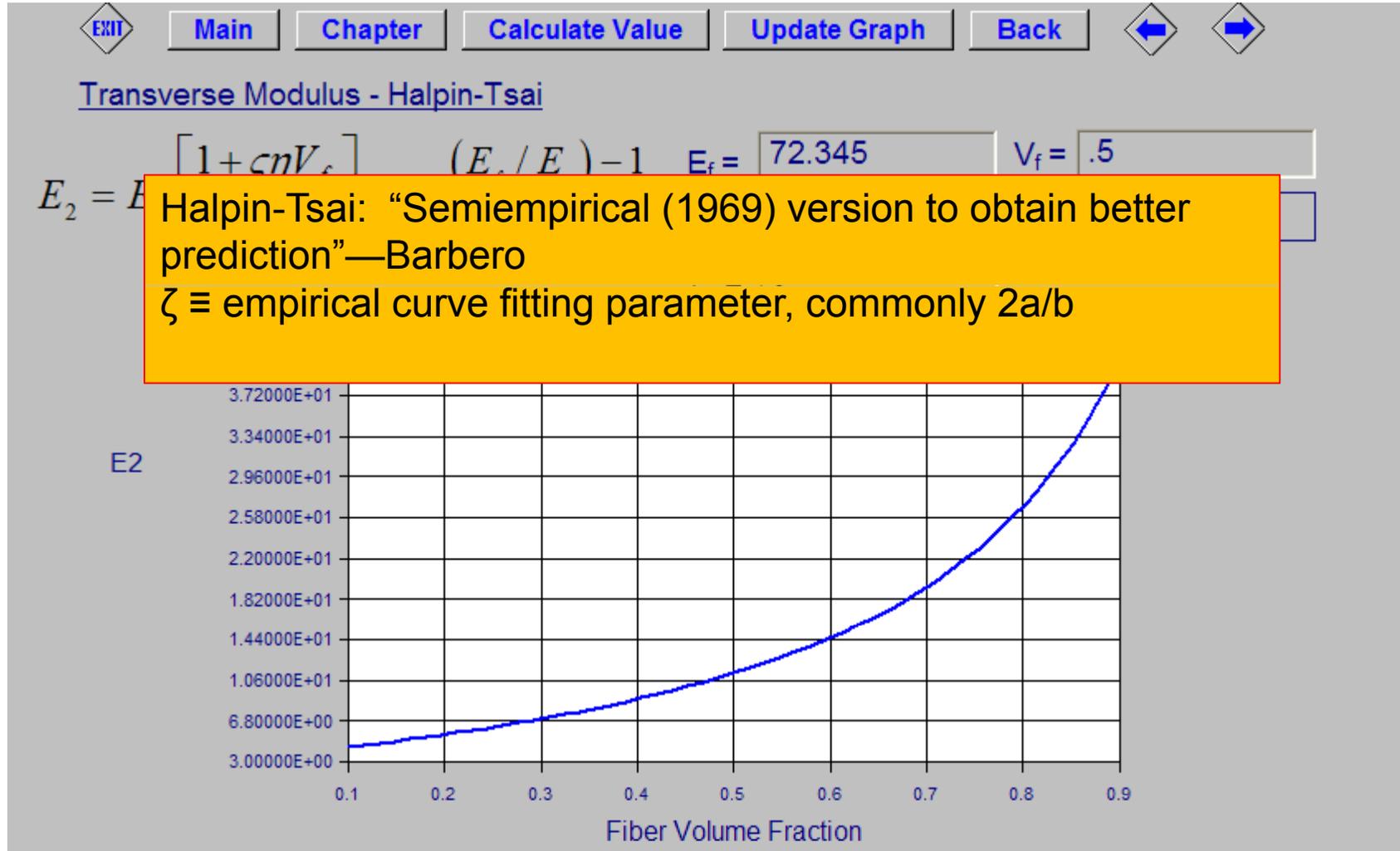


Micromechanics: Other Methods & Strengths

EXIT Main Chapter 4 - Micromechanics Back

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Micromechanics: Halpin-Tsai (E_2)



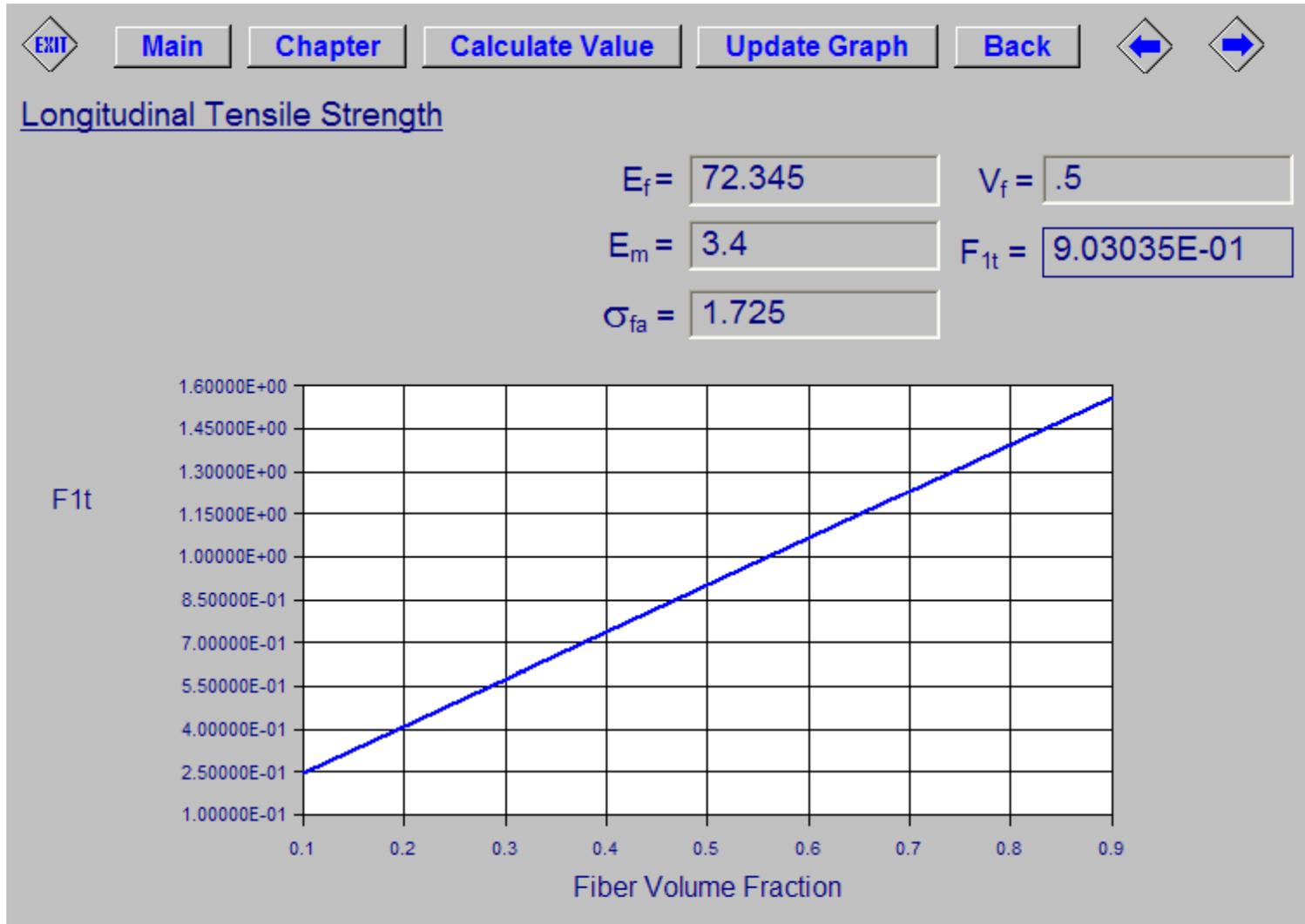
Micromechanics: Determining Properties

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	E_1, E_2 - Periodic Microstructure		Inplane Shear Strength

A yellow arrow points from the **G_{23} - Stress Partitioning** item in the left column to the **Longitudinal Tensile Strength** item in the right column.

