



SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)

Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE

www.sathyabama.ac.in

SCHOOL OF BIO & CHEMICAL ENGINEERING
DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – I – Heat Transfer – SCHA1301

1. Introduction to Heat Transfer

Practically all the operations that are carried out by the chemical engineers involve the production or absorption of energy in the form of heat. The study of temperature distribution and heat transfer is of great importance to engineers because of its almost universal occurrence in many branches of science and engineering. The first step in the optimal design of heat exchangers such as boilers, heaters, refrigerators and radiators is a detailed analysis of heat transfer. This is essential to determine the feasibility and cost of the undertaking, as well as the size of equipment required to transfer a specified amount of heat in a given time. Difference between thermodynamics and heat transfer Thermodynamic tells us (i) How much heat is transferred (ii) How much work is done (iii) Final state of the system.

Heat transfer tells us: (i) How much heat is transferred (with what modes) (ii) At what rate heat is transferred (iii) Temperature distribution inside the body.

The various modes of heat transfer are (i) conduction (ii) convection (iii) radiation. Conduction Heat transfer by the actual but invisible movement of molecules within the continuous substance due to temperature gradient is known as conduction. When a current or macroscopic particle of fluid crosses a specific surface, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called convection. Convection is the mode of heat transfer in which the heat flow is associated with the movement of fluid. Transfer of energy through space by electromagnetic waves is known as radiation.

1.1 Applications of heat transfer

Energy production and conversion -steam power plant, solar energy conversion etc.
Refrigeration and air-conditioning Domestic applications -ovens, stoves, toaster Cooling of electronic equipment Manufacturing / materials processing -welding, casting, soldering, laser machining Automobiles / aircraft design

1.1.1 Conduction: It is the transfer of internal energy by microscopic diffusion and collisions of particles or quasi-particles within a body due to a temperature gradient. The microscopically diffusing and colliding objects include molecules, electrons, atoms, and phonons. They transfer disorganized microscopic kinetic and potential energy, which are jointly known as internal energy. Conduction can only take place within an object or material, or between two objects that

are in direct or indirect contact with each other. On a microscopic scale, heat conduction occurs as hot, rapidly moving or vibrating atoms and molecules interact with neighboring atoms and molecules, transferring some of their energy (heat) to these neighboring particles. In other words, heat is transferred by conduction when adjacent atoms vibrate against one another, or as electrons move from one atom to another.

Fourier's law of heat Conduction

The rate of heat transfer due to conduction is governed by Fourier's Law, as shown

$$q = kA \left(\frac{\Delta T}{\Delta x} \right)$$

The terms in Eqn. 1 are:

q – rate of heat transfer (W)

k – thermal conductivity (W/m·K)

A – surface area across which heat is transferred (m²)

ΔT – difference in temperature over which heat is transferred (K)

Δx – distance over which heat is transferred (m)

Thermal conductivity indicates the ease of heat transfer through a material and is a material dependent property. The ΔT term is the driving force for heat transfer.

1.1.2 Convection

The rate of heat transfer due to convection is described by

$$q = hA(\Delta T)$$

In Eqn. 2 the new term is:

h – heat transfer coefficient (W/m²·K)

In Eqn. 2, the heat transfer coefficient replaces the $k/\Delta x$ term in Eqn. 1. The reason this happens is because convection has a mobile phase, and thickness is no longer an effective way of describing how the heat is transferred. The heat transfer coefficient can be thought of as the inverse of the resistance to heat transfer. Also, because temperature is a function of distance

from a surface, the ΔT term is calculated between the surface and the bulk temperature of the mobile phase.

1.1.3 Radiation

The concept for radiation is that all materials are constantly emitting infrared radiation that is absorbed by other materials. For this module, we will assume that radiation is emitted directly outward from the surface of objects. While conduction and convection are driven by a temperature gradient, radiation is only based on the temperature of the object emitting radiation.

The rate of heat transfer due to radiation can be described by

$$\text{Radiation Emitted: } q_{out} = \epsilon A \sigma (T_s^4)$$

$$\text{Radiation Absorbed: } q_{in} = \alpha A \sigma (T_o^4)$$

Heat transfer is the exchange of thermal energy between physical systems. The rate of heat transfer is dependent on the temperatures of the systems and the properties of the intervening medium through which the heat is transferred. The three fundamental modes of heat transfer are *conduction*, *convection* and *radiation*.

Heat transfer, the flow of energy in the form of heat, is a process by which a system changes its internal energy, hence is of vital use in applications of the First Law of Thermodynamics. Conduction is also known as diffusion, not to be confused with diffusion related to the mixing of constituents of a fluid. The direction of heat transfer is from a region of high temperature to another region of lower temperature, and is governed by the Second Law of Thermodynamics. Heat transfer changes the internal energy of the systems from which and to which the energy is transferred. Heat transfer will occur in a direction that increases the entropy of the collection of systems. Thermal equilibrium is reached when all involved bodies and the surroundings reach the same temperature. Thermal expansion is the tendency of matter to change in volume in response to a change in temperature.

Newton's law of cooling states that *the rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings*. As such, it is equivalent to a statement that the heat transfer coefficient, which mediates between heat losses and temperature

differences, is a constant. This condition is generally true in thermal conduction (where it is guaranteed by Fourier's law), but it is often only approximately true in conditions of convective heat transfer, where a number of physical processes make effective heat transfer coefficients somewhat dependent on temperature differences. Finally, in the case of heat transfer by thermal radiation, Newton's law of cooling is not true.

Thermal conductivity (often denoted k , λ , or κ) is the property of a material to conduct heat. It is evaluated primarily in terms of Fourier's Law for heat conduction. Heat transfer occurs at a lower rate across materials of low thermal conductivity than across materials of high thermal conductivity. Correspondingly, materials of high thermal conductivity are widely used in heat sink applications and materials of low thermal conductivity are used as thermal insulation. The thermal conductivity of a material may depend on temperature. The reciprocal of thermal conductivity is called thermal resistivity. Thermal conductivity is actually a tensor, which means it is possible to have different values in different directions.

Table 1: Thermal conductivity at room temperature for some metals and non-metals

Metals		Ag	Cu	Al	Fe	Steel	
k [W/m-K]		420	390	200	70	50	
Non-metals	H ₂ O	Air	Engine oil	H ₂	Brick	Wood	Cork
k [W/m-K]	0.6	0.026	0.15	0.18	0.4 - 0.5	0.2	0.04

1.2 Effect of temperature on thermal conductivity

Thermal conductivity is the physical property of the substance. It depends upon temperature gradient. For pure metals, thermal conductivity decreases with an increase in temperature. For gases and insulators, thermal conductivity increases with an increase in temperature. For small ranges of temperature, k may be considered constant. For larger temperature ranges, thermal conductivity can be approximated by an equation of the form $k = a + bT$, where a and b are empirical constants.

Steady-State Conduction It is the form of conduction which happens when the temperature difference driving the conduction is constant so that after an equilibrium time, the spatial distribution of temperatures (temperature field) in the conducting object does not change any further. In steady state conduction, the amount of heat entering a section is equal to amount of

heat coming out. **Unsteady state conduction** It is the form of conduction which happens when the temperature difference driving the conduction is not constant so that after an equilibrium time, the spatial distribution of temperatures (temperature field) in the conducting object changes as a function of time. Heat flux is denoted as q/A and it is defined as the rate of heat flow passing through a material per cross-sectional area and its unit is w/m^2 .

Silver is the material which possess highest thermal conductivity and being a solid it is composed of closed packing arrangement and due to this more molecular interactions within the molecules and hence the thermal conductivity is high.

Thermal conductivity is ability of a material to transport heat energy through it from high temperature region to low temperature region. The heat energy, Q , transported across a plane of area A in presence of a temperature gradient $\Delta T/\Delta l$ is given where k is the thermal conductivity of the material. It has units as $W/m.K$. It is a microstructure sensitive property. • Its value range o for metals 20-400 for ceramics 2-50 for polymers order of 0.3 Mechanisms - Thermal conductivity

Heat is transported in two ways – electronic contribution, vibrational (phonon) contribution. In metals, electronic contribution is very high. Thus metals have higher thermal conductivities. It is same as electrical conduction. Both conductivities are related through Wiedemann-Franz law where L – Lorentz constant (5.5×10^{-9} cal.ohm/sec.K²) • As different contributions to conduction vary with temperature, the above relation is valid to a limited extension for many metals. • With increase in temperature, both number of carrier electrons and contribution of lattice vibrations increase. Thus thermal conductivity of a metal is expected to increase. However, because of greater lattice vibrations, electron mobility decreases. The combined effect of these factors leads to very different behavior for different metals. Eg.: thermal conductivity of iron initially decreases then increases slightly; thermal conductivity decreases with increase in temperature for aluminium; while it increases for platinum.

1.3.1 HEAT TRANSFER THROUGH A PLANE WALL

Let us consider a plane wall of thickness L , thermal conductivity k , inside surface temperature T_i , outside surface temperature T_o . Let Q be the rate of heat transferred through the plane wall.

By Fourier's law of heat conduction

$$Q = -kA \frac{dT}{dx}$$

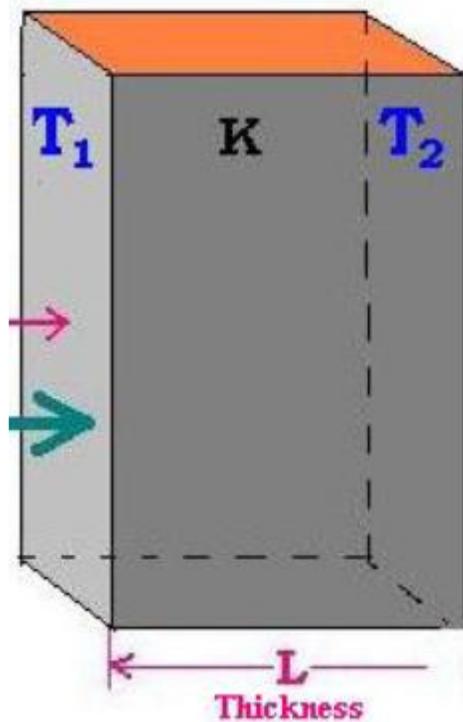


Fig. 1: Heat flow through a metal wall

$$Q \int_0^L dx = -k A \int_{T_i}^{T_o} dT$$

On integrating the above eqn, $Q = KA(T_i - T_o) / L$

$Q = KA \Delta T / L$ i.e $Q = \Delta T / (KA/L)$ where $KA/L = R_{th}$ where R_{th} – thermal resistance measured in $^{\circ}C / \text{Watts}$ or K / Watts . Hence $Q = \Delta T / R_{th}$.

1.3.2 HEAT TRANSFER THROUGH A HOLLOW CYLINDER

Let us consider a hollow cylinder. The inside radius of the cylinder is r_1 , the outside radius is r_2 , and the length of the cylinder is L . The thermal conductivity of the material of which the cylinder is made is k . The temperature of the outside surface is T_2 , and that of the inside surface is T_1 .

By Fourier’s law of heat conduction,

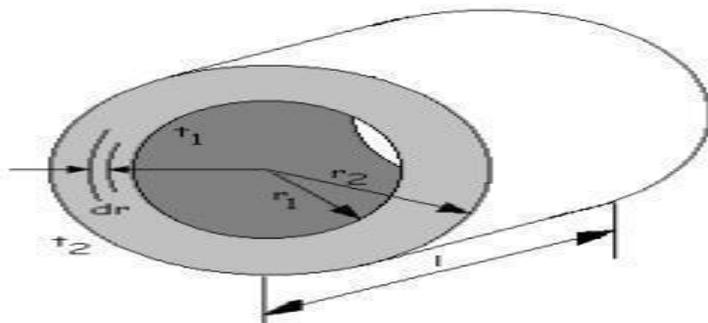


Fig. 2 Heat flow through a cylinder

$$Q = -kA \frac{dT}{dr}$$

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -k 2\pi L \int_{T_1}^{T_2} dT$$

integrating the above eqn,

$$Q = \frac{2\pi kL (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

1.3.2.1 LOGARITHMIC MEAN RADIUS AND ARITHMETIC MEAN RADIUS

Logarithmic mean radius is the radius that when applied to the integrated equation for a flat wall, will give the correct rate of heat flow through a thick walled cylinder. It is given by the expression

$$\bar{r}_L = \frac{r_o - r_i}{\ln\left(\frac{r_o}{r_i}\right)}$$

where \bar{r}_L is the logarithmic mean radius of the cylinder

r_o is the outer radius of the pipe and

r_i is the inner radius of the pipe.

Hence using the above expression $Q = 2\pi k L (T_i - T_o) * (r_o - r_i) / \ln (r_o/r_i) *(r_o - r_i)$

Using the Logarithmic mean radius expression in above, we get

$$Q = 2\pi k L \Delta T * r_{lm} / (r_o - r_i) \text{ hence, } Q = \Delta T / (r_o - r_i) / A_{lm} * k$$

Where $A_{lm} = 2\pi L * r_{lm}$, A_{lm} is the logarithmic mean area which is used for thin cylinders.

1.3.3 COMPOUND RESISTANCES IN SERIES

(I) Heat Transfer Through A Composite Plane Wall

Let us consider a flat wall constructed of a series of 3 layers as shown. Let the thickness of the layers be L_1, L_2, L_3 and the average thermal conductivities of the materials of which the layers are made be k_1, k_2, k_3 respectively. Let us consider a hot fluid at a temperature T_a and heat transfer coefficient h_a inside the wall and cold fluid at a temperature T_b and heat transfer coefficient h_b outside the wall. Let T_1, T_2, T_3 and T_4 be the interface temperatures. It is desired to derive an equation for calculating the rate of heat flow through the series of resistances.

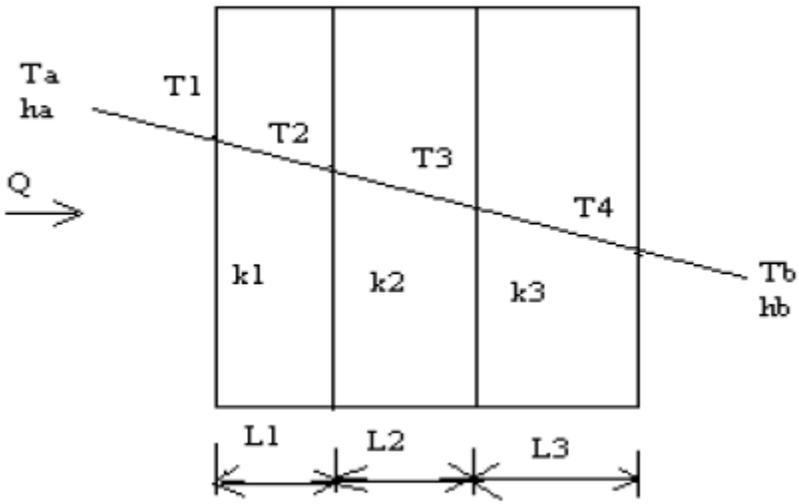


Fig. 3 Heat flow through a composite wall

Rate of heat flow from the hot fluid to the inner surface of the wall

By Newton's law of cooling

$$Q = h_a A (T_a - T_1)$$

By rearranging the above eqn, we get

$$Q = \frac{(T_a - T_1)}{\frac{1}{h_a A}}$$

Rate of heat flow through the I layer, by Fourier's law of heat conduction,

$$Q = KA(T_1 - T_2) / L \quad \text{On rearranging this,}$$

$$Q = (T_1 - T_2) / (L_1 / K_1 \cdot A)$$

Rate of heat flow through the II layer,

$$Q = (T_2 - T_3) / (L_2 / K_2 \cdot A)$$

Rate of heat flow through the III layer,

$$Q = (T_3 - T_4) / (L_3 / K_3 \cdot A)$$

Rate of heat flow from outer surface of the wall to the cold fluid By Newton's law of cooling

$$Q = h_b A (T_4 - T_b) \text{ and } Q = (T_4 - T_b) / 1/h_b A$$

Overall rate of heat flow = overall thermal resistance / overall temperature drop

Overall rate of heat flow

$$Q = \frac{(T_a - T_b)}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}}$$

In steady state heat conduction through a composite wall, this can be written as

$$Q = (T_1 - T_4) / (L_1/K_1.A + L_2/K_2.A + L_3/K_3.A)$$

i.e $Q = \Delta T / R_{th1} + R_{th2} + R_{th3}$

Hence, $Q = \Delta T / \sum R_{th}$

Rate of heat flow = $\frac{\text{Temperature difference}}{\text{Resistance}}$

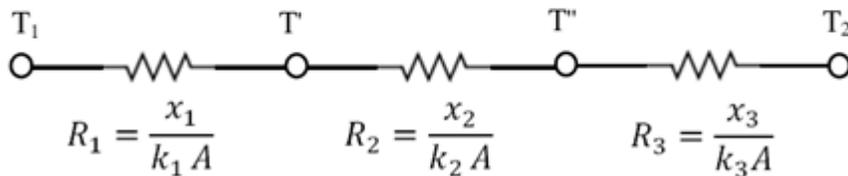


Table 2 Various flows and their driving forces

Flow	Driving force
Electricity flow	Electric potential gradient
Fluid flow	Pressure gradient
Heat flow	Temperature gradient

It was observed in the previous discussion that for the given plane wall the area for heat transfer was constant along the heat flow direction. The plane solid wall was one of the geometries but if we take some other geometry (tapered plane, cylindrical body, spherical body etc.) in which the area changes in the direction of heat flow. Now we will consider geometrical configuration which will be mathematically simple and also of great engineering importance like hollow cylinder and hollow sphere. In these cases the heat transfer area varies in the radial direction of heat conduction. We will take up both the cases one by one in the following sections.