Micromechanics: Longitudinal Tensile Strength

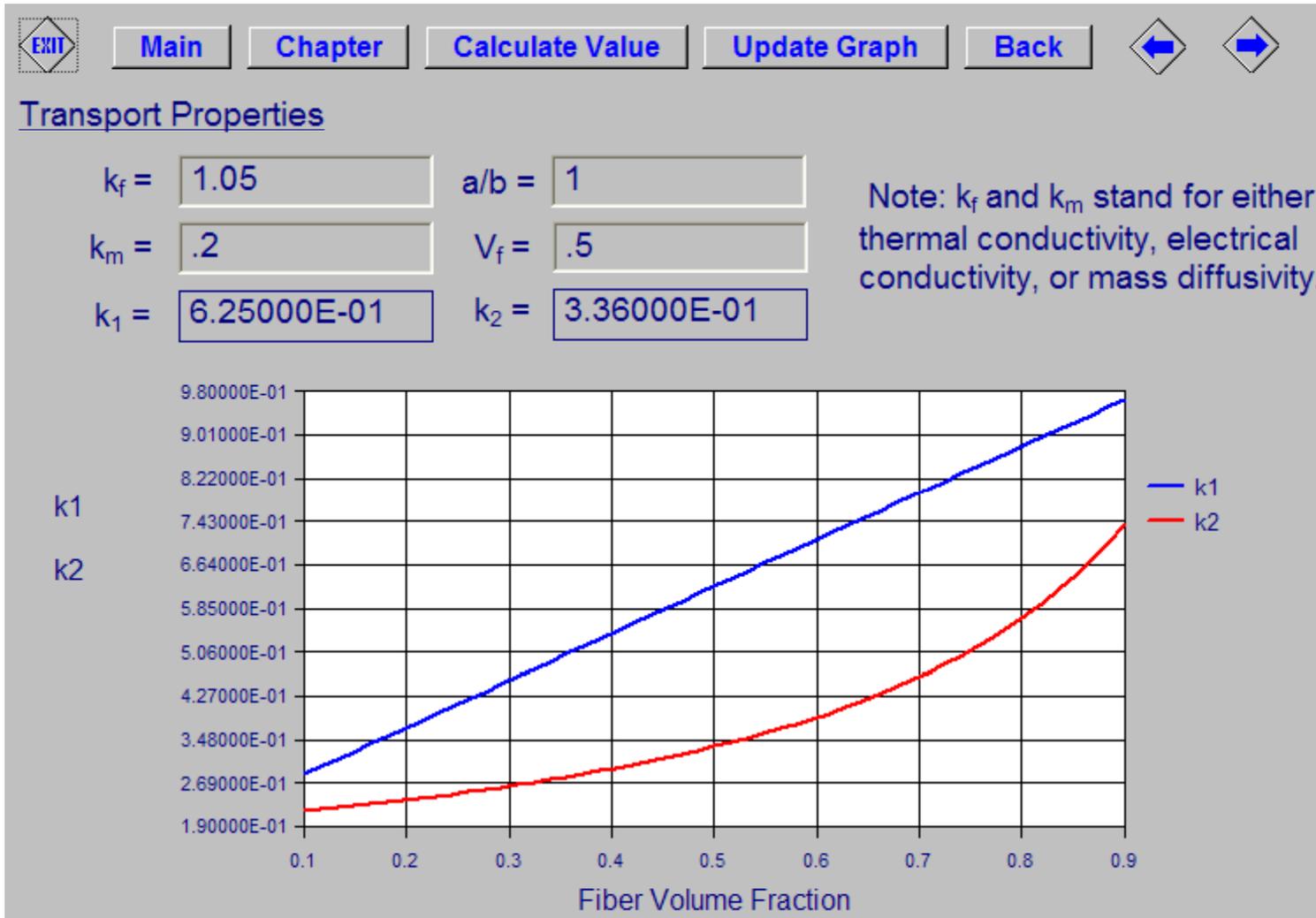


Micromechanics: Determining Properties

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	E_1, E_2 - Periodic Microstructure		Inplane Shear Strength

Micromechanics: Thermal & Electrical Cond



Ply Mechanics

- So what happens if we vary the fiber direction angle away from the 1-direction?
- CADEC uses Micromechanics results and fiber angle
 - Plane Stress
 - Transform stress/strain
 - Off-Axis Compliance/Stiffness
 - 3D Constitutive Equ's

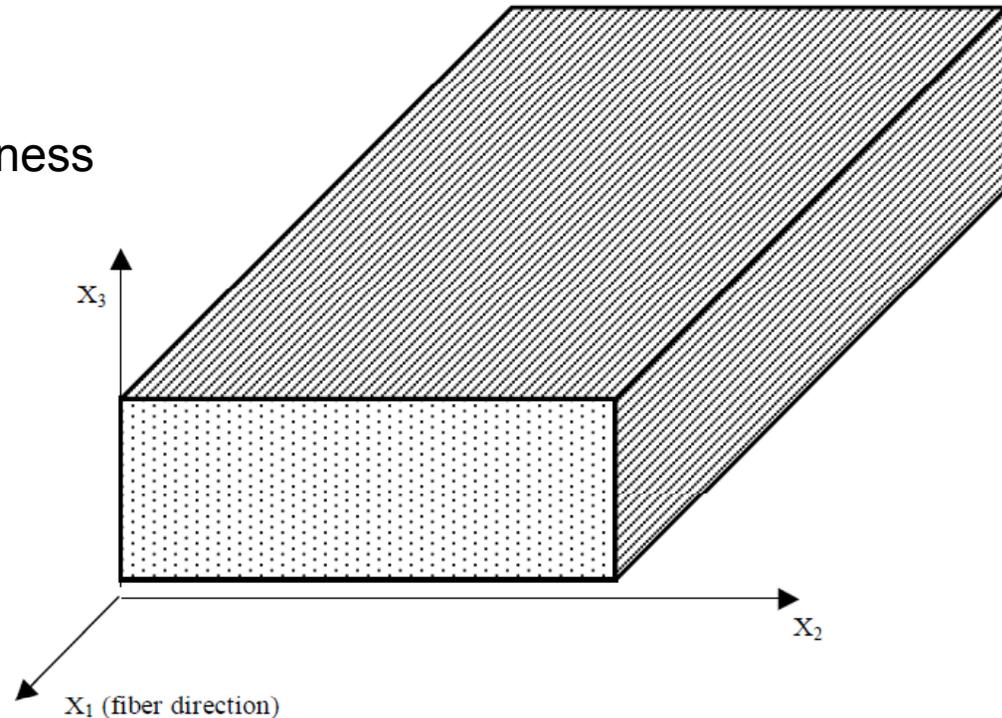


Figure 10.1 Idealized representation of a unidirectional fiber-reinforced material.

Ply Mechanics: CADEC

EXIT **Main** Chapter 5 - Ply Mechanics **Back** ◀ ▶

Plane Stress

- [S]** Compliance Matrix
- [S*]** Compliance Matrix
- [Q]** Stiffness Matrix
- [Q*]** Stiffness Matrix

Stress and Strain Transformations

- Strains - Material to Global
- Strains - Global to Material
- Stresses - Material to Global
- Stresses - Global to Material

Stiffness and Compliance Transformations

- [S̄]** Off-Axis Compliance Matrix
- [S̄*]** Off-Axis Compliance Matrix
- [Q̄]** Off-Axis Stiffness Matrix
- [Q̄*]** Off-Axis Stiffness Matrix

3D Constitutive Equations

- Orthotropic Material - Strains to Stresses
- Orthotropic Material - Stresses to Strains
- Transversely Isotropic Material - Strains to Stresses
- Transversely Isotropic Material - Stresses to Strains
- Isotropic Material - Strains to Stresses
- Isotropic Material - Stresses to Strains

Ply Mechanics: Compliance Plane Stress

EXIT Main Chapter Calculate Value Back

Compliance Equations - [S] Matrix

$\sigma_1 = 1.4e6$ $E_1 = 3.78725E+01$ $\epsilon_1 = 2.74606E+04$
 $\sigma_2 = 1.2e6$ $E_2 = 1.12711E+01$ $\epsilon_2 = 9.53771E+04$
 $\sigma_6 = .8e6$ $G_{12} = 3.33156E+00$ $\gamma_6 = 2.40128E+05$
 $\nu_{12} = 0.30000$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_6 \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}$$

Ply Mechanics: CADEC

EXIT Main Chapter 5 - Ply Mechanics Back

Plane Stress

- [S] Compliance Matrix
- [S*] Compliance Matrix
- [Q] Stiffness Matrix
- [Q*] Stiffness Matrix

Stiffness and Compliance Transformations

- [\bar{S}] Off-Axis Compliance Matrix
- [\bar{S}^*] Off-Axis Compliance Matrix
- [\bar{Q}] Off-Axis Stiffness Matrix
- [\bar{Q}^*] Off-Axis Stiffness Matrix

Stress and Strain Transformations

- Strains - Material to Global
- Strains - Global to Material
- Stresses - Material to Global
- Stresses - Global to Material

3D Constitutive Equations

- Orthotropic Material - Strains to Stresses
- Orthotropic Material - Stresses to Strains
- Transversely Isotropic Material - Strains to Stresses
- Transversely Isotropic Material - Stresses to Strains
- Isotropic Material - Strains to Stresses
- Isotropic Material - Stresses to Strains

Ply Mechanics: Transformations

Transform Material Stresses to Global Stresses

$\sigma_1 =$
 $\sigma_x =$

$\sigma_2 =$
 θ is in Degrees (°)
 $\sigma_y =$

$\sigma_6 =$
 $\theta =$
 $\sigma_{xy} =$

$\sigma_4 =$
 $\sigma_{yz} =$

$\sigma_5 =$
 $\sigma_{xz} =$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \begin{Bmatrix} \sigma_4 \\ \sigma_5 \end{Bmatrix} \quad \begin{matrix} m = \cos(\theta) \\ n = \sin(\theta) \end{matrix}$$

Ply Mechanics: CADEC

EXIT Main Chapter 5 - Ply Mechanics Back

Plane Stress

[S] Compliance Matrix

[S*] Compliance Matrix

[Q] Stiffness Matrix

[Q*] Stiffness Matrix

Stiffness and Compliance Transformations

[S̄] Off-Axis Compliance Matrix

[S̄*] Off-Axis Compliance Matrix

[Q̄] Off-Axis Stiffness Matrix

[Q̄*] Off-Axis Stiffness Matrix

Stress and Strain Transformations

3D Constitutive Equations

Strains - Material to Global

Strains - Global to Material

Stresses - Material to Global

Stresses - Global to Material

Orthotropic Material - Strains to Stresses

Orthotropic Material - Stresses to Strains

Transversely Isotropic Material - Strains to Stresses

Transversely Isotropic Material - Stresses to Strains

Isotropic Material - Strains to Stresses

Isotropic Material - Stresses to Strains

Ply Mechanics: Off-Axis Stiffness Matrices

Off-Axis Stiffness Matrix - $[\bar{Q}]$

$E_1 = 3.78725E+01$
 $E_2 = 1.12711E+01$
 $G_{12} = 3.33156E+00$
 $\nu_{12} = 0.30000$
 $\theta^\circ = 45$

$\epsilon_x = 1.50706E+00$
 $\epsilon_y = 2.05733E+00$
 $\gamma_{xy} = 6.42909E-01$

$\sigma_x = 4.95793E+01$
 $\sigma_y = 5.43611E+01$
 $\sigma_{xy} = 3.13561E+01$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = [\bar{Q}] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1.76928E+01 & 9.00294E+00 & 6.83338E+00 \\ 9.00294E+00 & 1.76928E+01 & 6.83338E+00 \\ 6.83338E+00 & 6.83338E+00 & 1.08868E+01 \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Ply Mechanics: CADEC

EXIT Main Chapter 5 - Ply Mechanics Back

Plane Stress

[S] Compliance Matrix

[S*] Compliance Matrix

[Q] Stiffness Matrix

[Q*] Stiffness Matrix

Stress and Strain Transformations

Strains - Material to Global

Strains - Global to Material

Stresses - Material to Global

Stresses - Global to Material

Stiffness and Compliance Transformations

[S̄] Off-Axis Compliance Matrix

[S̄*] Off-Axis Compliance Matrix

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3D Constitutive Equations

Orthotropic Material - Strains to Stresses

Orthotropic Material - Stresses to Strains

Transversely Isotropic Material - Strains to Stresses

Transversely Isotropic Material - Stresses to Strains

Isotropic Material - Strains to Stresses

Isotropic Material - Stresses to Strains

Ply Mechanics: Stress-Strain Relationships

- **Stress-Strain Relationship:** $\sigma_{ij} = C_{ij}\varepsilon_{ij}$
- **With 3 planes $\rightarrow C_{ij}$ has 81 terms, but since:** $\sigma_{ij} = \sigma_{ji}$
and: $\varepsilon_{ij} = \varepsilon_{ji}$ **only 36 terms**
- **Orthotropic material (2 planes of symmetry) reduces to 9 terms:**

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ \text{sym} & & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{Bmatrix}$$

Ply Mechanics: Orthotropic Material

Orthotropic Material - Stresses to Strains Activate Micromechanics

$\sigma_1 =$	<input type="text" value="0.1"/>	$E_1 =$	<input type="text" value="3.78725E+01"/>	$\varepsilon_1 =$	<input type="text" value="1.84831E-03"/>
$\sigma_2 =$	<input type="text" value="0.1"/>	$E_2 = E_3 =$	<input type="text" value="1.12711E+01"/>	$\varepsilon_2 =$	<input type="text" value="8.08012E-03"/>
$\sigma_3 =$	<input type="text" value="0"/>	$G_{12} = G_{13} =$	<input type="text" value="3.33156E+00"/>	$\varepsilon_3 =$	<input type="text" value="-5.87904E-03"/>
$\sigma_4 =$	<input type="text" value="0"/>	$\nu_{12} = \nu_{13} =$	<input type="text" value="0.30000"/>	$\gamma_4 =$	<input type="text" value="0.00000E+00"/>
$\sigma_5 =$	<input type="text" value="0"/>	$G_{23} =$	<input type="text" value="3.03426E+00"/>	$\gamma_5 =$	<input type="text" value="0.00000E+00"/>
$\sigma_6 =$	<input type="text" value="0"/>	$\nu_{23} =$	<input type="text" value="0.57335"/>	$\gamma_6 =$	<input type="text" value="0.00000E+00"/>

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}$$

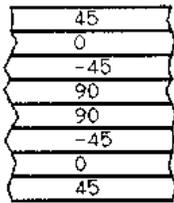
Macromechanics

- **What if there are multiple lamina at differing angles?**
- **CADEC uses Micromechanics and Ply mechanics to determine:**
 - Stiffness and Compliance Equations
 - Laminate Moduli
 - Global and Material Stresses and Strains
 - Strains and Curvatures
 - Thermal and Hygroscopic loads
 - For both Intact and Degraded materials
- **Assumes:**
 - Plane sections remain plane
 - Symmetry about a neutral surface
 - No shear coupling
 - Perfect bonding