1.4) Heat transfer through co-axial cylinder Provided with one layer of insulation

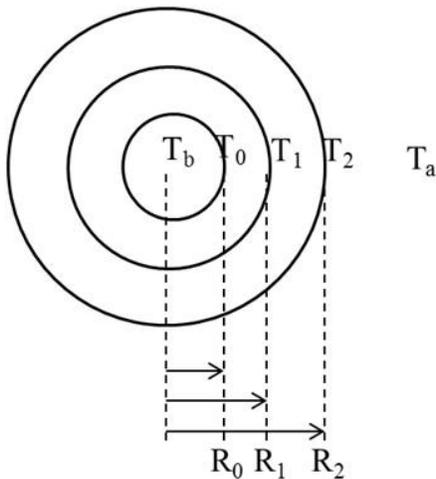


Fig. 4 Heat flow through a composite cylinder

Let us consider coaxial cylinders constructed of a series of 3 layers as shown in fig. Let R_0 , R_1 and R_2 be the radii of the cylinders and the average thermal conductivities of the materials of which the layers are made be k_1 and k_2 respectively. Let us consider a hot fluid at a temperature T_b and heat transfer coefficient h_b inside the cylinder and cold fluid at a temperature T_a and heat transfer coefficient h_a outside the cylinder. Let T_0 , T_1 and T_2 be the interface temperatures. It is desired to derive an equation for calculating the rate of heat flow through the composite cylinder provided with series of resistances.

Consider a very thin hollow cylinder of thickness d_r in the main geometry (fig.2.9a) at a radial distance r . If d_r is small enough with respect to r , then the area of the inner and outer surface of the thin cylinder may be considered to be of same area. In other words, for very small d_r with respect to r , the lines of heat flow may be considered parallel through the differential element in radial outward direction.

We may ignore the heat flow through the ends if the cylinder is sufficiently large. We may thus eliminate any dependence of the temperature on the axial coordinate and for one dimensional steady state heat conduction, the rate of heat transfer for the thin cylinder,

$$\dot{q} = -k \frac{dT}{dr} 2\pi r L$$

Where dT is the temperature difference between the inner and outer surface of the thin cylinder considered above and k is the thermal conductivity of the cylinder.

On rearranging,

$$\frac{dr}{r} = -\frac{2\pi L k}{\dot{q}} dT$$

To get the heat flow through the thick wall cylinder, the above equation can be integrated between the limits,

$$\int_{r_i}^{r_o} \frac{dr}{r} = -\frac{2\pi L k}{\dot{q}} \int_{T_i}^{T_o} dT$$

On solving,

$$\dot{q} = k(2\pi L) \frac{(T_i - T_o)}{\ln(r_o/r_i)}$$

$$\dot{q} = kA_{LM} \frac{(T_i - T_o)}{r_o - r_i}$$

Rate of heat flow from the hot fluid to the inner surface of the wall

By Newton's law of cooling,

$$Q = hb A (T_b - T_o)$$

$$Q = hb 2 \pi RL (T_b - T_o)$$

By rearranging the above,

$$Q = (T_b - T_o) / 1 / 2\pi R_o L$$

Rate of the heat flow through the cylinder By Fourier's law of heat conduction

$$Q = \frac{2\pi kL (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

using the above conditions from the fig,

$$Q = 2\pi k_1 L (T_o - T_1) / \ln (R_1/R_o)$$

By rearranging the above,

$$Q = (T_o - T_1) / \ln (R_1/R_o) / 2\pi k_1 L$$

Rate of the heat flow through the II layer By Fourier's law of heat conduction

$$Q = (T_1 - T_2) / \ln (R_2/R_1) / 2\pi k_2 L$$

Rate of heat flow from the outer surface of the wall to the cold fluid

By Newton's law of cooling

$$Q = ha A (T_2 - T_a)$$

$$Q = ha 2\pi R_2 L (T_2 - T_a) \text{ on rearranging, } Q = (T_2 - T_a) / 1 / ha 2\pi R_2 L$$

Overall rate of heat flow = *overall thermal resistance / overall temperature drop*

Hence overall rate of heat flow is given by

$$Q = (T_b - T_a) / (1/h_b + R_o) + (\ln(R_1/R_o) / 2\pi k_1 L) + (\ln(R_2/R_1) / 2\pi k_2 L) + (1/h_a + R_2/L)$$

For steady state conduction, heat transfer coefficients can be neglected and the heat flow is given by

$$Q = (T_b - T_a) / (\ln(R_1/R_o) / 2\pi k_1 L) + (\ln(R_2/R_1) / 2\pi k_2 L)$$

Hence $Q = \Delta T / R_{th1} + R_{th2}$

Where $R_{th1} = (\ln(R_1/R_o) / 2\pi k_1 L)$ and $R_{th2} = (\ln(R_2/R_1) / 2\pi k_2 L)$

$$Q = \Delta T / \sum R_{th}$$

1.5 Insulation

The addition of insulation material on a surface reduces the amount of heat flow to the ambient. There are certain instances in which the addition of insulation to the outside surface of cylindrical or spherical walls does not reduce the heat loss. Under certain circumstances it actually increases the heat loss up to a certain thickness of insulation. It is a well known fact that the rate of heat transfer will approach zero if an infinite amount of insulation is added. This means that there must be a value of radius for which the rate of heat transfer is maximum. This value is known as the critical radius of insulation, r_c .

Variation of thermal conductivity with respect to solids, liquids and gases

In solids, heat conduction is due to two effects: the lattice vibrational waves induced by the vibrational motions of the molecules positioned at relatively fixed positions in a periodic manner called a lattice, and the energy transported via the free flow of electrons in the solid. • The thermal conductivity of a solid is obtained by adding the lattice and electronic components. The relatively high thermal conductivities of pure metals are primarily due to the electronic component. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, diamond, which is a highly ordered crystalline solid has the highest known thermal conductivity at room temperature. Conduction crystalline solid, has the highest known thermal conductivity at room temperature.

The thermal conductivities of materials vary with temperature. The variation of thermal conductivity over certain temperature ranges is negligible for some materials, but significant for others. The thermal conductivities of certain solids exhibit dramatic increases at temperatures near absolute zero, when these solids become superconductors.

5.1 CRITICAL RADIUS OF INSULATION IN PIPES

Let us consider an insulating layer in the form of a hollow cylinder of length L . Let r_i and r_o be the inner and outer radii of insulation. The thermal conductivity of the material of which the layer is made be k . Let the inside surface of insulation be at a temperature T_i , and the outside surface at a temperature T_o be dissipating heat by convection to the surroundings at a temperature T_b with a heat transfer coefficient h . Then the rate of heat transfer Q through this insulation layer is

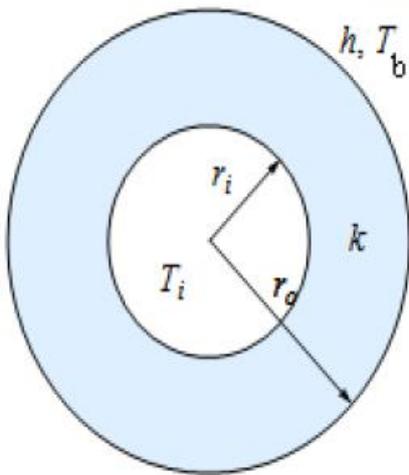


Fig. 5 Heat flow through a cylindrical pipe for critical radius

$$Q = \frac{2\pi L (T_i - T_b)}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{hr_o}} \quad (1)$$

The value of critical radius r_c , that is r_o for which Q is a maximum may be obtained by equating dQ/dr_o to zero.

$$\frac{dQ}{dr_o} = \frac{0 - (T_i - T_b) \left[\frac{1}{2\pi k L r_o} - \frac{1}{2\pi h L r_o^2} \right]}{\left[\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{2\pi h L r_o} \right]^2} \quad (2)$$

$(T_i - T_b) \neq 0$ (Since it is the driving force)

$$\therefore \frac{1}{2\pi k L r_o} - \frac{1}{2\pi h L r_o^2} = 0 \quad (3)$$

$$r_o = \frac{k}{h} = r_c$$

The radius at which the rate of heat transfer is maximum is known as the critical radius of insulation.

1.6 VARIABLE THERMAL CONDUCTIVITY

Let us a hollow cylinder. The inside radius of the cylinder is r_i , the outside radius is r_o , and the length of the cylinder is L . The thermal conductivity of the material of which the cylinder is varies with temperature as $k = k_o(1 + \alpha T)$. The temperature of the outside surface is T_o and that of the inside surface is T_i . This can be used with many equations such as

$$k = k_o (\alpha + \beta T)$$

$$k = k_o (\alpha + \beta T + \gamma T^2)$$

$$k = k_o (a + bT)$$

$$k = k_o (a + bT + cT^2)$$

By Fourier's law of heat conduction

$$Q = -kA \frac{dT}{dr} \quad (1)$$

$$Q = -k_0 (1 + \beta T) A \frac{dT}{dr}$$

$$Q \int_{r_i}^{r_o} \frac{dr}{r} = -k_0 2\pi L \int_{T_i}^{T_o} (1 + \beta T) dT \quad (2)$$

$$Q \ln \frac{r_o}{r_i} = k_0 2\pi L \left(1 + \beta \frac{[T_i + T_o]}{2} \right) (T_i - T_o)$$

$$Q = \frac{k_0 2\pi L \left(1 + \beta \frac{[T_i + T_o]}{2} \right) (T_i - T_o)}{\ln \frac{r_o}{r_i}} \quad (3)$$

1.7 Introduction to Unsteady state heat transfer

A solid body is said to be in a steady state if its temperature does not vary with time. If however there is an abrupt change in its surface temperature or environment it takes some time before the body to attain an equilibrium temperature or steady state. During this interim period the temperature varies with time and the body is said to be in an unsteady or transient state. The analysis of unsteady state heat transfer is of great interest to engineers because of its widespread occurrence such as in boiler tubes, rocket nozzles, automobile engines, cooling of IC engines, cooling and freezing of food, heat treatment of metals by quenching, etc. For practical purposes it is necessary to know the time taken to attain a certain temperature when the environment suddenly changes. The solution of an unsteady state problem will be more complex than that of steady state one because of the presence of another variable time, t .

Transient heat conduction problems can be divided into periodic heat flow and non periodic heat flow problems. Periodic heat flow problems are those in which the temperature varies on a regular basis, eg., the variation of temperature of the surface of the earth during a twenty

four hour period.. In the non periodic type, the temperature at any point within the system varies non linearly with time.

Introduction to this point, we have considered conductive heat transfer problems in which the temperatures are independent of time. In many applications, however, the temperatures are varying with time, and we require the understanding of the complete time history of the temperature variation. For example, in metallurgy, the heat treating process can be controlled to directly affect the characteristics of the processed materials. Annealing (slow cool) can soften metals and improve ductility. On the other hand, quenching (rapid cool) can harden the strain boundary and increase strength. In order to characterize this transient behavior, the full unsteady equation is needed.

$$\frac{1}{a} \cdot \frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k}$$

Where $\alpha = \frac{k}{\rho c}$ is the thermal diffusivity. Without any heat generation and considering spatial variation of temperature only in x-direction, the above equation reduces to:

1.8 Systems with negligible internal resistance – Lumped Heat Analysis

Heat transfer in heating and cooling of a body is dependent upon both the internal and surface resistances. The simplest unsteady state problem is one in which the internal resistance is negligible, that is, the convective resistance at the surface boundary is very large when compared to the internal resistance due to conduction. In other words, the solid has an infinite thermal conductivity so that there is no variation of temperature inside the solid and temperature is a function of time only. This situation cannot exist in reality because all the solids have a finite thermal conductivity and there will always be a temperature gradient inside whenever heat is added or removed. Problems such as heat treatment of metals by quenching, time response of thermocouples and thermometers, etc can be analyzed by this idealization of negligible internal resistance. The process in which the internal resistance is ignored being negligible in comparison with its surface resistance is called the Newtonian heating and cooling process. In Newtonian heating and cooling process the temperature

throughout the solid is considered to be uniform at a given time. Such an analysis is called the lumped heat capacity analysis.

1.9 Systems with negligible surface resistance

Another class of transient problems met with in practice is one in which the surface resistance is negligible compared to the overall resistance. This amounts to saying that the convective heat transfer coefficient at the surface is infinity. For such a process the surface temperature remains constant for all the time and its value is equal to that of ambient temperature.

Dimensionless parameters:

$$\frac{T_i - T_s}{T_s - T_\infty} = \frac{\bar{h}L}{k} = \text{Biot number}$$

$$\mathbf{Bi} = \frac{\mathbf{resistance\ to\ internal\ heat\ flow}}{\mathbf{resistance\ to\ external\ heat\ flow}}$$

The Biot number is dimensionless, and it can be thought of as the ratio to the internal and external heat flows. Whenever the Biot number is small, the internal temperature gradients are also small and a transient problem can be treated by the “lumped thermal capacity” approach. The lumped capacity assumption implies that the object for analysis is considered to have a single mass-averaged temperature.

In general, a characteristic length scale may be obtained by dividing the volume of the solid by its surface area: $L = V/A_s$

Using this method to determine the characteristic length scale, the corresponding Biot number may be evaluated for objects of any shape, for example a plate, a cylinder, or a sphere. As a thumb rule, if the Biot number turns out to be less than 0.1, lumped capacity assumption is applied. In this context, a *dimensionless time*, known as the Fourier number, can be obtained by multiplying the dimensional time by the thermal diffusivity and dividing by the square of the characteristic length:

$$\text{dimensionless time} = \frac{\alpha t}{L^2} = \text{Fo}$$

Lumped heat capacity

analysis: temperature distribution inside or outside the solid is neglected.

The cases considered so far have been those in which the heat conducting solid is free of internal heat generation. However, the situations where the internal heat is generated are very common cases in chemical industries for example, the exothermic reaction in the solid pallet of a catalyst.

We have learnt that how the Fourier equation is used for the steady-state heat conduction through the composites of three different geometries that were not having any heat source in it. However, the heat generation term would come into the picture for these geometries. It would not be always easier to remember and develop heat conduction relations for different standard and non-standard geometries. Therefore, at this point we should learn how to develop a general relation for the heat conduction that should be applicable to the entire situation such as steady-state, unsteady state, heat source, different geometry, heat conduction in different direction, etc. Again here we will consider that the solid is isotropic in nature, which means the thermal conductivity of the material is same in all the direction of heat flow.

To get such a general equation the differential form of the heat conduction equation is most important. For simplicity, we would consider an infinitesimal volume element in a Cartesian coordinate system. The dimensions of the infinitesimal volume element are d_x , d_y , and d_z in the respective direction as shown in the fig

The simplest situation in an unsteady heat transfer process is to use the lump assumption, wherein we neglect the temperature distribution inside the solid as with the heat transfer between the solid and the ambient fluids. In other we assuming that the temperature inside the solid is constant and is equal to temperature.