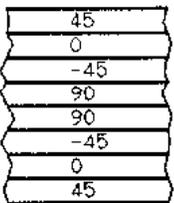
Shorthand Laminate Orientation Code

Tapes or Undirectional Tapes

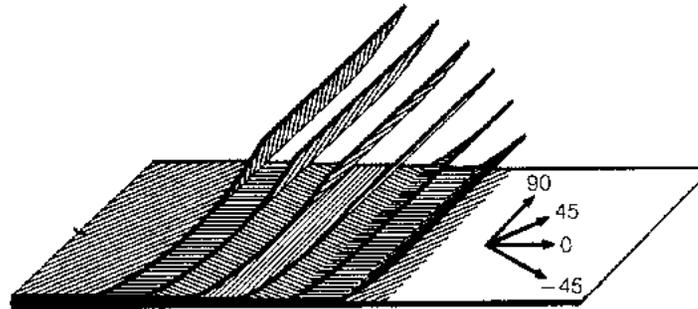
[45/0/-45/90₂/-45/0/45



[45/0/-45/90]_s



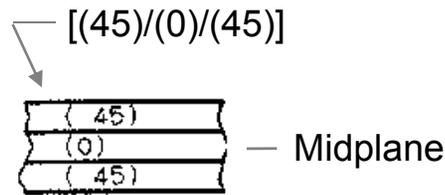
Tapes or undirectional tapes



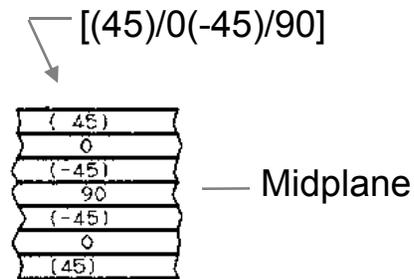
- Each lamina is labeled by its ply orientation.
- Laminae are listed in sequence with the first number representing the lamina to which the arrow is pointing.
- Individual adjacent laminae are separated by a slash if their angles differ.
- Adjacent laminae of the same angle are depicted by a numerical subscript indicating the total number of laminae which are laid up in sequence at that angle.
- Each complete laminate is enclosed by brackets.
- When the laminate is symmetrical and has an even number on each side of the plane of symmetry (known as the midplane) the code may be shortened by listing only the angles from the arrow side to the midplane. A subscript "S" is used to indicate that the code for only one half of the laminate is shown.

Shorthand Laminate Orientation Code

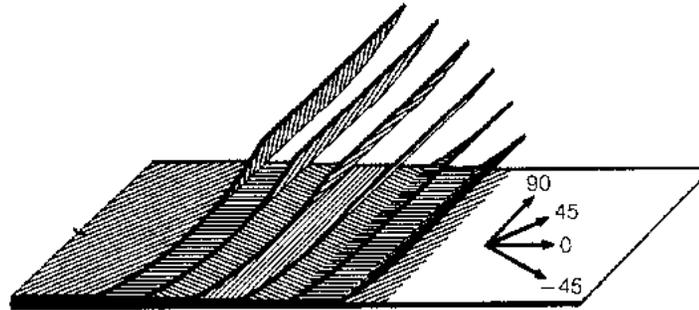
Fabrics and Tapes and Fabrics



Fabrics



Tapes & Fabrics



- When plies of fabric are used in a laminate. The angle of the fabric warp is used as the ply direction angle. The fabric angle is enclosed in parentheses to identify the ply as a fabric ply.
- When the laminate is composed of both fabric and tape plies (a hybrid laminate). The parentheses around the fabric plies will distinguish the fabric plies from the tape plies.
- When the laminate is symmetrical and has an odd number of plies, the center ply is overlined to indicate that it is the midplane.

Macromechanics: CADEC



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These values are calculated and cannot be directly changed by the user.

Macromechanics: CADEC

The screenshot displays the CADEC software interface for Chapter 6 - Macromechanics. The interface is divided into two main sections: "Analysis of Intact Material" on the left and "Analysis of Degraded Material" on the right. Both sections offer a list of analysis options, including Mid-Strains & Curvatures, Thermal & Hygroscopic Loads, Reduced Stiffness Matrices, [A], [B], [D] and [H] Matrices, Global Strains, Material Strains, Global Stresses, and Material Stresses. A yellow arrow points from the "Global Strains" option in the intact material section to a dialog box. The dialog box contains the text: "Before displaying results, please go to Laminate Definition, build a new laminate, and analyze it." Below the text are two buttons: "Laminate Definition" and "Stay Here". At the bottom right of the interface, there are two buttons: "Laminate Definition" and "To Chapter 7".

EXIT **Main** Chapter 6 - Macromechanics **Back**

Analysis of Intact Material

- Mid-Strains & Curvatures
- Thermal & Hygroscopic Loads
- Reduced Stiffness Matrices
- [A], [B], [D] and [H] Matrices
- Global Strains
- Material Strains
- Global Stresses
- Material Stresses

Analysis of Degraded Material

- Mid-Strains & Curvatures
- Thermal & Hygroscopic Loads
- Reduced Stiffness Matrices
- [A], [B], [D] and [H] Matrices
- Global Strains
- Material Strains
- Global Stresses
- Material Stresses

Before displaying results, please go to Laminate Definition, build a new laminate, and analyze it.

Laminate Definition **Stay Here**

Laminate Definition **To Chapter 7**

Macromechanics: Defining Laminate

Filename: (1/2)

Laminate Definition for:

Number of Layers: Number of Materials: Total Thickness:

Layer Thicknesses:

Layup Angles:

Layer Materials:

Loading:

Nx	<input type="text" value="-1e-9"/>	Mx	<input type="text" value="0"/>	Qx	<input type="text" value="0"/>
Ny	<input type="text" value="0"/>	My	<input type="text" value="0"/>	Qy	<input type="text" value="0"/>
Nxy	<input type="text" value="0"/>	Mxy	<input type="text" value="0"/>		

Temperature Change:

Moisture Concentration:

Safety Factor:





Macromechanics: Defining Laminate

Filename: (1/2)

Laminate Definition for:

Number of Layers: Number of Materials: Total Thickness:

Layer Thicknesses:

Layup Angles:

Layer Materials:

Loading:

Nx	<input type="text" value="-1e-9"/>	Mx	<input type="text" value="0"/>	Qx	<input type="text" value="0"/>
Ny	<input type="text" value="0"/>	My	<input type="text" value="0"/>	Qy	<input type="text" value="0"/>
Nxy	<input type="text" value="0"/>	Mxy	<input type="text" value="0"/>		

Temperature Change:

Moisture Concentration:

Safety Factor:



Macromechanics: Material Properties

Number of Materials: 1
Material Number 1

Load Data File: Unidirectional CSM

E_1	<input type="text" value="3.78725E+01"/>	E_2	<input type="text" value="1.12711E+01"/>	G_{12}	<input type="text" value="3.33156E+00"/>	G_{13}	<input type="text" value="3.33156E+00"/>
G_{23}	<input type="text" value="3.03426E+00"/>	ν_{12}	<input type="text" value="0.30000"/>	F_{1t}	<input type="text" value="9.03035E-01"/>	F_{1c}	<input type="text" value="0.37881096899"/>
F_{2t}	<input type="text" value="4.36240E-02"/>	F_{2c}	<input type="text" value="6.73614E-02"/>	F_6	<input type="text" value="4.36240E-02"/>	F_4	<input type="text" value="7.59000E-02"/>
F_5	<input type="text" value="4.36240E-02"/>	F_{12}^*	<input type="text" value="-0.5"/>	α_1	<input type="text" value="6.50423E-06"/>	α_2	<input type="text" value="2.20427E-05"/>
β_1	<input type="text" value="8.30418E-02"/>	β_2	<input type="text" value="1.25159E+00"/>	df	<input type="text" value="0.2"/>		

Macromechanics: CADEC Quirkiness

The screenshot displays the CADEC software interface for Chapter 6 - Macromechanics. The interface is divided into two main sections: "Analysis of Intact Material" and "Analysis of Degraded Material". Each section contains a list of analysis options with corresponding icons. A dialog box is overlaid on the interface, displaying the message: "Before displaying results, please wait until the icon LAMINATE in the taskbar disappears." with an "It's Gone" button. The dialog box is titled with a close button (X) in the top right corner.