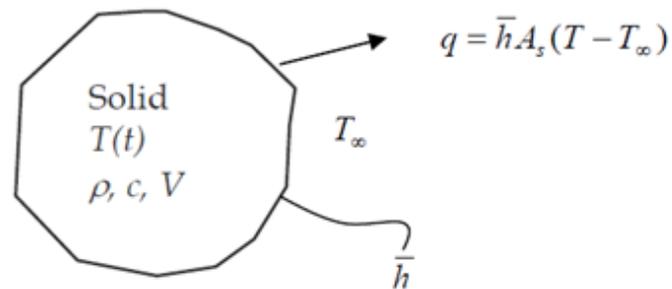


Fig. 6

The solid object shown in figure 5.2 is a metal piece which is being cooled in forming. Thermal energy is leaving the object from all elements of the surface shown for simplicity by a single arrow. The first law of thermodynamics applied to this problem is

$$\left(\begin{array}{l} \text{heat out of object} \\ \text{during time } dt \end{array} \right) = \left(\begin{array}{l} \text{decrease of internal thermal} \\ \text{energy of object during time } dt \end{array} \right)$$

Now, if Biot number is small and temperature of the object can be considered to be uniform, this equation can be written as

$$\bar{h}A_s[T(t) - T_\infty]dt = -\rho cVdT$$

or,

$$\frac{dT}{(T - T_\infty)} = -\frac{\bar{h}A_s}{\rho cV} dt$$

Integrating and applying the initial condition $T(0) = T_i$,

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{\bar{h}A_s}{\rho cV} t$$

Taking the exponents of both sides and rearranging,

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

where

$$b = \frac{\bar{h}A_s}{\rho cV} \quad (1/s)$$

$$b = \frac{\bar{h}A_s}{\rho cV} \quad (1/s)$$

b is a positive quantity having dimension (time)⁻¹. The reciprocal of b is usually called *time constant*, which has the dimension of time.

1.10 Heat conduction through spherical surface 6

Heat Conduction in a Spherical Shell

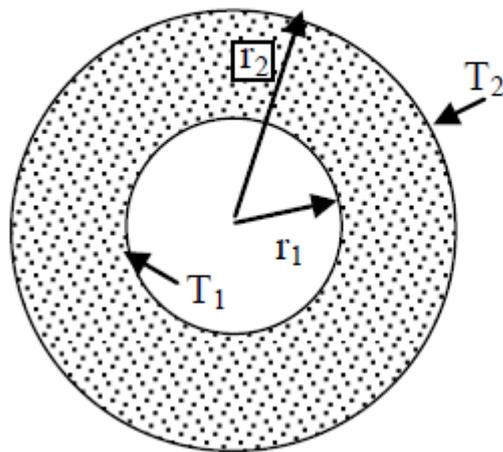


Fig. 7 Heat flow through a spherical surface

Consider a spherical shell with inside radius r_1 and outside radius r_2 . Let T_1 be the inside temperature and T_2 be the outside temperature. K be the thermal conductivity of the material. Q be the heat flow through the spherical shell with a cross sectional area to be A . based on Fourier's law of heat conduction and following the assumptions,

- (i) Heat flow is uniform
- (ii) Heat flow is normal to the surface
- (iii) The material is uniform and possess constant thermal conductivity
- (iv) Heat flow is uni-directional

Let us consider a differential element of thickness dr which is lying between inside and outside radius. For such an element the heat flow is given by

$Q = -kA \, dT/dr$ where area of the sphere is $A = 4\pi r^2$

Substituting the area of a sphere

Integrating, between $r = r_1$ and r_2 , and T_1 and T_2 ,

$$\frac{q_r}{4\pi} \left| -\frac{1}{r} \right|_{r_1}^{r_2} = -k \left| T \right|_{T_1}^{T_2}$$

$$\frac{q_r (r_2 - r_1)}{4\pi r_1 r_2} = -k (T_2 - T_1)$$

$$q_r = \frac{4\pi k r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}$$

The thermal resistance is expressed as

$Q = \Delta T / (r_2 - r_1) / 4\pi k r_1 r_2$ where the resistance is given by $(r_2 - r_1) / 4\pi k r_1 r_2$

Geometric mean radius is given by $r_m^2 = r_1 r_2$ and hence

$R_{th} = (r_2 - r_1) / A_m \cdot k$ where $A_m = 4\pi r_m^2$ and A_m is the geometric mean area.

Similarly, rate of heat flow for a composite spherical shell with one layer of insulation is given by $Q = \Delta T / [(r_2 - r_1) / 4\pi k_1 r_1 r_2 + (r_3 - r_2) / 4\pi k_2 r_2 r_3]$

In a plane wall the area perpendicular to the direction of heat flow adding more insulation to a wall always decreases heat transfer. If thicker is insulation, the lower is the heat transfer rate. This is due to the fact the outer surface have always the same area.

But in cylindrical and spherical coordinates, the addition of insulation also increases the outer surface, which decreases the convection resistance at the outer surface. Moreover, in some cases, a decrease in the convection resistance due to the increase in surface area can be more important than an increase in conduction resistance due to thicker insulation. As a result the total resistance may actually decrease resulting in increased heat flow.

The thickness upto which heat flow increases and after which heat flow decreases is termed as critical thickness. In the case of cylinders and spheres it is called critical radius. It can be derived the critical radius of insulation depends on the thermal conductivity of the insulation k and the external convection heat transfer coefficient h .

Optimum thickness of insulation

The radius of insulation at which the total annual cost is minimum corresponds to optimum thickness of insulation.

Assume a steel pipe of $r_1 = 10$ mm, which is exposed to natural convection at $h = 50$ W/m².K. This pipe is insulated by material of thermal conductivity $k = 0.5$ W/m.K. Determine the critical thickness of this combination:

$$r_{cr} = \frac{k}{h} = \frac{0.5}{50} = 0.01 \text{ m}$$

where

k is the materials conductivity [W.m⁻¹.K⁻¹]

h is the heat transfer coefficient [W.m⁻²]

Hence $r_{cr} > r_1$ and heat transfer will increase with the addition of insulation up to a thickness of $r_{cr} - r_1 = (0.010 - 0.005)\text{m} = 0.005$ m

P.No. 1. A furnace wall consists of two layers, 22.5cm of fire brick($k=1.2\text{kcal/hr m}^\circ\text{C}$)and 12.5cm of insulating brick ($k=0.15\text{kcal/hr m}^\circ\text{C}$) . The temperature inside the furnace is 1650°C and the inside heat transfer coefficient is $60\text{kcal/hr m}^\circ\text{C}$. The temperature of the surrounding atmosphere is 27°C and the outside heat transfer coefficient is $10\text{kcal/hr m}^2^\circ\text{C}$. Determine the rate of heat of loss per square meter of the wall.

Solution:

$$L_1 - 22.5 \times 10^{-2} \text{ m}$$

$$L_2 - 12.5 \times 10^{-2} \text{ m}$$

$$k_1 - 1.2 \text{ kcal/hr m}^\circ\text{C}$$

$$k_2 - 0.15 \text{ kcal/hr m}^\circ\text{C}$$

$$h_a - 60 \text{ kcal/hr m}^2^\circ\text{C}$$

$$h_b - 10\text{kcal/hr m}^2^\circ\text{C}$$

$$T_a - 1650^\circ\text{C}$$

$$T_b - 27^\circ\text{C}$$

$$Q = \frac{(T_a - T_b)}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_b A}}$$

$$Q / A = 1426.8 \text{ W / m}^2$$

P.No.2. A pipe carrying steam at 220°C has an I.D. of 15cm. The convection coefficient on the inside wall is $60\text{W/m}^2\text{K}$. The pipe wall thickness is 15mm and the thermal conductivity is 35W/mK . The outside is exposed to a chemical at 130°C with a convection coefficient of $15\text{W/m}^2\text{K}$. If the pipe wall is covered with two insulation layers, the first 3cm thickness with $k=0.12\text{W/mK}$ and the second 4cm thickness with $k= 0.35\text{W/m K}$. Determine the rate of heat transfer.

Solution :

$$r_1 - 75 \times 10^{-3} \text{ m}$$

$$r_2 - 90 \times 10^{-3} \text{ m}$$

$$r_3 - 120 \times 10^{-3} \text{ m}$$

$$r_4 - 160 \times 10^{-3} \text{ m}$$

$$k_1 - 35 \text{ W / m K}$$

$$k_2 = 0.12 \text{ W / m K}$$

$$k_3 = 0.35 \text{ W / m K}$$

$$h_a = 60 \text{ W / m}^2 \text{ K}$$

$$h_b = 15 \text{ W / m}^2 \text{ K}$$

$$Q = \frac{2\pi L (T_a - T_b)}{\frac{1}{h_a r_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_3} + \frac{1}{h_b r_4}}$$

$$Q = 146.32 \text{ W}$$

1.11 Analogy between heat flow and electricity:

Heat flow is represented by Fourier's law of heat conduction whereas electrical flow is represented by ohm's law.

Heat flow = Temperature gradient / thermal resistance

Ohm's law is given by

Electrical flow = voltage drop / electrical resistance

The various parameters analogous to each other in both the laws are heat flow and electrical flow, voltage drop with temperature gradient, thermal and electrical resistance.



SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE

www.sathyabama.ac.in

SCHOOL OF BIO & CHEMICAL ENGINEERING

DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – II – Heat Transfer – SCHA1301

2.0 Convective Heat transfer

When a current or macroscopic particle of fluid crosses a specific surface, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called convection. Convection can refer to the flow of heat associated with the movement of fluid, such as when hot air from a furnace enters a room, or to the transfer of heat from a hot surface to a flowing fluid. The two types of convection are Natural convection and forced convection.

2.1 Natural convection: If the convection currents are the result of buoyancy forces generated by the differences in density and the differences in density are in turn caused by temperature gradients in the fluid mass, the action is called natural convection.

2.2 Forced convection: If the convection currents are set in motion by the action of a mechanical device such as pump or agitator, the flow is independent of density gradients, it is called forced convection. **Heat flux, average temperature of fluid stream** Heat flux: Heat transfer calculations are based on the area of the heating surface area. The rate of heat transfer per unit area is called the heat flux. **Bulk mean temperature:** When a fluid is heated or cooled the temperature will vary throughout the cross section of the stream. Because of these temperature gradients through out the stream it is necessary to state what is meant by the temperature of the stream. It is the temperature that will be at a fluid stream flowing across the section are withdrawn and mixed adiabatically to a uniform temperature. This is also called as average or mixing cup stream temperature.

Film temperature: It is the average between the temperature of the surface and the fluid. indeed if the entire Overall Heat Transfer Coefficient Let us consider a plane wall of thickness x_w and thermal conductivity k_w . The warm fluid at a mean temperature of T_h is flowing through the inside surface of the wall. The cold fluid at a mean temperature of T_c is flowing through the outside surface of the wall. The inside surface temperature is T_{wh} and outside surface temperature is T_{wc} . The overall heat transfer coefficient is constructed from the individual coefficients and the resistances of the wall in the following manner. (i) Overall Heat transfer Coefficient based on outside surface area. The rate of heat transfer from the warm fluid to the inner surface of the wall in differential form:

$$\frac{dq}{dA_i} = h_i(T_h - T_{wh})$$

By rearranging the above,

$$dq = \frac{(T_h - T_{wh})}{\frac{1}{h_i dA_i}}$$

. The rate of heat transfer through the wall in differential form is given by

$$\frac{dq}{dA_L} = \frac{k_w(T_{wh} - T_{wc})}{x_w}$$

on rearranging,

$$dq = \frac{(T_{wh} - T_{wc})}{\frac{x_w}{k_w dA_L}}$$

the rate of heat transfer from the outer surface of the wall to the cold fluid in differential form is given by

$$\frac{dq}{dA_o} = h_o(T_{wc} - T_c)$$

on rearranging,

$$dq = \frac{(T_{wc} - T_c)}{\frac{1}{h_o dA_o}}$$

If the eqns are solved for the temperature differences and the temperature differences added, the result is

$$(T_h - T_{wh}) + (T_{wc} - T_c) + (T_{wc} - T_c) = T_h - T_c = \Delta T = dQ \left(\frac{1}{h_i dA_i} + \frac{x_w}{k_w dA_L} + \frac{1}{h_o dA_o} \right)$$

Assume that the heat transfer rate is arbitrarily based on the outside area. If the eqn is solved for dQ, and if both sides of the resulting equations are divided by dAo, the result is

$$\frac{dQ}{dA_o} = \frac{T_h - T_c}{\frac{1}{h_i} \left(\frac{dA_o}{dA_i} \right) + \frac{x_w}{k_w} \left(\frac{dA_o}{dA_L} \right) + \frac{1}{h_o} \left(\frac{dA_o}{dA_o} \right)}$$

$$\frac{dA_o}{dA_i} = \frac{r_o}{r_i} ; \quad \frac{dA_o}{dA_L} = \frac{r_o}{r_L}$$

Then the above Eqn becomes

$$\frac{dQ}{dA_o} = \frac{T_h - T_c}{\frac{1}{h_i} \left(\frac{r_o}{r_i} \right) + \frac{r_o}{k_w} \ln \left(\frac{r_o}{r_i} \right) + \frac{1}{h_o}}$$

2.3 Overall heat transfer coefficient based on inside surface area The rate of heat transfer from the warm fluid to the inner surface of the wall in differential form:

$$\frac{dq}{dA_i} = h_i (T_h - T_{wh})$$

On rearranging,

$$dq = \frac{(T_h - T_{wh})}{\frac{1}{h_i dA_i}}$$

The rate of heat transfer through the wall in differential form

$$\frac{dq}{dA_L} = \frac{k_w (T_{wh} - T_{wc})}{x_w}$$

By rearranging the above,

$$dq = \frac{(T_{wh} - T_{wc})}{\frac{x_w}{k_w dA_L}}$$

The rate of heat transfer from the outer surface of the wall to the cold fluid in differential form:

$$\frac{dq}{dA_o} = h_o (T_{wc} - T_c)$$

On rearranging

$$dq = \frac{(T_{wc} - T_c)}{\frac{1}{h_o dA_o}}$$

If the eqns to are solved for the temperature differences and the temperature differences added, the result is

$$(T_h - T_{wh}) + (T_{wc} - T_c) + (T_{wc} - T_c) = T_h - T_c = \Delta T = dQ \left(\frac{1}{h_i dA_i} + \frac{x_w}{k_w dA_L} + \frac{1}{h_o dA_o} \right)$$

Assume that the heat transfer rate is arbitrarily based on the inside area. If the Eqn is solved for dQ, and if both sides of the resulting equations are divided by dAi, the result is

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i} \left(\frac{dA_i}{dA_i} \right) + \frac{x_w}{k_w} \left(\frac{dA_i}{dA_L} \right) + \frac{1}{h_o} \left(\frac{dA_i}{dA_o} \right)}$$

$$\frac{dA_i}{dA_o} = \frac{r_i}{r_o} ; \quad \frac{dA_i}{dA_L} = \frac{r_i}{r_L}$$

The above eqn becomes

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i} \left(\frac{r_i}{r_i} \right) + \frac{r_i}{k_w} \ln \left(\frac{r_o}{r_i} \right) + \frac{1}{h_o} \left(\frac{r_i}{r_o} \right)}$$

the overall heat transfer coefficient based on outside surface area is,

$$U_o = \frac{1}{\frac{1}{h_i} \left(\frac{r_o}{r_i} \right) + \frac{r_o}{k_w} \ln \left(\frac{r_o}{r_i} \right) + \frac{1}{h_o}}$$

Overall heat transfer coefficient based on inside surface area

The rate of heat transfer from the warm fluid to the inner surface of the wall in differential form

$$\frac{dq}{dA_i} = h_i (T_h - T_{wh})$$

On rearranging,

$$dq = \frac{(T_h - T_{wh})}{\frac{1}{h_i dA_i}}$$

The rate of heat transfer from the outer surface of the wall to the cold fluid in differential form:

$$\frac{dq}{dA_o} = h_o (T_{wc} - T_c)$$

on rearranging,

$$dq = \frac{(T_{wc} - T_c)}{\frac{1}{h_o dA_o}}$$

substituting the above equations,

$$(T_h - T_{wh}) + (T_{wc} - T_c) + (T_{wc} - T_c) = T_h - T_c = \Delta T = dQ \left(\frac{1}{h_i dA_i} + \frac{x_w}{k_w dA_L} + \frac{1}{h_o dA_o} \right)$$

Assume that the heat transfer rate is arbitrarily based on the inside area. If the eqn is solved for dQ, and if both sides of the resulting equations are divided by dAi, the result is

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i} \left(\frac{dA_i}{dA_i} \right) + \frac{x_w}{k_w} \left(\frac{dA_i}{dA_L} \right) + \frac{1}{h_o} \left(\frac{dA_i}{dA_o} \right)}$$

$$\frac{dA_i}{dA_o} = \frac{r_i}{r_o} ; \quad \frac{dA_i}{dA_L} = \frac{r_i}{r_L}$$

$$\frac{dQ}{dA_i} = \frac{T_h - T_c}{\frac{1}{h_i} \left(\frac{r_i}{r_i} \right) + \frac{r_i}{k_w} \ln \left(\frac{r_o}{r_i} \right) + \frac{1}{h_o} \left(\frac{r_i}{r_o} \right)}$$

Hence, the overall heat transfer coefficient based on inside area is

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k_w} \ln \left(\frac{r_o}{r_i} \right) + \frac{1}{h_o} \left(\frac{r_i}{r_o} \right)}$$

2.4 Fouling factors

In actual service, heat transfer surfaces do not remain clean. Scale, dirt and other solid deposits form on one or both the sides of the tubes, provide additional resistances to heat flow and reduce the overall coefficient. The effect of such deposits is taken into account as fouling factors in design calculation of heat exchangers. h_{di} , h_{do} are the fouling factors for the scale deposits on the inside and outside tube surfaces.

Overall heat transfer coefficient based on outside surface area

$$U_o = \frac{1}{\frac{1}{h_i} \left(\frac{r_o}{r_i} \right) + \frac{1}{h_{di}} \left(\frac{r_o}{r_i} \right) + \frac{r_o}{k_w} \ln \left(\frac{r_o}{r_i} \right) + \frac{1}{h_{do}} + \frac{1}{h_o}}$$

Overall heat transfer coefficient based on inside surface area

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{1}{h_{di}} + \frac{r_i}{k_w} \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_{do}} \left(\frac{r_i}{r_o}\right) + \frac{1}{h_o} \left(\frac{r_i}{r_o}\right)}$$

Concept of heat transfer by convection, natural and forced convection

When a current or macroscopic particle of fluid crosses a specific surface, it carries with it a definite quantity of enthalpy. Such a flow of enthalpy is called convection. Convection can refer to the flow of heat associated with the movement of fluid, such as when hot air from a furnace enters a room, or to the transfer of heat from a hot surface to a flowing fluid.

The two types of convection are Natural convection and forced convection.

Natural convection: If the convection currents are the result of buoyancy forces generated by the differences in density and the differences in density are in turn caused by temperature gradients in the fluid mass, the action is called natural convection.

Forced convection: If the convection currents are set in motion by the action of a mechanical device such as pump or agitator, the flow is independent of density gradients, it is called forced convection.

2.5 Newton's law of cooling

According to Newton's law of cooling, convective heat flux is proportional to the difference between the surface temperature and the temperature of the fluid.

$$Q = h A (T_s - T_f)$$

Where

Q = rate of heat transfer

h= heat transfer coefficient

A = heat transfer area

T = (T_s - T_f) = temperature difference

T_s = surface temperature

T_f = bulk temperature of fluid

2.6 Application of dimensional analysis for convection

Many engineering problems can't be solved completely by theoretical or mathematical methods. Problems of this type are common in fluid flow, heat flow. One method of attacking a problem for which no mathematical equation can be derived is that of empirical experimentation. The empirical method of obtaining an equation, is laborious, and it is difficult to organize or correlate the results so

obtained into a useful relationship for calculation. There exists a method intermediate between mathematical equation and empirical equation. It is based on the fact that if a theoretical equation does exist among the variables involved in the process, that equation must be dimensionally homogeneous. It is possible to group many factors into a smaller number of dimensionless groups of variables. The groups themselves appear in the final equation. This method is called dimensional analysis.

(i) Natural convection

Let us consider the case of natural convection from a vertical plane wall to an adjacent fluid.

Variables	Symbol	Dimension
Length	L	L
Fluid density	ρ	ML^{-3}
Fluid viscosity	μ	$ML^{-1}t^{-1}$
Fluid heat capacity	C_p	$L^2t^{-2}\theta^{-1}$
Fluid coefficient of thermal expansion	β	θ^{-1}
Acceleration due to gravity	g	Lt^{-2}
Temperature difference	ΔT	θ
Heat transfer coefficient	h	$Mt^{-3}\theta^{-1}$
Fluid thermal conductivity	k	$MLt^{-3}\theta^{-1}$

Number of π groups = number of variables(N) – number of fundamental dimensions(m)

Number of π groups = 9-4 = 5

$$\pi_1 = L^{a_1} \mu^{b_1} k^{c_1} g^{d_1} \rho$$

$$\pi_2 = L^{a_2} \mu^{b_2} k^{c_2} g^{d_2} C_p$$

$$\pi_3 = L^{a_3} \mu^{b_3} k^{c_3} g^{d_3} \beta$$

$$\pi_4 = L^{a_4} \mu^{b_4} k^{c_4} g^{d_4} \Delta T$$

$$\pi_5 = L^{a_5} \mu^{b_5} k^{c_5} g^{d_5} h$$

$$\pi_1 = (L)^{a_1} (ML^{-1}t^{-1})^{b_1} (MLt^{-3}\theta^{-1})^{c_1} (Lt^{-2})^{d_1} ML^{-3}$$

By equating the net dimensions of mass, length, time and temperature to zero, we get

$$b_1 + c_1 + 1 = 0$$

$$a_1 - b_1 + c_1 + d_1 - 3 = 0$$

$$-b_1 - 3c_1 - 2d_1 = 0$$

$$-c_1 = 0$$

By solving these equations, we get

$$c_1 = 0$$

$$b_1 = -1$$

$$d_1 = \frac{1}{2}$$

$$a_1 = \frac{3}{2}$$

$$\pi_1 = L^{3/2} g^{1/2} \rho / \mu$$

Squaring of both sides

$$\pi_1 = \frac{L^3 g \rho^2}{\mu^2}$$

Hence,

$$\pi_2 = (L)^{a_2} (ML^{-1}t^{-1})^{b_2} (MLt^{-3}\theta^{-1})^{c_2} (Lt^{-2})^{d_2} L^2 t^{-2} \theta^{-1}$$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_2 = \frac{\mu C_p}{k} = N_{Pr}$$

$$\pi_3 = (L)^{a3} (ML^{-1}t^{-1})^{b3} (MLt^{-3}\theta^{-1})^{c3} (Lt^{-2})^{d3} \theta^{-1}$$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_3 = \frac{L\mu g\beta}{k}$$

$$\pi_4 = (L)^{a4} (ML^{-1}t^{-1})^{b4} (MLt^{-3}\theta^{-1})^{c4} (Lt^{-2})^{d4} \theta$$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_4 = \frac{k\Delta T}{L\mu g}$$

$$\pi_5 = (L)^{a5} (ML^{-1}t^{-1})^{b5} (MLt^{-3}\theta^{-1})^{c5} (Lt^{-2})^{d5} Mt^{-3}\theta^{-1}$$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_5 = \frac{hL}{K} = N_{Nu}$$

Combining π_1, π_3, π_4 ;

$$\frac{L^3 \rho^2 g \beta \Delta T}{\mu^2} = N_{Gr}$$

$$\therefore \mathbf{N_{Nu} = f(N_{Gr}, N_{pr})}$$

Where N_{Nu} = Nusselt number

N_{Gr} = Grashof number

N_{pr} = Prandtl number

(ii) Forced Convection

Let us consider the flow of fluid through a hot tube.

Variables	Symbol	Dimension
Tube Diameter	D	L
Fluid density	ρ	ML^{-3}
Fluid velocity	u	Lt^{-1}
Fluid viscosity	μ	$ML^{-1}t^{-1}$
Fluid heat capacity	Cp	$L^2t^{-2}\theta^{-1}$
Fluid thermal conductivity	k	$MLt^{-3}\theta^{-1}$
Heat transfer coefficient	h	$Mt^{-3}\theta^{-1}$

Number of π groups = number of variables (N) – number of fundamental dimensions(m)

Number of $[\]$ groups = 7-4 = 3

$$\pi_1 = D^{a_1} \mu^{b_1} u^{c_1} k^{d_1} \rho$$

$$\pi_2 = D^{a_1} \mu^{b_1} u^{c_1} k^{d_1} Cp$$

$$\pi_3 = D^{a_1} \mu^{b_1} u^{c_1} k^{d_1} h$$

$$\pi_1 = (L)^{a_1} (ML^{-1}t^{-1})^{b_1} (Lt^{-1})^{c_1} (MLt^{-3}\theta^{-1})^{d_1} ML^{-3}$$

By equating the net dimensions of mass,length,time and temperature to zero, we get

$$b_1 + d_1 + 1 = 0$$

$$a_1 - b_1 + c_1 + d_1 - 3 = 0$$

$$-b_1 - c_1 - 3d_1 = 0$$

$$-d_1 = 0$$

By solving these equations, we get

$$d_1 = 0$$

$$b_1 = -1$$

$$c_1 = 1$$

$$a_1 = 1$$

$$\pi_1 = \frac{D\rho u}{\mu} = N_{Re}$$

$$\pi_2 = (L)^{a_2} (ML^{-1}t^{-1})^{b_2} (Lt^{-1})^{c_2} (MLt^{-3}\theta^{-1})^{d_2} L^2t^{-2}\theta^{-1}$$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_2 = \frac{\mu C_p}{k} = N_{Pr}$$

$$\pi_3 = (L)^{a_3} (ML^{-1}t^{-1})^{b_3} (Lt^{-1})^{c_3} (MLt^{-3}\theta^{-1})^{d_3} Mt^{-3}\theta^{-1}$$

By equating the net dimensions of mass, length, time and temperature to zero and solving those equations, we get

$$\pi_3 = \frac{hD}{k} = N_{Nu}$$

$$\therefore \mathbf{N_{Nu} = f(N_{Re}, N_{Pr})}$$

Where N_{Nu} = Nusselt number

N_{Re} = Reynolds number

N_{Pr} = Prandtl number

2.7 Heat transfer coefficient for convective transfer processes

Heat transfer coefficient calculation for forced convection:

Empirical equations for laminar flow:

An empirical equation for moderate Graetz numbers is

$$Nu = 2 Gz^{1/3}$$

For viscous liquids with large temperature drops, a modification of the above equation is required to account for differences between heating and cooling. Therefore a correction factor is added with that equation to give the final equation for laminar flow heat transfer.

$$Nu = 2 Gz^{1/3} \phi_v = 2 \left(\frac{mCp}{kL} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}$$

Empirical equations for turbulent flow:

One empirical correlation for long tubes with sharp-edged entrances is the **Dittus-Boelter** equation.

$$Nu = \frac{h_i D}{k} = 0.023 Re^{0.8} Pr^n$$

$n = 0.4$ when the fluid is heated, $n = 0.3$ when the fluid is cooled

A better relationship for turbulent flow is known as the **Sieder-Tate** equation.

$$Nu = \frac{h_i D}{k} = 0.023 Re^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}$$

The correction factor $\left(\frac{\mu}{\mu_w} \right)^{0.14}$ accounts for the heating and cooling for fluid. μ is

the absolute of viscosity of fluid at bulk mean temperature and μ_w is the absolute of viscosity of fluid at surface temperature.

Heat transfer coefficient calculation for natural convection

Equations for heat transfer in natural convection between fluids and solids of definite geometric shape are of the form

$$Nu = b (Gr Pr)^n$$

b,n are constants.

2.8 Heat transfer to fluids without phase change

Heat transfer from condensing vapours, dropwise and film-type condensation,

A vapour may condense on a cold surface in one of two ways, which are drop-wise and film-wise condensation. In film wise condensation, the liquid condensate forms a film, or continuous layer of liquid that flows over the surface of the tube under the action of gravity.

In drop-wise condensation, the condensate begins to form at microscopic nucleation sites. The drops grow and coalesce with their neighbors to form visible fine drops. The fine drops in turn, coalesce into rivulets, which flow down the tube under the action of gravity.

In drop-wise condensation, large areas of the tube surface are covered with an extremely thin film of liquid of negligible thermal resistance. Because of this the heat transfer coefficient at these areas is very high the average coefficient for Drop-wise condensation may be 5 – 8 times that for film-type condensation. For normal design, film-type condensation is assumed.



Fig. 1 Film-wise and drop-wise condensation

Heat transfer coefficient in drop-wise condensation is 10 times higher than in Film-wise condensation.

Much of the experimental work on the drop-wise condensation of steam is summarized as follows:

- (1) Film-type condensation of water occurs on tubes of the common metals if both the steam and the tube are clean, in the presence or absence of air, on rough or on the polished surfaces.**
- (2) Drop-wise condensation is obtainable only when the cooling surface is not wetted by the liquid. In the condensation of steam it is often induced by contamination of the vapour with droplets of oil. It is more easily maintained on a smooth surface than on a rough surface.**
- (3) The quantity of contaminant or promoter required to cause drop-wise condensation is minute, and apparently only a monomolecular film is necessary.**
- (4) Effective drop promoters are strongly adsorbed by the surface, and substances that merely prevent wetting are ineffective. Some properties are especially effective on certain metals.**
- (5) The average coefficient obtainable in pure drop-wise condensation may be as high as 115 kw/m²K.**

2.9 The effect of non condensable gases on condensation.

The presence of even small amounts of non-condensing gas in a condensing vapour seriously reduces the rate of condensation. When a vapour containing non-condensable gas condenses, the non- condensable gas is left at the surface.

Any further condensation will occur only after the incoming vapour has diffused through this non-condensable gas which does not move toward the condensate. As condensation proceeds, the relative amount of this inert gas in the vapour phase increases significantly. The non-condensable gas acts as a thermal resistance to the condensation process.

Heat transfer coefficients calculation for film-type condensation

The assumptions used in Nusselt's equation for condensation to determine film thickness.

- (i) The vapour and liquid at the outside boundary of the liquid layer are in thermodynamic equilibrium.**
- (ii) The only resistance to heat flow is offered by the layer of condensate flowing in laminar flow.**
- (iii) The velocity of the liquid at the wall is zero.**
- (iv) The temperatures of the wall and the vapour are constant.**
- (v) Condensate is assumed to leave the tube at the condensing temperature.**

(vi) The fluid properties are taken at the mean film temperature.

Film wise condensation on horizontal pipe

$$h = 0.729 \left[\frac{k_f^3 \rho_f^2 g \lambda}{\Delta T D \mu_f} \right]^{1/4}$$

Film wise condensation on vertical pipe

$$h = 0.943 \left[\frac{k_f^3 \rho_f^2 g \lambda}{\Delta T L \mu_f} \right]^{1/4}$$

Where

k_f = thermal conductivity of condensate

ρ_f = density of condensate

g = acceleration due to gravity

λ = latent heat of condensation

μ_f = absolute viscosity of condensate

D = Pipe diameter

L = pipe length

ΔT = temperature difference

2.10 Heat Transfer To Boiling Liquids

Heat transfer to a boiling liquid is a necessary step in evaporation, distillation, and steam generation, and it may be used to control the temperature of a chemical reactor. Boiling occurs at the solid–liquid interface when a liquid is brought into contact with a surface maintained at a temperature sufficiently above the saturation temperature of the liquid.

Saturated Boiling Boiling of liquid When the temperature of the liquid is equal to the saturation temperature.

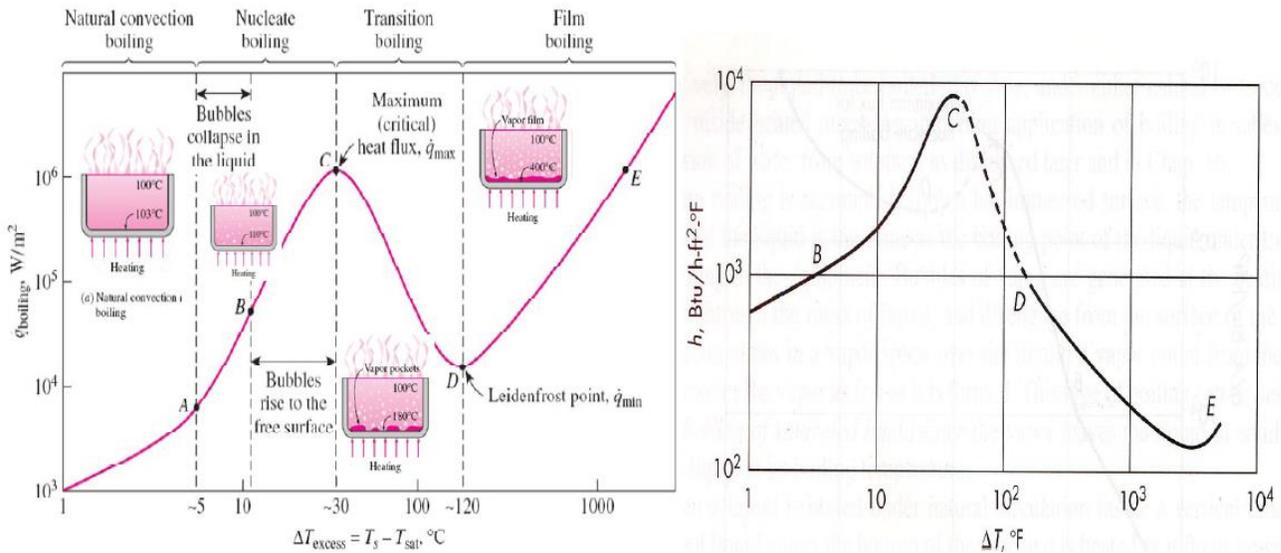


Fig. 2 Regimes of boiling

(a) q/A Vs ΔT diagram (b) h Vs diagram

When boiling is accomplished by a hot immersed surface, the temperature of the mass of the liquid is same as the boiling point of the liquid under the pressure existing in the equipment. Bubbles of vapour are generated at the heating surface, rise through the mass of liquid, and disengage from the surface of the liquid. Vapour accumulates in a vapour space over the liquid, a vapour outlet from the vapour space removes the vapour as fast as it is formed. This type of boiling can be described as pool boiling of saturated liquid since the vapour leaves the liquid in equilibrium with the liquid at its boiling temperature.