EXIT **Main** Chapter 6 - Macromechanics **Back**

Analysis of Intact Material

- Mid-Strains & Curvatures**
- Thermal & Hygroscopic Loads**
- Reduced Stiffness Matrices**
- [A], [B], [D] and [H] Matrices**
- Global Strains**
- Material Strains**
- Global Stresses**
- Material Stresses**

Analysis of Degraded Material

- Mid-Strains & Curvatures**
- Thermal & Hygroscopic Loads**
- Reduced Stiffness Matrices**
- [B], [D] and [H] Matrices**
- Global Strains**
- Material Strains**
- Global Stresses**
- Material Stresses**

Before displaying results, please wait until the icon LAMINATE in the taskbar disappears.

It's Gone

Plate Stiffness and Compliance

- Stiffness Equations**
- Compliance Equations**
- Laminate Moduli**

Laminate Definition

To Chapter 7

Macromechanics: Review Outputs

The screenshot shows a software interface titled "Chapter 6 - Macromechanics". At the top, there are buttons for "EXIT", "Main", "Back", and navigation arrows. The interface is divided into three main sections:

- Analysis of Intact Material:** Contains seven menu items: "Mid-Strains & Curvatures" (with a K icon), "Thermal & Hygroscopic Loads" (with a thermometer and water drop icon), "Reduced Stiffness Matrices" (with a $[Q]$ icon), "[A], [B], [D] and [H] Matrices" (with a matrix icon), "Global Strains" (with a globe icon), "Material Strains" (with a grid icon), and "Global Stresses" (with a globe icon). A yellow arrow points to the "Global Stresses" option.
- Analysis of Degraded Material:** Contains the same seven menu items as the intact material section.
- Plate Stiffness and Compliance:** Contains three menu items: "Stiffness Equations" (with a matrix icon), "Compliance Equations" (with a matrix icon), and "Laminate Moduli" (with a grid icon).

At the bottom right, there are two buttons: "Laminate Definition" and "To Chapter 7".

Macromechanics: Global Stresses

Ply Face		Global Stresses - Intact Material					
Hygrothermal Stress		σ_x	σ_y	σ_{xy}	σ_{yz}	σ_{xz}	Print
8	TOP	.000	.000	.000	.000	.000	
8	BOT	.000	.000	.000	.000	.000	
7	TOP	.000	.000	.000	.000	.000	
7	BOT	.000	.000	.000	.000	.000	
6	TOP	.000	.000	.000	.000	.000	
Mechanical Stress		σ_x	σ_y	σ_{xy}	σ_{yz}	σ_{xz}	Print
8	TOP	-.684E-07	.635E-07	.383E-23	.000	.000	
8	BOT	-.684E-07	.635E-07	.387E-23	.000	.000	
7	TOP	-.918E-07	-.332E-07	-.297E-07	.000	.000	
7	BOT	-.918E-07	-.332E-07	-.297E-07	.000	.000	
6	TOP	-.918E-07	-.332E-07	.297E-07	.000	.000	
Total Stress		σ_x	σ_y	σ_{xy}	σ_{yz}	σ_{xz}	Print
8	TOP	-.684E-07	.635E-07	.383E-23	.000	.000	
8	BOT	-.684E-07	.635E-07	.387E-23	.000	.000	
7	TOP	-.918E-07	-.332E-07	-.297E-07	.000	.000	
7	BOT	-.918E-07	-.332E-07	-.297E-07	.000	.000	
6	TOP	-.918E-07	-.332E-07	.297E-07	.000	.000	

Macromechanics: ABD Matrices

- **Stiffness of composite**

where:

- $[A]$ = in-plane stiffness.
- $[D]$ = bending stiffness.
- $[B]$ relates in-plane strains to bending moments and curvatures to in-plane forces—bending-extension coupling.
- $[H]$ relates transverse shear strains to transverse forces.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (6.15)$$

$$\begin{Bmatrix} V_y \\ V_x \end{Bmatrix} = \begin{bmatrix} H_{44} & H_{45} \\ H_{45} & H_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

with

$$A_{ij} = \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k - z_{k-1}) = \sum_{k=1}^N (\bar{Q}_{ij})_k t_k \quad i, j = 1, 2, 6$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) = \sum_{k=1}^N (\bar{Q}_{ij})_k t_k \bar{z}_k \quad i, j = 1, 2, 6$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) = \sum_{k=1}^N (\bar{Q}_{ij})_k \left(t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right) \quad i, j = 1, 2, 6$$

$$H_{ij} = \frac{5}{4} \sum_{k=1}^N (\bar{Q}_{ij})_k \left[t_k - \frac{4}{t^2} \left(t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right) \right] \quad i, j = 4, 5 \quad (6.16)$$

Macromechanics: ABD Matrices

EXIT **Main** Chapter 6 - Macromechanics **Back**

Analysis of Intact Material

- Mid-Strains & Curvatures
- Thermal & Hygroscopic Loads
- Reduced Stiffness Matrices
- [A], [B], [D] and [H] Matrices**
- Global Strains
- Material Strains
- Global Stresses
- Material Stresses

Analysis of Degraded Material

- Mid-Strains & Curvatures
- Thermal & Hygroscopic Loads
- Reduced Stiffness Matrices
- [A], [B], [D] and [H] Matrices
- Global Strains
- Material Strains
- Global Stresses
- Material Stresses

Laminate Definition

To Chapter 7

Plate Stiffness and Compliance

- Stiffness Equations**
- Compliance Equations**
- Laminate Moduli**

Macromechanics: ABD Matrices

EXIT Main Chapter Save Matrices Back ◀ ▶

Analysis for Intact Material

[A] =
$$\begin{bmatrix} .171763 & .580162\text{E-}01 & -.884037\text{E-}19 \\ .580162\text{E-}01 & .171763 & .349677\text{E-}17 \\ -.884037\text{E-}19 & .349677\text{E-}17 & .568736\text{E-}01 \end{bmatrix}$$

[B] =
$$\begin{bmatrix} .677626\text{E-}20 & .169407\text{E-}20 & -.309413\text{E-}21 \\ -.169407\text{E-}20 & .000000 & -.609111\text{E-}21 \\ -.309413\text{E-}21 & -.609111\text{E-}21 & .169407\text{E-}20 \end{bmatrix}$$

[D] =
$$\begin{bmatrix} .618290\text{E-}06 & .279199\text{E-}06 & .546670\text{E-}07 \\ .279199\text{E-}06 & .127429\text{E-}05 & .546670\text{E-}07 \\ .546670\text{E-}07 & .546670\text{E-}07 & .273105\text{E-}06 \end{bmatrix}$$

[H] =
$$\begin{bmatrix} .209407\text{E-}01 & -.929063\text{E-}04 \\ -.929063\text{E-}04 & .214981\text{E-}01 \end{bmatrix}$$

Macromechanics: Stiffness Equations

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Analysis of Intact Material

-  **Mid-Strains & Curvatures**
-  **Thermal & Hygroscopic Loads**
-  **Reduced Stiffness Matrices**
-  **[A], [B], [D] and [H] Matrices**
-  **Global Strains**
-  **Material Strains**
-  **Global Stresses**
-  **Material Stresses**

Analysis of Degraded Material

-  **Mid-Strains & Curvatures**
-  **Thermal & Hygroscopic Loads**
-  **Reduced Stiffness Matrices**
-  **[A], [B], [D] and [H] Matrices**
-  **Global Strains**
-  **Material Strains**
-  **Global Stresses**
-  **Material Stresses**