Consider a horizontal, electrically heated wire immersed in a vessel containing a boiling liquid. Assume that q , heat flux, and ΔT , the difference between the temperature of the wire surface, T_w and that of the boiling liquid T are measured. Start with a very low temperature drop. Now raise T_w and increase the temperature drop by steps, measuring q and ΔT at each step, until very large values of ΔT are reached. A plot of q vs ΔT on logarithmic coordinates will give a curve of the type shown in figure. This curve can be divided into four segments.

Each of the four segments of the graph corresponds to a definite mechanism of boiling. In the first section, at low temperature drops, the mechanism is that of heat transfer to a liquid in natural convection. Bubbles form on the surface of the heater, are released from it, rise to the surface of the liquid, and are disengaged into the vapour space, but they are too few to disturb

the normal currents of free convection. This segment is called natural convection zone. At larger temperature drops the rate of bubble production is large enough for the stream of bubbles moving up through the liquid to increase the velocity of circulation currents in the mass of liquid, and the coefficient of heat transfer becomes greater than in undisturbed natural convection. As ΔT is increased, the rate of bubble formation increases and the coefficient increases rapidly.

The action occurring at temperature drops below the critical temperature drop is called nucleate boiling, in reference to the formation of tiny bubbles on the heating surface. During nucleate boiling, the bubbles occupy a small portion of the heating surface at a time, and most of the surface is in direct contact with the liquid. The bubbles are generated at localized active sites, usually small pits or scratches on the heating surface. As the temperature drop is raised, more sites become active, improving the agitation of the liquid and increasing the heat flux and the heat transfer coefficient.

Eventually, however, so many bubbles are present that they tend to coalesce and cover portions of the heating surface with a layer of insulating vapour. This layer has a highly unstable surface. This type of action is called transition boiling. The heat flux and the heat transfer coefficient both fall as the temperature drop is raised. Near the Leidenfrost point another distinct change in mechanism occurs. The hot surface becomes covered with the film vapour, through which heat is transferred by conduction and by radiation. The random explosion characteristic of transition boiling disappears and is replaced by the slow and orderly formation of bubbles at the interface between the liquid and the film of hot vapour. As temperature drop increases, the heat flux rises, slowly at first and then more rapidly as radiation heat transfer becomes important. The boiling action in this region is known as film boiling.

Sub-cooled Boiling

In some types of forced-circulation equipment, the temperature of the mass of the liquid is below that of its boiling point, but the temperature of the heating surface is considerably above the boiling point of the liquid. Bubbles form on the heating surface, but on release from the surface are absorbed by the mass of the liquid. This type of heat transfer is called subcooled boiling, even though the fluid leaving the heat exchanger is entirely liquid.

2.11 Extended surfaces : Heat transfer through fins

Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law: $q = hA(T_s - T_a)$, where T_s is the surface temperature and T_a is the fluid temperature. Therefore, to increase the convective heat transfer, one can increase the temperature difference ($T_s - T_a$) between the surface and the fluid. Increase the convection coefficient h . This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the h . Example: a cooling fan. Increase the contact surface area A . Example: a heat sink with fins. Many times, when the first option is not in our control and the second option (i.e. increasing h) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces. Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid. Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere.

Heat Transfer From a Fin

Fins are used in a large number of applications to increase the heat transfer from surfaces. Typically, the fin material has a high thermal conductivity. The fin is exposed to a flowing fluid, which cools or heats it, with the high thermal conductivity allowing increased heat being conducted from the wall through the fin. The design of cooling fins is encountered in many situations and we thus examine heat transfer in a fin as a way of defining some criteria for design.

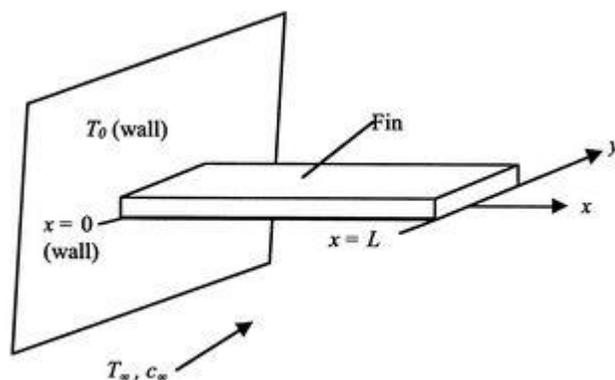


Fig. 3 Geometry of heat transfer through longitudinal fin

A model configuration is shown in Figure. The fin is of length L . The other parameters of the problem are indicated. The fluid has velocity c_∞ and temperature T_∞ . We assume (using the Reynolds analogy or other approach) that the heat transfer coefficient for the fin is known and has value h . The end of the fin can have a different heat transfer coefficient, which we can call h_L .

The approach taken will be quasi-one-dimensional, in that the temperature in the fin will be assumed to be a function of x only. This may seem a drastic simplification, and it needs some explanation. With a fin cross-section equal to A and a perimeter P , the characteristic dimension in the transverse direction is A/P (For a circular fin, for example, $A/P = r/2$).

The regime of interest will be taken to be that for which the Biot number is much less than unity, $Bi = h(A/P)/k \ll 1$, which is a realistic approximation in practice.

The physical content of this approximation can be seen from the following. Heat transfer per unit area out of the fin to the fluid is roughly of magnitude $\sim h(T_w - T_\infty)$ per unit area. The heat transfer per unit area within the fin in the transverse direction is (again in the same approximate terms)

$$k \frac{(T_1 - T_w)}{A/P},$$

where T_1 is an internal temperature. These two quantities must be of the same magnitude.

If $h(A/P)/k \ll 1$, then $(T_1 - T_w)/(T_w - T_\infty) \ll 1$. In other words, if $Bi \ll 1$, there is a much larger capability for heat transfer per unit area across the fin than there is between the fin and the fluid, and thus little variation in temperature inside the fin in the transverse direction. To emphasize the point, consider the limiting case of zero heat transfer to the fluid,

i.e., an insulated fin. Under these conditions, the temperature within the fin would be uniform and equal to the wall temperature.

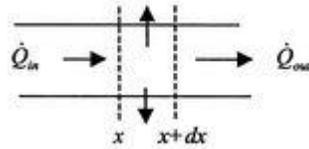


Fig 4: Element of fin showing heat transfer

If there is little variation in temperature across the fin, an appropriate model is to say that the

$$T = T(x)$$

temperature within the fin is a function of x only, and use a quasi-one-dimensional approach. To do this, consider an element, dx , of the fin as shown. There is heat

$$\dot{Q}_{in}$$

flow of magnitude at the left-hand side and heat flow out of magnitude

$$\dot{Q}_{out} = \dot{Q}_{in} + \frac{d\dot{Q}}{dx}dx$$

at the right hand side. There is also heat transfer around the perimeter

on the top, bottom, and sides of the fin. From a quasi-one-dimensional point of view, this is a situation similar to that with internal heat sources, but here, for a cooling fin, in each elemental

$$Pdxh(T - T_{\infty})$$

slice of thickness dx there is essentially a heat sink of magnitude ,

where Pdx is the area for heat transfer to the fluid.

$$\dot{Q} = \dot{q}A$$

The heat balance for the element can be written in terms of the heat flux using , for a fin of constant area:

$$\dot{q}A = Ph(T - T_{\infty})dx + \left(\dot{q}A + \frac{d\dot{q}}{dx}dxA \right).$$

$$\frac{d^2T}{dx^2} - \frac{Ph}{Ak}(T - T_{\infty}) = 0.$$

$$A \frac{d\dot{q}}{dx} + Ph(T - T_\infty) = 0.$$

The quantity of interest is the temperature difference $(T - T_\infty)$, and we can change variables

$$\frac{d}{dx}(T - T_\infty) = \frac{dT}{dx}.$$

$$\frac{d^2}{dx^2}(T - T_\infty) - \frac{Ph}{Ak}(T - T_\infty) = 0.$$

the temperature variation along the fin. It is a second order equation and needs two boundary conditions. The first of these is that the temperature at the end of the fin that joins the wall is equal to the wall temperature.

$$(T - T_\infty)_{x=0} = T_0 - T_\infty.$$

The second boundary condition is at the other end of the fin. We will assume that the heat transfer from this end is negligible. The boundary condition at $x = L$ is

$$\left. \frac{d}{dx}(T - T_\infty) \right|_{x=L} = 0.$$

The relation between derivatives that is needed to cast the equation in terms of ξ is

$$\frac{d}{dx} = \frac{d\xi}{dx} \frac{d}{d\xi} = \frac{1}{L} \frac{d}{d\xi}.$$

$$\frac{d^2 \Delta \tilde{T}}{d\xi^2} - \left(\frac{hP}{kA} L^2 \right) \Delta \tilde{T} = 0.$$

There is one non-dimensional parameter and it is m

$$m^2 L^2 = \frac{hPL^2}{kA}.$$

The equation for the temperature distribution we have obtained is

$$\frac{d^2 \Delta \tilde{T}}{d\xi^2} - m^2 L^2 \Delta \tilde{T} = 0.$$

This second order equation has the solution

$$\Delta \tilde{T} = ae^{mL\xi} + be^{-mL\xi},$$

the boundary condition at $\xi = 0$ is given by

$$\Delta \tilde{T}(0) = a + b = 1.$$

The boundary condition at $\xi = 1$ is that the temperature gradient is zero or

$$\frac{d\Delta \tilde{T}}{d\xi}(L) = mLae^{mL} - mLbe^{-mL} = 0.$$

Solving the two equations given by the boundary conditions for a and b gives an expression for $\Delta \tilde{T}$ in terms of the hyperbolic cosine or \cosh :

$$\cosh x = \frac{(e^x + e^{-x})}{2},$$

$$\Delta \tilde{T} = \frac{\cosh mL(1 - \xi)}{\cosh mL}.$$

This is the solution to above Equation for a fin with no heat transfer at the tip. In terms of the actual heat transfer parameters it is written as

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = \frac{\cosh\left(\left(1 - \frac{x}{L}\right)\sqrt{\frac{hP}{kA}}L\right)}{\cosh\left(\sqrt{\frac{hP}{kA}}L\right)}$$

The amount of heat removed from the wall due to the fin, which is the quantity of interest, can be found by differentiating the temperature and evaluating the derivative at the wall, $x = 0$

$$\dot{Q} = -kA \frac{dT}{dx} \Big|_{x=0}$$

$$\frac{\dot{Q}L}{kA(T_0 - T_{\infty})} = -\frac{d\Delta\tilde{T}}{d\xi} \Big|_{\xi=0} = \frac{mL \sinh(mL)}{\cosh(mL)} = mL \tanh(mL),$$

$$\frac{\dot{Q}}{\sqrt{kAhP}(T_0 - T_{\infty})} = \tanh(mL).$$

this is the expression for heat flow through a longitudinal fin.

It can also be derived for fin with an insulation and fin provided with free convection off the end.

PROBLEMS

Water heated to 80°C flows through a 2.54cm I.D and 2.88cm O.D steel tube ($k = 50 \text{ W/m K}$).

The tube is exposed to an environment which is known to provide an average convection coefficient of $h_o = 30800 \text{ W/m}^2 \text{ K}$ on the outside of the tube. The water velocity is 50 cm/s.

Calculate the overall heat transfer coefficient based on the outer area of the pipe. Properties of

water at bulk mean temperature of 80 °C: $\rho = 974 \text{ kg/m}^3$, $\gamma = 0.364 \times 10^{-6} \text{ m}^2/\text{sec}$, $k = 668.7 \times 10^{-3} \text{ W/m K}$, $\text{Pr} = 2.20$.

$$U_o = \frac{1}{\frac{1}{h_i} \left(\frac{r_o}{r_i} \right) + \frac{1}{h_{di}} \left(\frac{r_o}{r_i} \right) + \frac{r_o}{k_w} \ln \left(\frac{r_o}{r_i} \right) + \frac{1}{h_{do}} + \frac{1}{h_o}}$$

r_i	-	0.0127 m
r_o	-	0.0144 m
h_o	-	30800 W/m ² K
k_w	-	50 W/m K

Dittus-Boelter equation

$$N_{Nu} = 0.023(N_{Re})^{0.8}(N_{Pr})^{0.3} = 125.48$$

$$N_{Re} = \frac{D_i \rho u}{\mu} = 34890.10$$

$$h_i = \frac{N_{Nu} k}{D_i} = 3303.48 \text{ W/m}^2 \text{ K}$$

$$U_o = 2428.23 \text{ W/m}^2 \text{ K}$$

2. Air at 2 atm and 200°C is heated as it flows at a velocity of 12m/s through a tube with a diameter of 3cm. The tube wall temperature is 20°C above the air temperature all along the length of the tube. Calculate the rate of heat transfer per unit length of the tube. The properties of air at bulk mean temperature are; $\text{Pr} = 0.681$; $\mu = 2.57 \times 10^{-5} \text{ kg/ms}$; $k = 0.0386 \text{ W/m K}$ and $C_p = 1.025 \text{ kJ/kg K}$

Solution:

D	-	$3 \times 10^{-2} \text{ m}$
u	-	12m/s
Tb	-	200°C
Pr	-	0.681
μ	-	$2.57 \times 10^{-5} \text{ kg/ms}$
k	-	0.0386 W/mK
Cp	-	1.025 kJ/kg K

$$N_{Re} = \frac{D_i \rho u}{\mu} = \mathbf{21137}$$

Dittus-Boelter equation

$$N_{Nu} = 0.023(N_{Re})^{0.8} (N_{Pr})^{0.3} = \mathbf{59.12}$$

$$h = \frac{N_{Nu} k}{D_i} = \mathbf{76.07 \text{ W/m}^2 \text{ K}}$$

Newton's law of cooling

$$Q = h A \Delta T = h \pi D L \Delta T$$

$$Q/L = \mathbf{143.32 \text{ W/m}}$$

3. A horizontal cylinder, 3.0 cm in diameter and 0.8 m length is suspended in water at 20°C.

Calculate the rate of heat transfer if the cylinder surface is at 55°C. Given $Nu = 0.53 (Gr \times Pr)$

0.25. The properties of water at average temperature are as follows: Density = 990 kg/m³,

Viscosity = 2.47 kg/hr.m, k = 0.534 kcal/hr.m·C, Cp = 1 kcal/kg·C,

Solution:

D	-	$3 \times 10^{-2} \text{m}$
L	-	0.8m
T_b	-	20°C
T_s	-	55°C
ρ	-	990 kg/m^3
μ	-	2.47 kg/hr.m
k	-	$0.534 \text{ kcal/hr.m.}^\circ\text{C}$
C_p	-	$1 \text{ kcal/kg}^\circ\text{C}$

$$N_{Gr} = \frac{D^3 \rho^2 g \beta \Delta T}{\mu^2} = \mathbf{6.216 \times 10^7}$$

$$N_{Pr} = \frac{\mu C_p}{k} = \mathbf{4.6255}$$

$$Nu = 0.53 (Gr \times Pr)^{0.25} = \mathbf{69.01}$$

$$h = \frac{N_{Nu} k}{D} = \mathbf{1228.4 \text{ kcal/h m}^2\text{K}}$$

Newton's law of cooling

$$Q = h A \Delta T = h \pi D L \Delta T = \mathbf{3240.03 \text{ kcal/h}}$$

4. Liquid sodium is to be heated from 120°C to 149°C at a rate of 2.3 kg/sec in a 2.5 cm diameter electrically heated tube (constant heat flux). Calculate the heat transfer coefficient.

The properties of sodium at 134.5°C are: density = 916 kg/m^3 , $C_p = 1.3565 \text{ kJ/kg K}$, $\nu = 0.594 \times 10^{-6} \text{ m}^2/\text{sec}$, $k = 84.90 \text{ W/m K}$, $Pr = 0.0087$. Given $N_{Nu} = 4.82 + 0.0185 N_{Pe}^{0.827}$

Solution: Calculate h

$$h = N_{Nu} * k$$

using Peclet number, $N_{Nu} = 14.22$

$$N_{Pe} = N_{Pr} * N_{Re}$$

$$N_{Re} = Di \rho u / \mu = 215067.34 \quad N_{Pr} = 0.0087$$

$$N_{Pe} = 1871.08 \text{ (Reynolds * Prandtl)}$$

Hence $h = 48291.12 \text{ w/m}^2 \text{ k}$.

Reynolds and Colburn Analogy:

Reynolds has taken the following assumptions to find the analogy between heat and momentum transport.

1. Gradients of the dimensionless parameters at the wall are equal.
2. The diffusivity terms are equal. That is

$$(v + \epsilon_M) = (\alpha + \epsilon_H)$$

References:

1. McCabe M.L. and Smith J.C., Unit Operations in Chemical Engineering, 7 th Edition, McGraw Hill, 2009.



SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)

Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE

www.sathyabama.ac.in

SCHOOL OF BIO & CHEMICAL ENGINEERING
DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – III – Heat Transfer – SCHA1301

1.0 Heat Exchangers

In any process industry, we need to transfer heat for different operations (like cooling, heating, vaporizing, or condensing) to or from various fluid streams in various equipment like condensers, water heaters, re-boilers, air heating or cooling devices etc., where heat exchanges between the two fluids. In a chemical process industry, the heat exchanger is frequently used for such applications. A heat exchanger is a device where two fluids streams come into thermal contact in order to transfer the heat from hot fluid to cold fluid stream.

In this chapter, we will discuss about the technical analysis of the heat exchangers along with the method for predicting heat exchanger performance and operational parameters. Moreover the discussion on heat exchanger size will also be discussed. However, we will not discuss the economics (though discuss the heat exchanger size) of the heat exchanger. We will consider that the heat transfer will primarily be taken by conduction and convection only. We will describe the commonly used heat exchangers and their important characteristics.

In general heat exchangers may be categorized into two general classes depending on the relative orientation of the flow direction of the two fluid streams. If the two streams cross one another in space, usually at right angles, the heat exchangers are known as cross flow heat exchanger as shown in the fig.1(a).

In the second class of heat exchanger the two streams move in parallel direction in space. The usual shell and tube heat exchanger or concentric pipe exchanger or double tube exchanger is the most frequently used exchanger in the class. Two situations may arise when the fluid flow in the parallel direction, one in which the fluids flow in same direction and the other in which the fluids flow in opposites direction. “Parallel –flow” or “Co-current flow” is used when the flow is in same direction and counter current is used when the fluid flow is in the opposite direction.

Before understanding the principle of heat exchanger we would first understand it from the point of construction.

1.1 Elements of shell and tube heat exchanger

We will discuss shell and tube heat exchanger as they or the most commonly used heat exchangers in the chemical process industries. Schematic of a typical shell and tube heat exchanger is shown in fig.2.

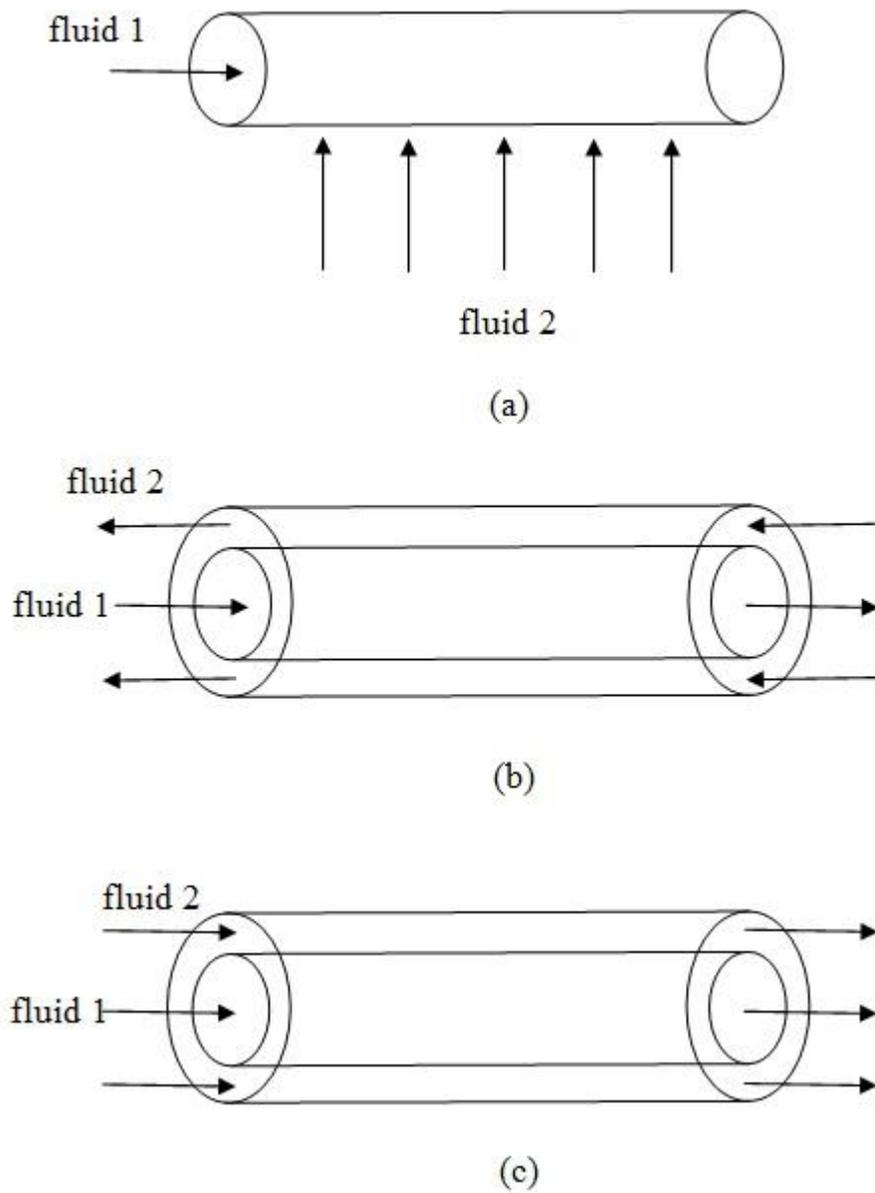


Fig.1: Orientation of fluid stream in heat exchanger (a) cross flow (b) counter current flow (c) parallel flow

The heat exchanger as shown in fig.2 consists of a bundle of tubes properly secured at either ends in tube sheets. The tube sheets are drilled plates into which the tubes are fixed up using different technique to have leak proof joints. The entire tube bundles shown in the fig.3 is placed inside a closed shell, which seals around the tube sheet periphery to form the two immiscible zones for hot and cold fluids are shown in fig.4.

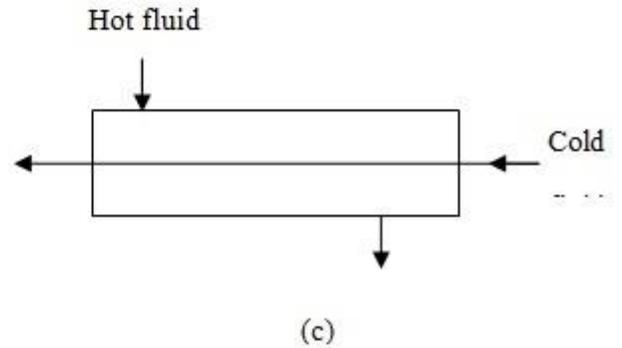
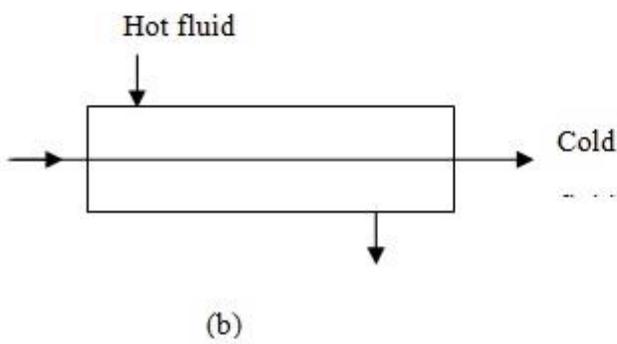
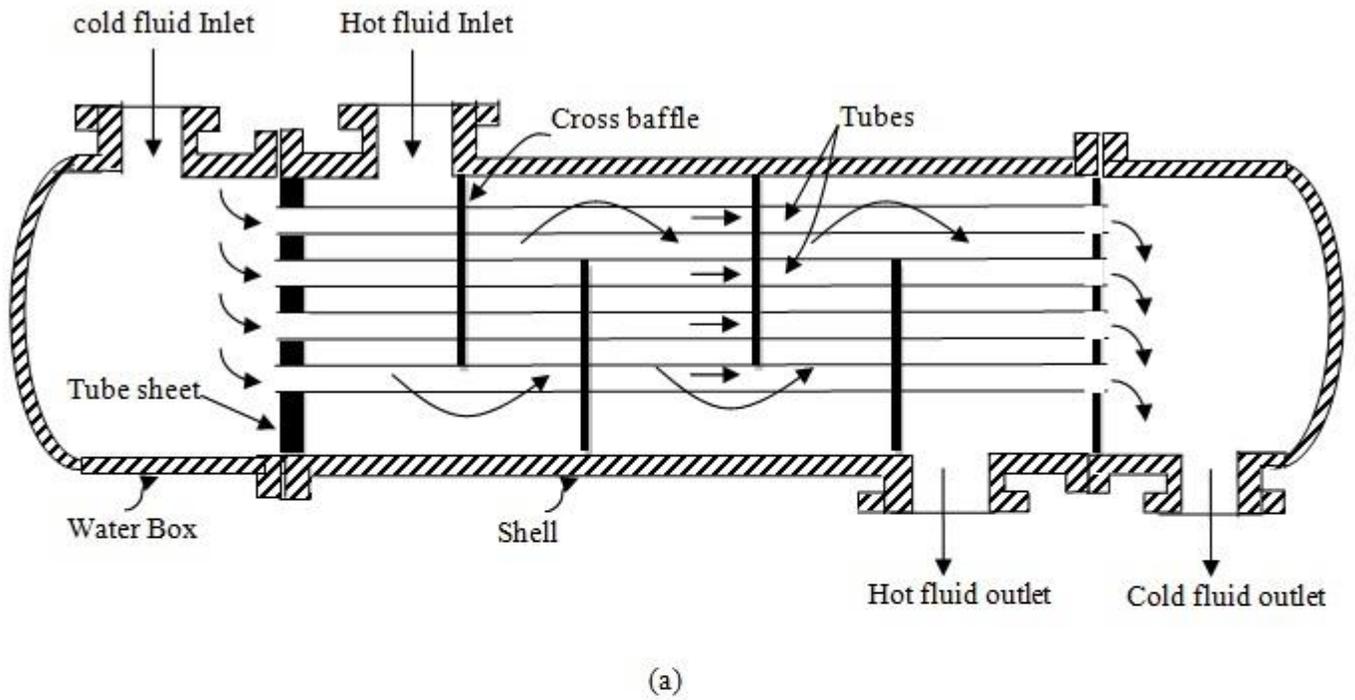


Fig.2: A schematic of (a) one-shell pass, one tube pass heat exchanger; (b) parallel flow; and (c) counter flow

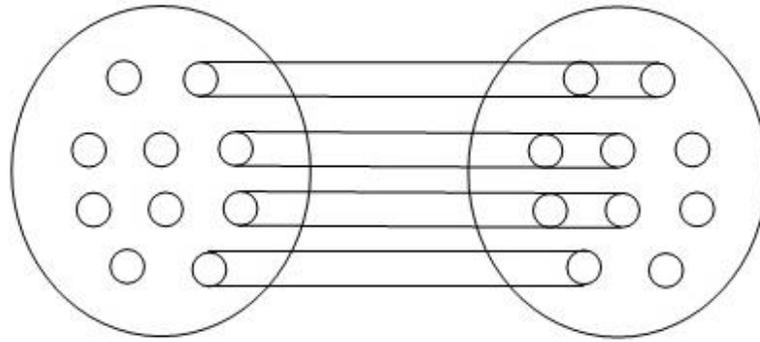


Fig. 3 Tube bundle fitted in two sheets

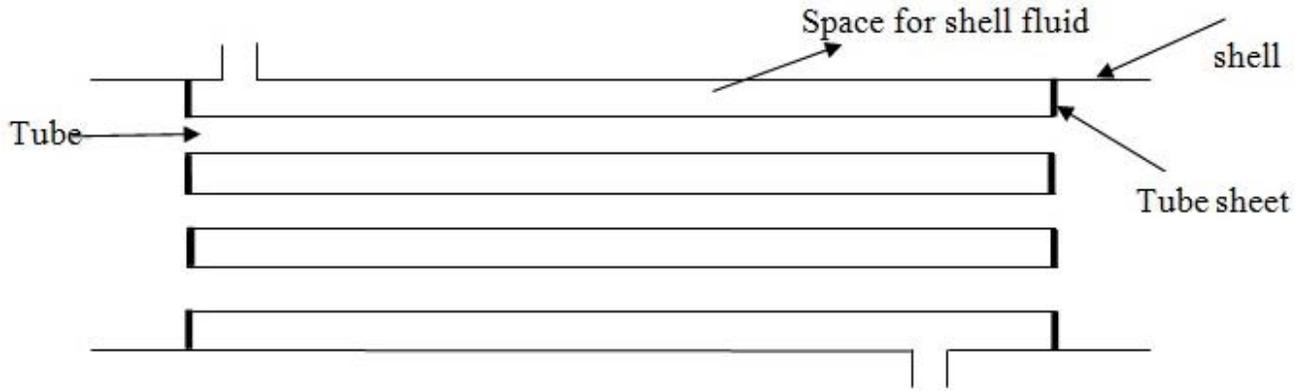


Fig.4: Tube bundle inside a shell

One fluid flows through the tubes while the other fluid flows around the outside of the tubes, it is the space between the tube sheets and enclosed by the outer shell.

For a thorough distribution of the shell side fluid, baffles are placed normal to the tube bundle. This baffle creates turbulence in the shell side fluid and enhances the transfer coefficients for the shell side flow.

Fig.2 shows the simplified diagram of a shell and tube heat exchanger, showing a few of the important components. Infact, the present heat exchanger used in the process industry are quite complex and have improved design such as factors for thermal expansion stresses, tube fouling due to contaminated fluids, ease of assembly and disassembly, size, weight, etc.

The heat exchanger shown in fig.2 is having one shell and one tube pass since both the shell and tube side fluid make a single traverse through the heat exchanger. Thus, this type of shell-and-tube heat exchanger is designated as 1-1 exchanger. If we desire to pass the tube fluid twice, then it is designated as 1-2 exchanger. Similarly, if there are 2 shell pass and 4 tube pass, the designation will be 2-4 exchanger. The number of pass in tube side is done by the pass partition plate. A pass partition plate or pass divider as shown in fig.5. The shell side pass can be created by a flat plate as shown in fig.6.