Plate Stiffness and Compliance

-  **Stiffness Equations**
-  **Compliance Equations**
-  **Laminate Moduli**

[Laminate Definition](#) 

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Macromechanics: Stiffness Equations

Plate Stiffness Equations

$N_x =$	<input type="text" value="-9.99687E-10"/>	<input type="text" value="1.72E-01"/>	<input type="text" value="5.80E-02"/>	<input type="text" value="-8.84E-20"/>	<input type="text" value="6.78E-21"/>	<input type="text" value="1.69E-21"/>	<input type="text" value="-3.09E-22"/>	$\epsilon_x^0 =$	<input type="text" value="-0.657E-08"/>
$N_y =$	<input type="text" value="1.47426E-13"/>	<input type="text" value="5.80E-02"/>	<input type="text" value="1.72E-01"/>	<input type="text" value="3.50E-18"/>	<input type="text" value="-1.69E-21"/>	<input type="text" value="0.00E+00"/>	<input type="text" value="-6.09E-22"/>	$\epsilon_y^0 =$	<input type="text" value="0.222E-08"/>
$N_{xy} =$	<input type="text" value="-1.67775E-29"/>	<input type="text" value="-8.84E-20"/>	<input type="text" value="3.50E-18"/>	<input type="text" value="5.69E-02"/>	<input type="text" value="-3.09E-22"/>	<input type="text" value="-6.09E-22"/>	<input type="text" value="1.69E-21"/>	$\gamma_{xy}^0 =$	<input type="text" value="-0.147E-24"/>
$M_x =$	<input type="text" value="3.06089E-32"/>	<input type="text" value="6.78E-21"/>	<input type="text" value="1.69E-21"/>	<input type="text" value="-3.09E-22"/>	<input type="text" value="6.18E-07"/>	<input type="text" value="2.79E-07"/>	<input type="text" value="5.47E-08"/>	$\kappa_x =$	<input type="text" value="0.786E-22"/>
$M_y =$	<input type="text" value="-2.22681E-29"/>	<input type="text" value="-1.69E-21"/>	<input type="text" value="0.00E+00"/>	<input type="text" value="-6.09E-22"/>	<input type="text" value="2.79E-07"/>	<input type="text" value="1.27E-06"/>	<input type="text" value="5.47E-08"/>	$\kappa_y =$	<input type="text" value="-0.254E-22"/>
$M_{xy} =$	<input type="text" value="1.12259E-32"/>	<input type="text" value="-3.09E-22"/>	<input type="text" value="-6.09E-22"/>	<input type="text" value="1.69E-21"/>	<input type="text" value="5.47E-08"/>	<input type="text" value="5.47E-08"/>	<input type="text" value="2.73E-07"/>	$\kappa_{xy} =$	<input type="text" value="-0.131E-22"/>
$V_y =$	<input type="text" value="0.00000E+00"/>	<input type="text" value="2.09E-02"/>	<input type="text" value="-9.29E-05"/>					$\gamma_{yz} =$	<input type="text" value="0.00000E+00"/>
$V_x =$	<input type="text" value="0.00000E+00"/>	<input type="text" value="-9.29E-05"/>	<input type="text" value="2.15E-02"/>					$\gamma_{xz} =$	<input type="text" value="0.00000E+00"/>

Matrices [ABD] and [H] are based on the current intact material, or the last matrix file loaded.

Macromechanics: Laminate Moduli

 [Main](#) Chapter 6 - Macromechanics [Back](#)  

Analysis of Intact Material

-  **Mid-Strains & Curvatures**
-  **Thermal & Hygroscopic Loads**
-  **Reduced Stiffness Matrices**
-  **[A], [B], [D] and [H] Matrices**
-  **Global Strains**
-  **Material Strains**
-  **Global Stresses**
-  **Material Stresses**

Analysis of Degraded Material

-  **Mid-Strains & Curvatures**
-  **Thermal & Hygroscopic Loads**
-  **Reduced Stiffness Matrices**
-  **[A], [B], [D] and [H] Matrices**
-  **Global Strains**
-  **Material Strains**
-  **Global Stresses**
-  **Material Stresses**

Plate Stiffness and Compliance

-  **Stiffness Equations**
-  **Compliance Equations**
-  **Laminate Moduli**

[Laminate Definition](#) 

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Macromechanics: Laminate Moduli

EXIT Main Chapter Back ← →

LaminateModuli: Inplane Bending

$E_x =$	2.278E+1	$E_x^b =$	3.243E+1
$E_y =$	1.779E+1	$E_y^b =$	1.567E+1
$G_{xy} =$	6.329E+0	$G_{xy}^b =$	3.774E+0
$\nu_{xy} =$	0.335	$\nu_{xy}^b =$	0.244

Macromechanics: Degraded Material

EXIT Main Chapter 6 - Macromechanics Back

Analysis of Intact Material  **Analysis of Degraded Material**

	Mid-Strains & Curvatures		Mid-Strains & Curvatures
	Thermal & Hygroscopic Loads		Thermal & Hygroscopic Loads
	Reduced Stiffness Matrices		Reduced Stiffness Matrices
	[A], [B], [D] and [H] Matrices		[A], [B], [D] and [H] Matrices
	Global Strains		Global Strains
	Material Strains		Material Strains
	Global Stresses		Global Stresses
	Material Stresses		Material Stresses

Plate Stiffness and Compliance

	Stiffness Equations		Compliance Equations		Laminate Moduli
---	---------------------	---	----------------------	---	-----------------

Laminate Definition →
To Chapter 7 →

Macromechanics: Degraded Material

- What is a degraded material?