The three most common types of shell-and-tube exchangers are (1) fixed tubesheet design, (2) U-tube design, and (3) floating-head type. In all three types, the front-end head is stationary while the rear-end head can be either stationary or floating depending on the thermal stresses in the shell, tube, or tubesheet, due to temperature differences as a result of heat transfer. The exchangers are built in accordance with three mechanical standards that specify design, fabrication, and materials of unfired shell-and-tube heat exchangers. Class R is for the generally severe requirements of petroleum and related processing applications. Class C is for generally moderate requirements for commercial and general process applications. Class B is for chemical process service. The exchangers are built to comply with the applicable ASME Boiler and Pressure Vessel Code, Section VIII (1998), and other pertinent codes and/or standards. The TEMA standards supplement and define the ASME code for heat exchanger applications. In addition, state and local codes applicable to the plant location must also be met. The TEMA standards specify the manufacturing tolerances for various mechanical classes, the range of tube sizes and pitches, baffling and support plates, pressure classification, tubesheet thickness formulas, and so on, and must be consulted for all these details. In this book, we consider only the TEMA standards where appropriate, but there are other standards, such as DIN 28 008. Tubular exchangers are widely used in industry for the following reasons. They are custom designed for virtually any capacity and operating conditions, such as from high 14

CLASSIFICATION OF HEAT EXCHANGERS FIGURE (a) Shell-and-tube exchanger (BEM) with one shell pass and one tube pass; (b) shell-and-tube exchanger (BEU) with one shell pass and two tube passes. vacuum to ultrahigh pressure [over 100 MPa (15,000 psig)], from cryogenics to high temperatures [about 1100°C (2000°F)] and any temperature and pressure differences between the fluids, limited only by the materials of construction. They can be designed for special operating conditions: vibration, heavy fouling, highly viscous fluids, erosion, corrosion, toxicity, radioactivity, multi-component mixtures, and so on. They are the most versatile exchangers, made from a variety of

metal and nonmetal materials (such as graphite, glass, and Teflon) and range in size from small [0.1 m² (1 ft²) to supergiant [over 105 m² (106 ft²)] surface area. They are used extensively as process heat exchangers. in the petroleum-refining and chemical industries; as steam generators, condensers, boiler feed water heaters, and oil coolers in power plants; as condensers and evaporators in some air-conditioning and refrigeration applications; in waste heat recovery applications with heat recovery from liquids and condensing fluids; and in environmental control

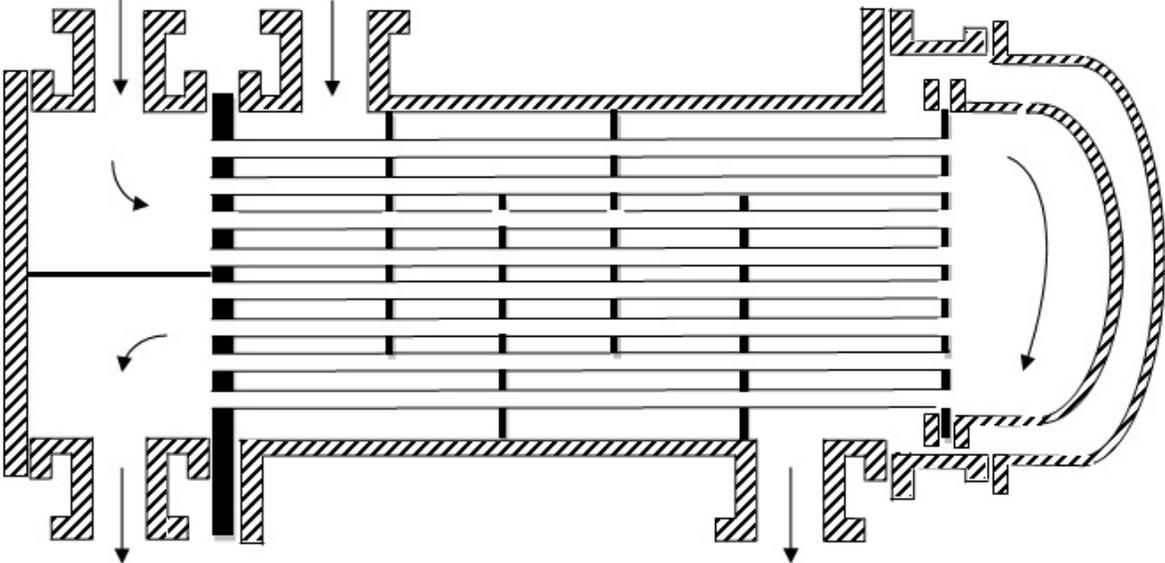


Fig. 5: 1-2 exchanger showing pass partition plate

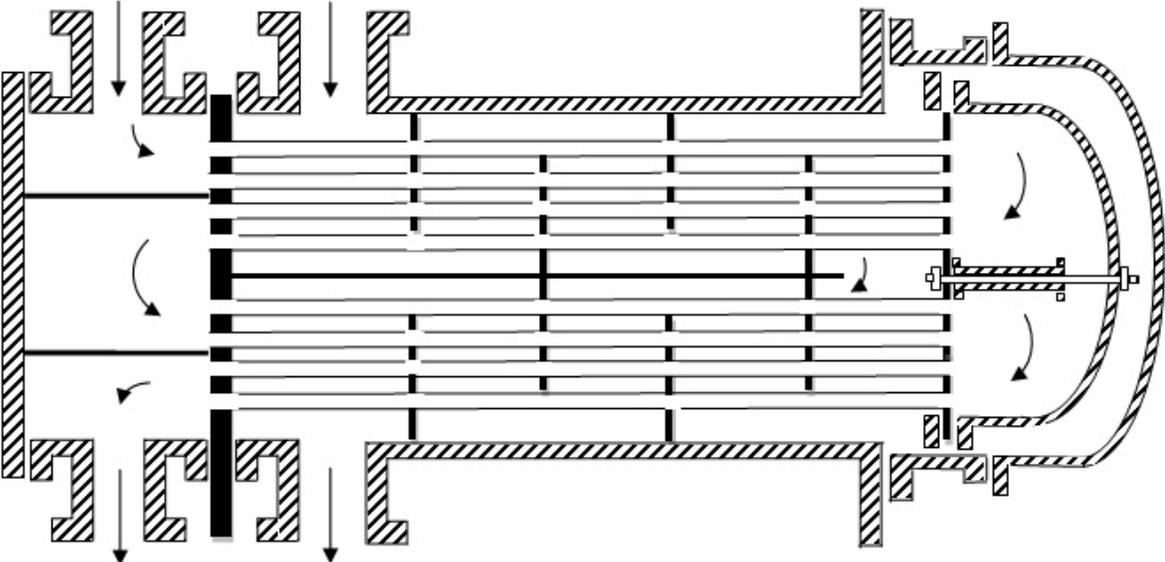
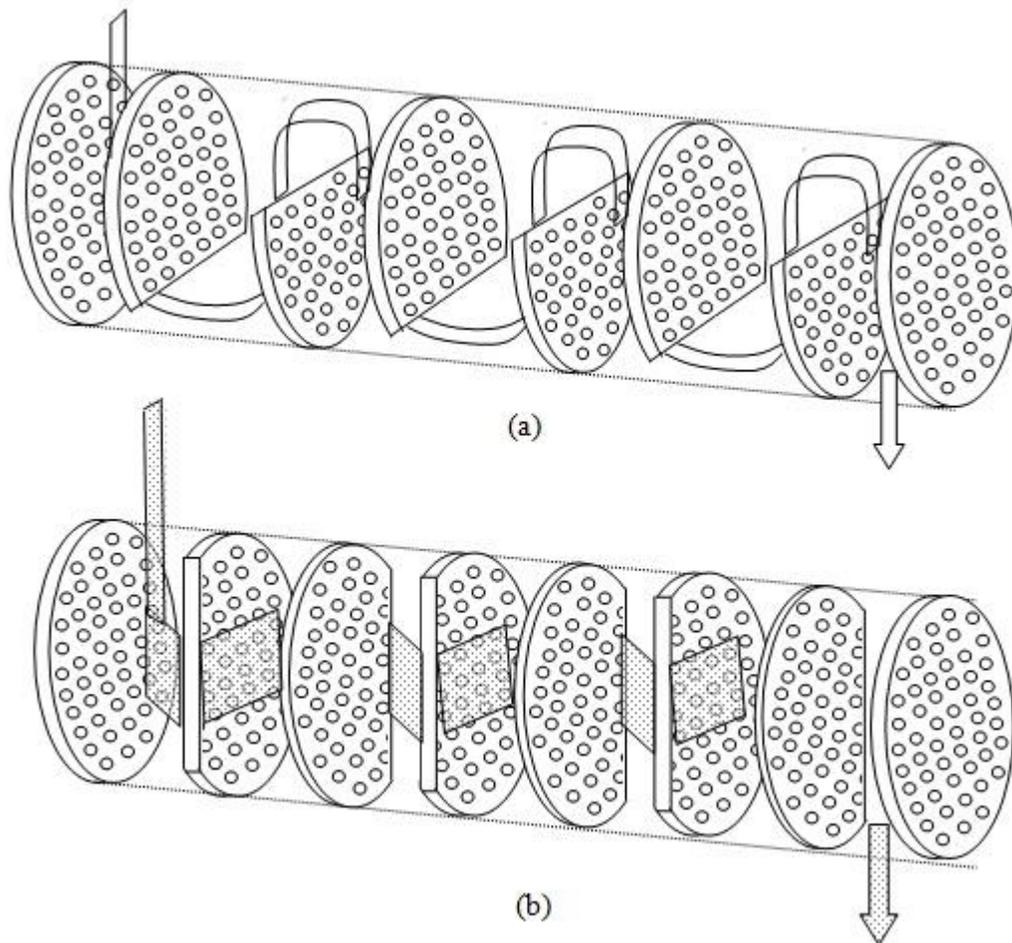


Fig 6: 2-4 exchanger showing shell and tube passes

It can be understood that for a given number of tubes; the area available for flow of the tube side fluid is inversely proportional to the number of passes. Thus, on increasing the pass the area reduces and as a result the velocity of fluid in the tube increases and henceforth the Reynolds number increases. It would result in increased heat transfer coefficient but at the expense of high pressure drop. Generally, even numbers of tube passes are used for the multi-pass heat exchangers.

Figure 2 shows some baffles. These baffles (or shell-side baffles) are a metal plate usually in the form of the segment of a circular disc having holes to accommodate tubes. Shell-side baffles have two functions. The first is to create turbulence in the shell side fluid by changing the flow pattern parallel or cross flow to the tube bundles and thus increases the shell side heat transfer coefficient. The second major function of these baffles is to support the tube all along its length otherwise the tube may bend. Moreover, these baffles may have horizontal or vertical cuts (segmental baffle) as shown in fig.7.



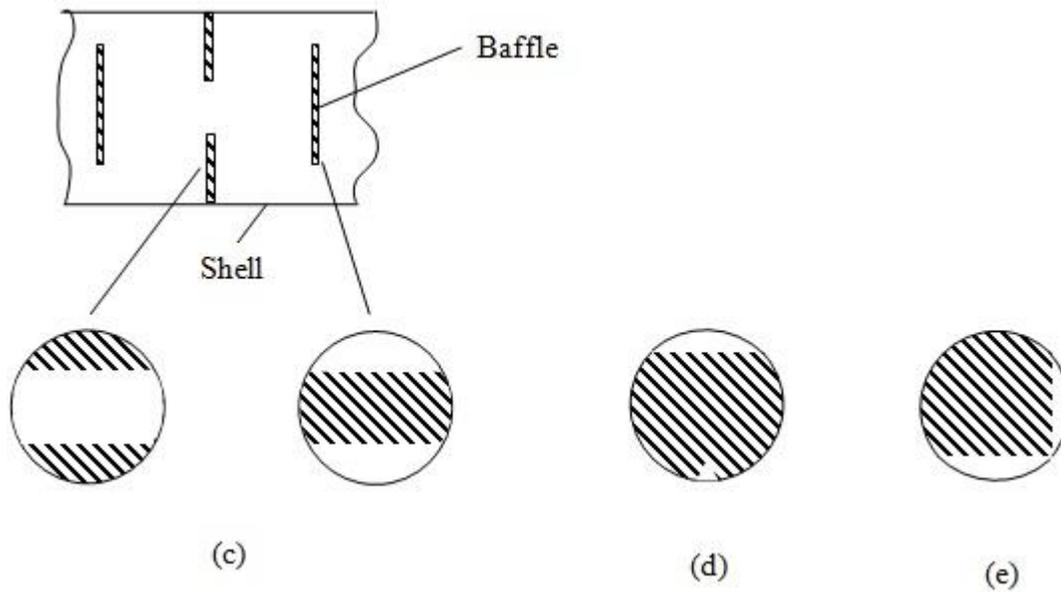


Fig.7: Baffles; (a) horizontal cut baffles; (b) Vertical cut baffles; (c, d and e)the shaded region show the baffle area

The cut portion of the baffle, which is called baffle window, provides the area for flow of the shell fluid. The baffle window area ranges from 15% to 50%. At 25% cut segmental baffle means that the area of the cut-out portion is 25% of the area of the baffle. The spacing between the baffles is an important aspect. A larger baffles spacing reduces the shell side pressure drop but at the same time decreases the turbulence and heat transfer coefficient. Smaller baffle spacing increases the turbulence and heat transfer coefficient. However, the pressure drop may increase significantly, thus the advantage attained due to the higher heat transfer coefficient may be nullified. Therefore baffle spacing is selected considering the allowable shell side pressure drop and the heat transfer coefficient desired. A rule of thumb is that the minimum spacing of segmental baffles is one by fifth of the shell diameter or 5 cm, whichever is larger.

1.2 Thermal design of heat exchangers

The mechanical design is done by the mechanical engineers on the inputs of chemical engineers and using the codes. The most widely used code in Tubular Exchanger Manufacturers Associations (TEMA). This USA code along with ASME selection VIII (unfired pressure vessel) code is used

together for the mechanical design of the heat exchanger. The Indian code for the heat exchanger design IS 4503.

Here we would discuss about the process design (or thermal design) leading to the sizing of the heat exchanger. Before understanding design steps, it is necessary to understand the following for the heat exchanger.

1.3 Overall heat transfer coefficient

As understood by the previous discussion that generally heat exchangers are tubular in nature (Note: we are not discussing about plate type heat exchangers). Thus we can easily find out the overall heat transfer coefficient based on our previous knowledge. Figure.8 shows a simplest form (double pipe heat exchanger) of tubular heat exchanger, where fluid A is being heated by fluid B in a co-current flow pattern. The inside and outside radii of the inner tube is represented r_i and r_o . The length of the exchanger for heat transfer is considered as L for section 1 to 2.

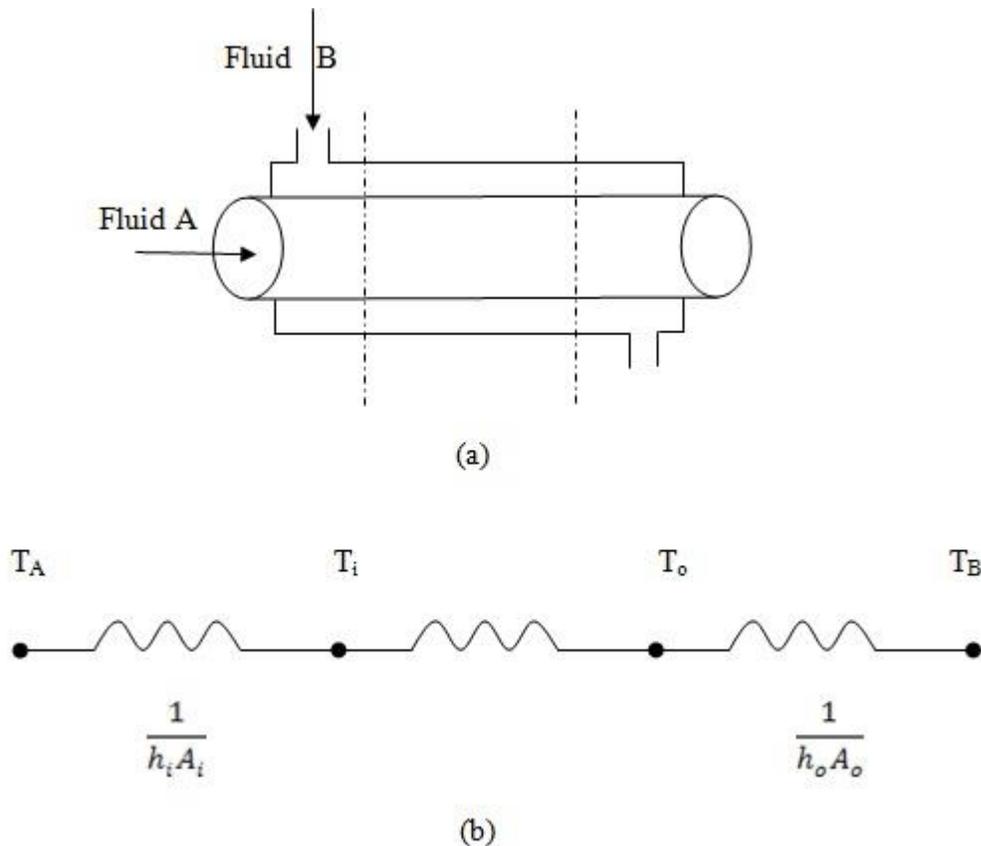


Fig.8: (a) Schematic of a double pipe heat exchanger (b) thermal resistance network for overall heat transfer

Thus the rate of heat transfer from the hot fluid to the cold fluid will be represented by eq.1,

$$q = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o A_o}} \quad (1)$$

The overall heat transfer coefficient;

Based on inside area of the inner pipe (eq.2)

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi k L} + \frac{A_i}{h_o A_o}} \quad (2)$$

Based on outer side area of the outer pipe (eq.3)

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o \ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o}} \quad (3)$$

1.4 Fouling factor or dirt factor

Over a time period of heat exchanger operation the surface of the heat exchanger may be coated by the various deposits present in the flow system. Moreover, the surfaces may become corroded or eroded over the time. Therefore, the thickness of the surface may get changed due to these deposits. These deposits are known as scale. These scales provide another resistance and usually decrease the performance of the heat exchangers. The overall effect is usually represented by dirt factor or fouling factor, or fouling resistance, R_f (Table.1) which must have included all the resistances along with the resistances due to scales for the calculation of overall heat transfer coefficient.

The fouling factor must be determined experimentally using eq.4,

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \quad (4)$$

Thus to determine the R_f , it is very important to know U_{clean} for the new heat exchanger. The U_{clean} must be kept securely to obtain the R_f , at any time of the exchanger's life.