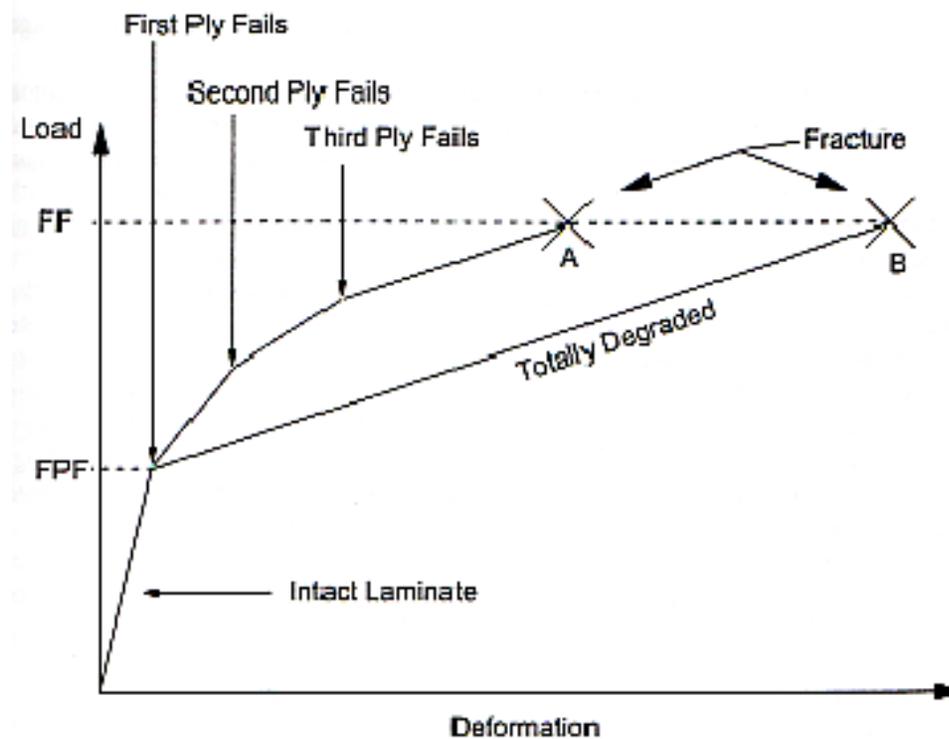
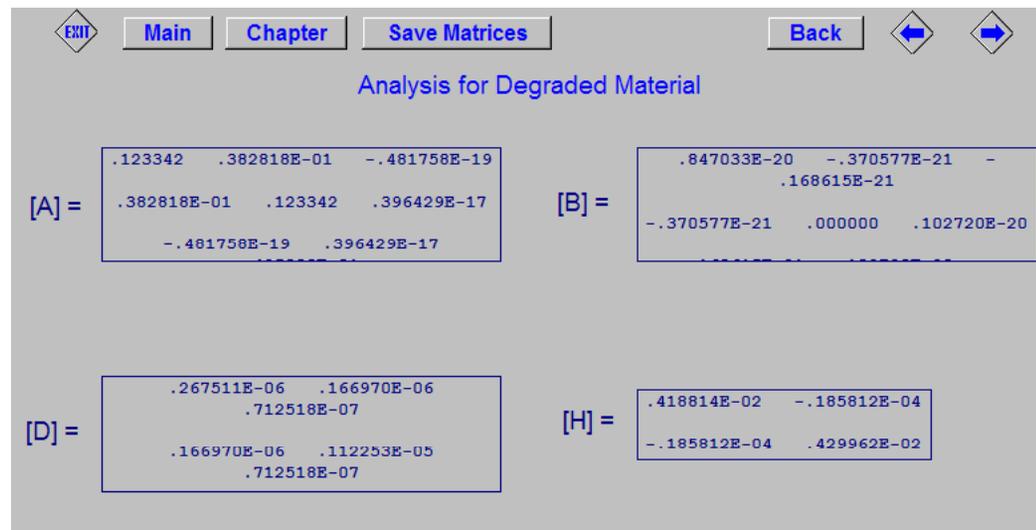
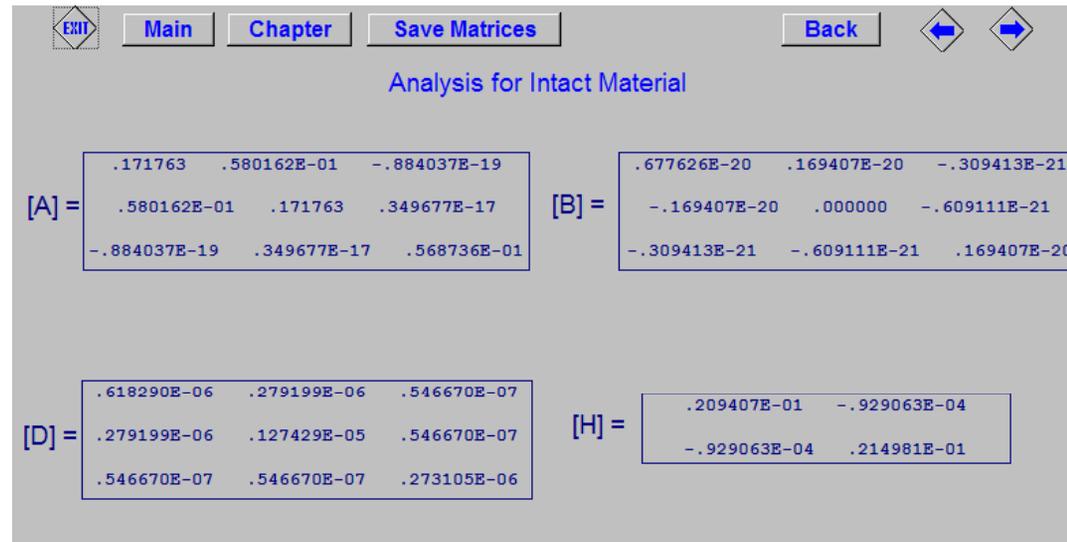


Figure 7.7 Determination of FF load by an incremental and two-step approach.

Macromechanics: ABD Comparison



Macromechanics: CADEC Alt Methods

 **Main** **Chapter** **Load Laminate** **Save Laminate** **Back**  

Filename: (1/2)

Laminate Definition for:

Number of Layers: Number of Materials: Total Thickness:

Layer Thicknesses:

Layup Angles:

Layer Materials:

Loading:

Nx	<input type="text" value="0"/>	Mx	<input type="text" value="0"/>	Qx	<input type="text" value="0"/>
Ny	<input type="text" value="0"/>	My	<input type="text" value="0"/>	Qy	<input type="text" value="0"/>
Nxy	<input type="text" value="0"/>	Mxy	<input type="text" value="0"/>		

Temperature Change:

Moisture Concentration:

Safety Factor:



Failure Theories

- Many failure criteria, most popular:
 - Maximum stress criterion
 - Maximum strain criterion
 - Tsai-Hill failure criterion
 - Tsai-Wu failure criterion

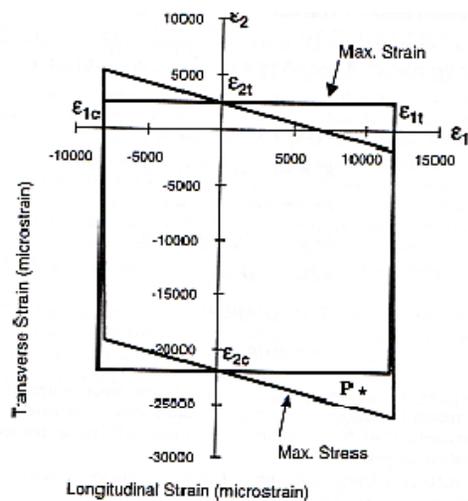


Figure 7.3 Failure envelopes using the maximum strain and maximum stress criteria in strain space $\epsilon_1 - \epsilon_2$ for carbon-epoxy.

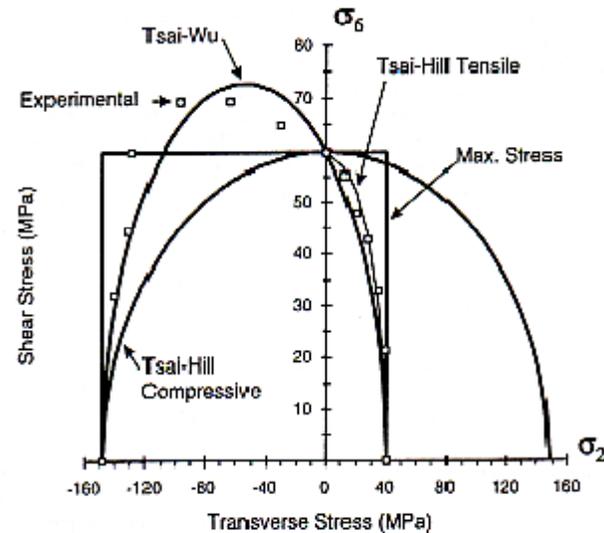
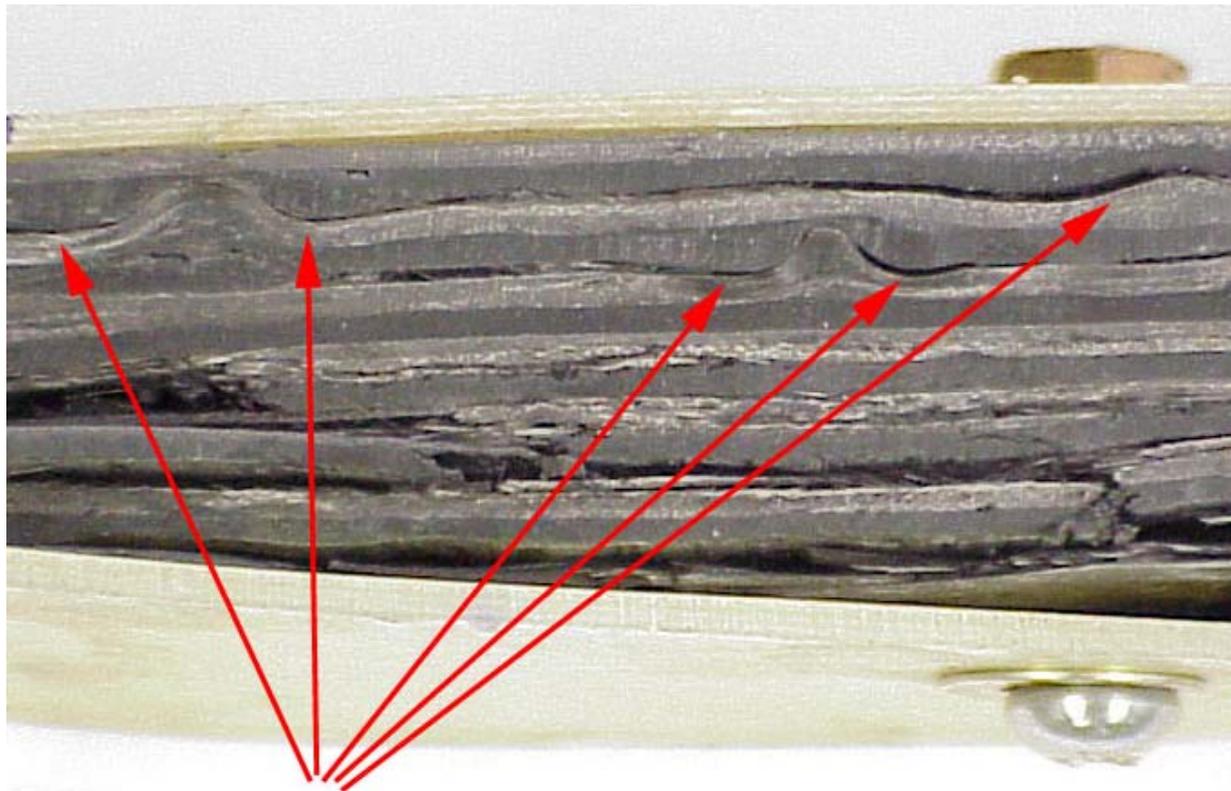


Figure 7.5 Various failure envelopes in $\sigma_6 - \sigma_2$ space compared with experimental data.

Not Just An Academic Exercise



DELAMINATIONS ASSOCIATED WITH WAVES

Consequence of Misalignment in Large, Composite Structure
Design and Analysis of Aircraft Structures

Failure Theories: CADEC

 **Main** Chapter 6 - Macromechanics **Back**  

Analysis of Intact Material

-  **Mid-Strains & Curvatures**
-  **Thermal & Hygroscopic Loads**
-  **Reduced Stiffness Matrices**
-  **[A], [B], [D] and [H] Matrices**
-  **Global Strains**
-  **Material Strains**
-  **Global Stresses**
-  **Material Stresses**

Analysis of Degraded Material

-  **Mid-Strains & Curvatures**
-  **Thermal & Hygroscopic Loads**
-  **Reduced Stiffness Matrices**
-  **[A], [B], [D] and [H] Matrices**
-  **Global Strains**
-  **Material Strains**
-  **Global Stresses**
-  **Material Stresses**

Plate Stiffness and Compliance

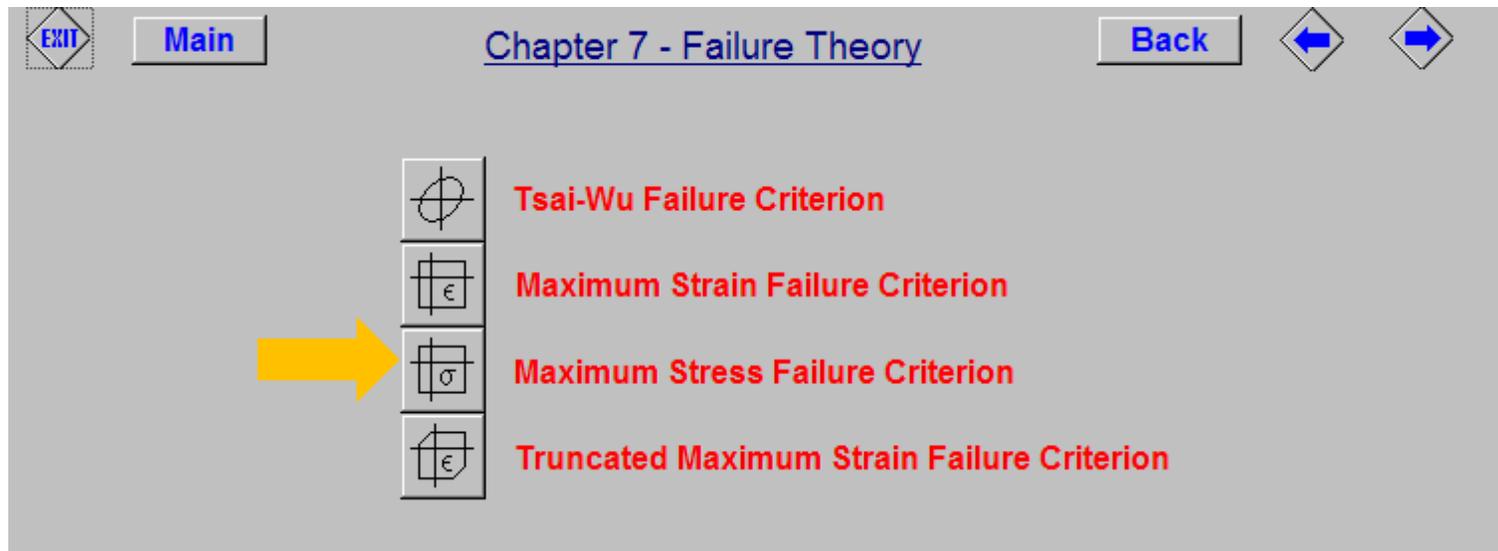
-  **Stiffness Equations**
-  **Compliance Equations**
-  **Laminate Moduli**

Laminate Definition 

To Chapter 7 



Failure Theories: CADEC



Failure Theories: Max Stress Criterion

Maximum Stress Failure Criterion

Ply	Angle	<u>Intact Laminate</u>		<u>Degraded Laminate</u>	
		Rint-Top	Rint-Bot	Rdeg-Top	Rdeg-Bot
4	-10.	1000.(1)	1000.(1)	1000.(1)	1000.(1)
3	10.	1000.(1)	1000.(1)	1000.(1)	1000.(1)
2	-10.	1000.(1)	1000.(1)	1000.(1)	1000.(1)
1	10.	1000.(1)	1000.(1)	1000.(1)	1000.(1)

Print FPF 1000. FF 1000.

Failure Theories: Tsai-Wu Criterion

EXIT Main Chapter Back

Tsai-Wu Failure Criterion

Ply	Angle	Intact Laminate		Degraded Laminate	
		Rint-Top	Rint-Bot	Rdeg-Top	Rdeg-Bot
4	-10.	.000	.000	.000	.000
3	10.	.000	.000	.000	.000
2	-10.	.000	.000	.000	.000
1	10.	.000	.000	.000	.000

Print FPF .000 FF .000

CADEC Demo

Concluding Remarks

- **Composite design fairly simple**
 - Assumptions lead to simplified analysis
 - Idealized
 - Real-world?
- **CADEC**
 - Begin with component properties
 - Micromechanic, Ply and Macromechanic analysis
 - Apply loads and match against failure criteria
 - Simple structures (Not covered)
 - Software options: COMPRO, MSEExcel, Matlab, MathCAD, etc.
- **Composites still require significant analysis and physical testing**
- **Parts/Structures are only as good as the manufacturing**
 - “You can never make good parts with bad materials, but you can easily make bad parts with good materials!”