Table-1 Fouling factor of a few of the industrial fluids

	Fouling factor (or resistance) $\frac{hm^2 \text{ } ^\circ C}{kcal} \times 10^3$
Liquid	
Fuel oil	1.024
Refrigerant liquids	0.102
Mono-and di-ethanolamine solution	0.409
Gasoline , naphtha and kerosene	0.205
Light gas oil	0.409
Heavy gas oil	0.615
Gases and Vapour	
Solvent vapour	0.205
Air	0.102-0.205
Flue gases	0.205-0.615
Steam (Saturated, oil free)	0.102-0.307
Water	
River water (treated, velocity > 0.6 m/s)	0.205-0.409
Treated boiler feed water	0.102-0.205
Process water	0.205-0.409

1.5 Temperature profiles in heat exchangers

Fig.9 shows the temperature profile along the length of a 1-2 exchangers and 2-4 exchangers.

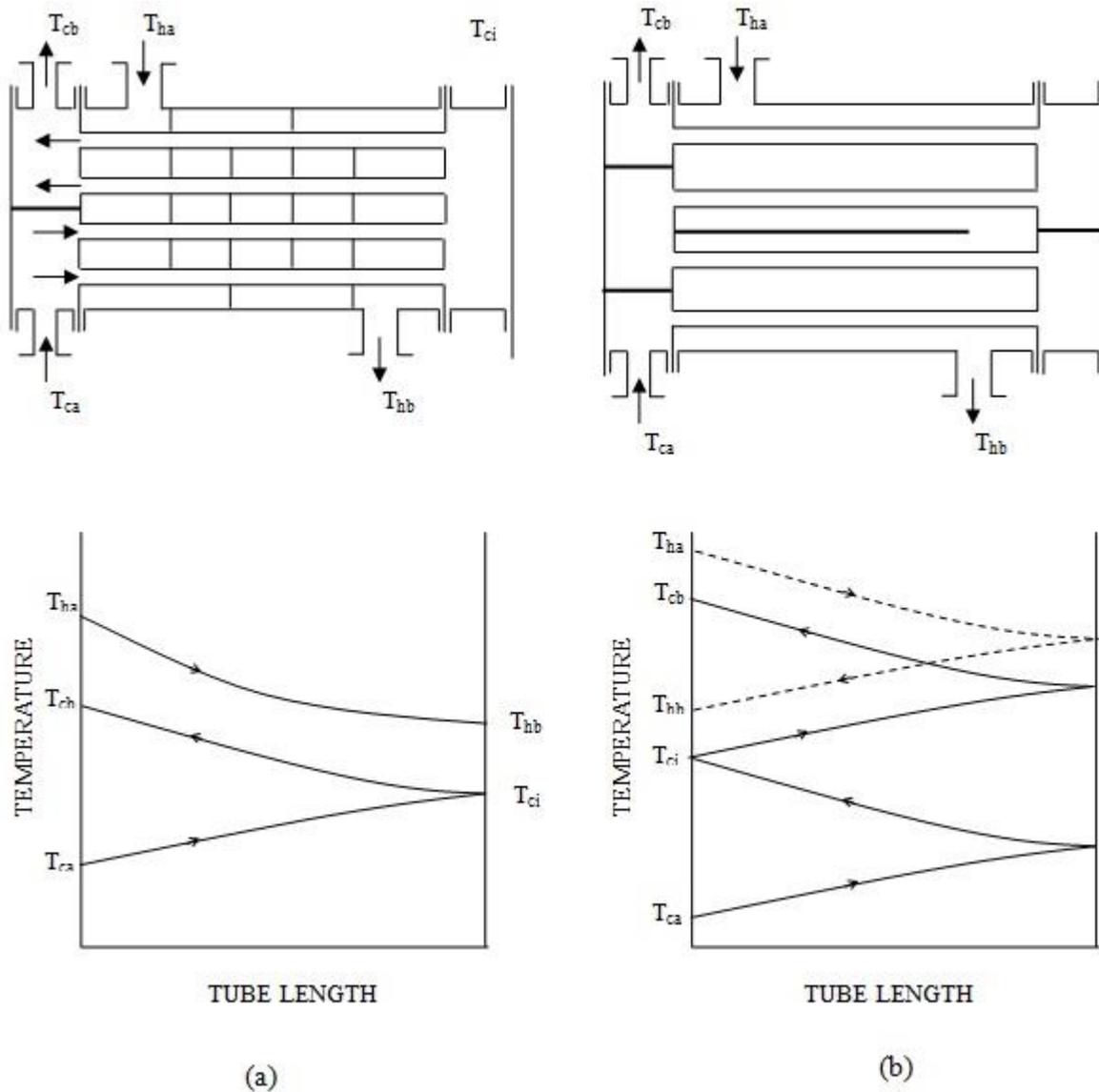


Fig.9 Temperature-length curve corresponding to (a) 1-2 exchanger ; (b) 2-4 exchanger

The nomenclature used in the fig.9 is described below

- T_{ha} : Inlet temperature of hot fluid
- T_{hb} : Outlet temperature of hot fluid
- T_{ca} : Inlet temperature of cold fluid
- T_{cb} : Outlet temperature of cold fluid
- T_{ci} : Intermediate temperature of cold fluid

In the above arrangement it is assumed that the hot fluid is flowing in the shell side and cold fluid is flowing in the tube side of the exchangers. The fig.9 (a) shows the 1-2 exchangers in which the hot fluid enter into the exchanger from the left side and exits from the right side. The cold fluid enters concurrently that is from the left side to the tube of the exchangers and goes up to right end of the exchangers and returns back to make two tube pass, and exits from the left end of the exchangers. The temperature profile all along the length of the exchanger is shown in the corresponding temperature length profile. Figure.9 (b) shows the flow direction and corresponding temperature length profile for 2-4 exchangers. The shell side fluid two passes and the tube side fluid has 4-passes in the exchangers.

It can be easily understood that whenever the number of passes is more than one, the flow cannot be truly co-current or counter current. Thus it will be a mix of co-current and counter current flows in any multi pass heat exchangers.

Though the temperature profile of the hot and cold streams can be easily predictable for single pass heat exchangers but for the complex flow modes, the prediction of temperature distribution will be difficult as shown in fig.9. As can be seen when 1-2 exchangers was (fig.9 (a)) used in co-current mode, the temperature profile was given in the figure. However, if the fluid streams enter in counter current mode a temperature cross may occur sometimes. Temperature cross is described as the positive temperature difference between the cold and the hot fluid, when these fluids leave the exchangers. In that case the cold fluid will attain the maximum temperature inside the exchanger instead of at the exit (fig.10).

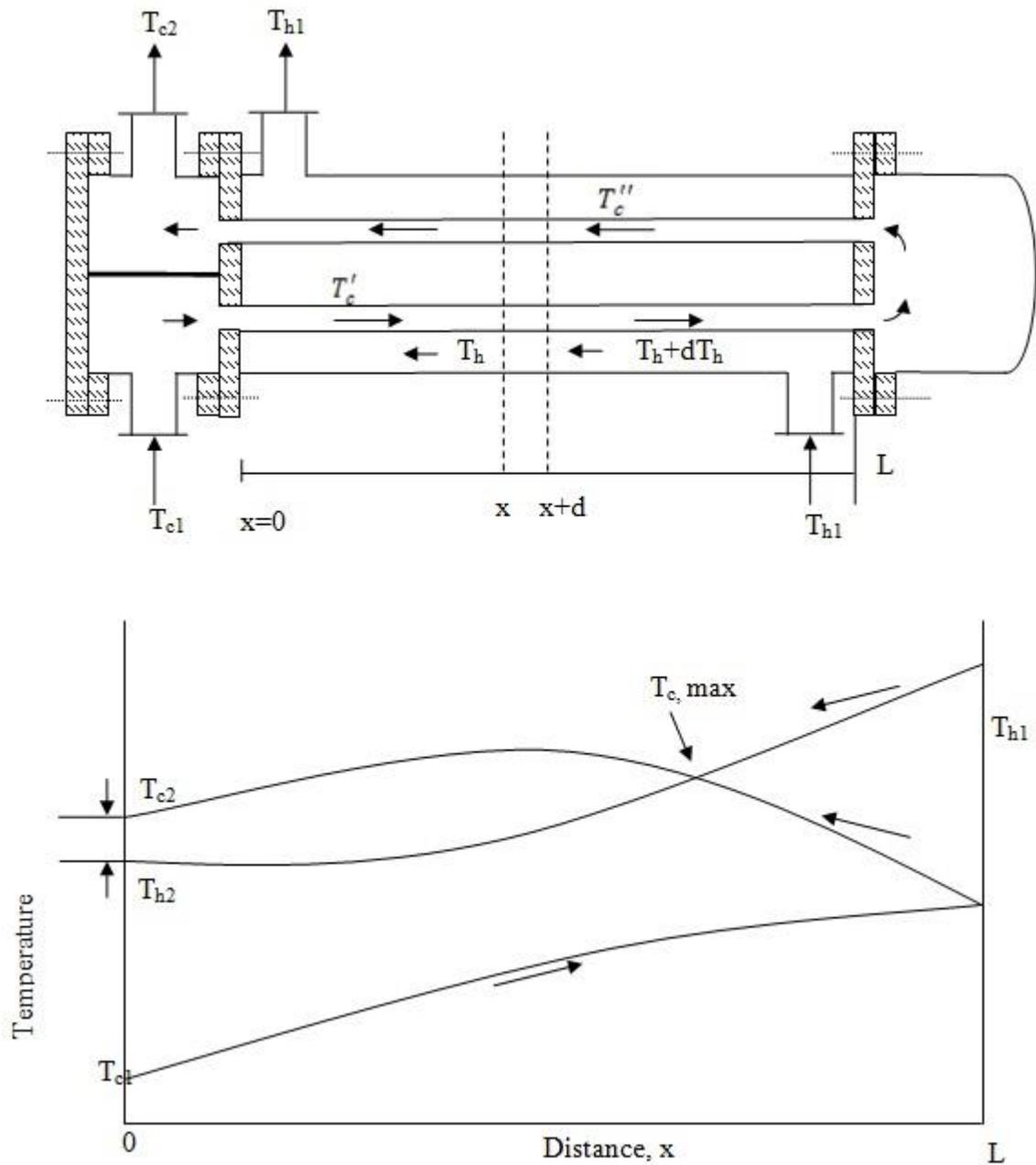


Fig. 10: 1-2 flow pattern and temperature profile in exchanger showing cross flow

At this temperature cross, the cold fluid temperature reaches the maximum at a point inside the exchanger and not at its exits. This temperature cross point also coincides with the point of intersection of the temperature profile of the hot fluid and the co-current zone of the cold fluid. The difference $(T_{c2} - T_{h2})$ is called the temperature cross of the exchanger. However, if the temperature cross does not appear then the $(T_{c2} - T_{h2})$ is called the approach. Moreover, on careful evaluation it can be seen that

for the multi shell side pass a significant length of the exchanger have cross flow pattern in the tube flow when the shell side fluid is migrating from one shell pass to another shell pass. Although the parallel flow or counter flow are quite similar, the parallel flow and counter flow heat exchangers differ greatly in the manner in which the fluid temperatures vary as the fluid pass through. The difference can be understood in the figure 11.

The fig.11 shows an important parameter, $m c_p$, the product of mass flow rate (m) and the specific heat, c_p , of the fluids. The product $m c_p$ is called the rate of heat capacity. The overall energy balance of the heat exchanger gives the total heat transfer between the fluids, q , expressed by eq.5,

$$q = \dot{m}_c c_{pc} (T_{co} - T_{ci}) = \dot{m}_h c_{ph} (T_{hi} - T_{ho}) \quad (5)$$

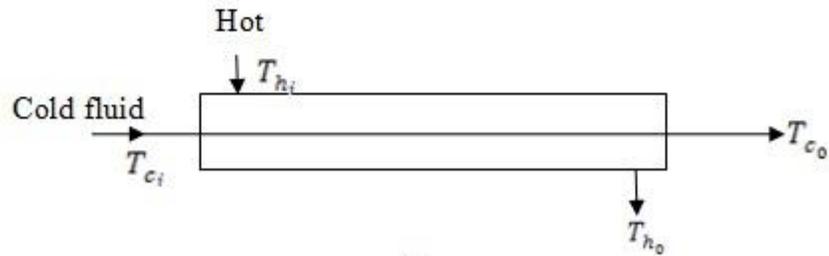
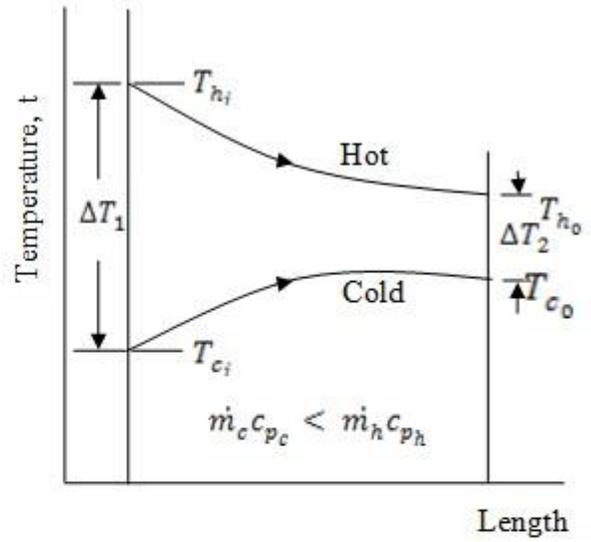
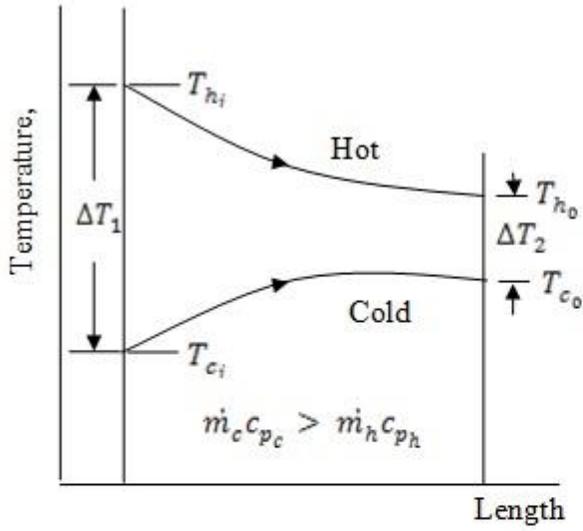
$$\Rightarrow \frac{\dot{m}_c c_{pc}}{\dot{m}_h c_{ph}} = \frac{(T_{hi} - T_{ho})}{(T_{co} - T_{ci})}$$

The fig.11 shows the relative variation of the two fluid temperatures through the heat exchanger, which is influenced by whether $\dot{m}_c c_{pc}$ is greater or less than $\dot{m}_h c_{ph}$. In particular, for counter flow, examination of the sketches in fig.11 shows that limiting condition for maximum heat transfer is determine by whether $\dot{m}_c c_{pc}$ is greater or less than $\dot{m}_h c_{ph}$. When, $\dot{m}_c c_{pc} > \dot{m}_h c_{ph}$ the maximum possible heat transfer is determined by the fact that the hot fluid can be cooled to the temperature of the cold fluid inlet. Thus, for $\dot{m}_c c_{pc} > \dot{m}_h c_{ph}$.

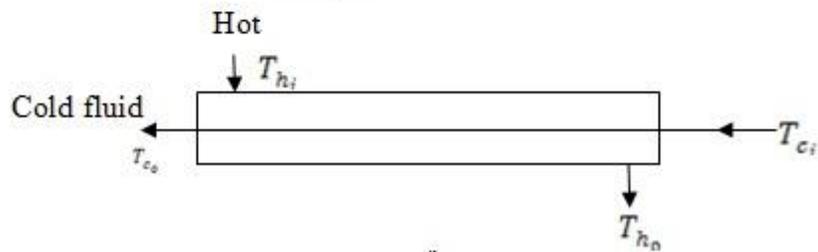
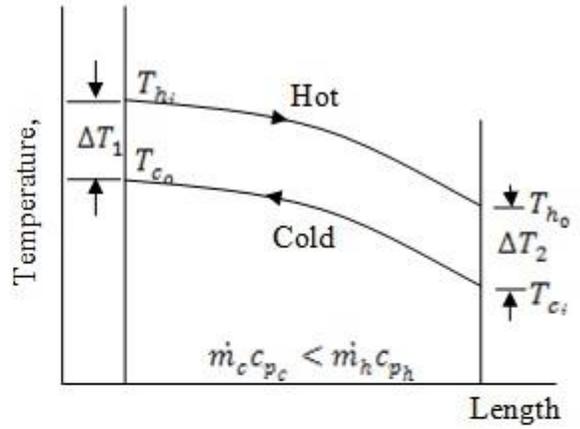
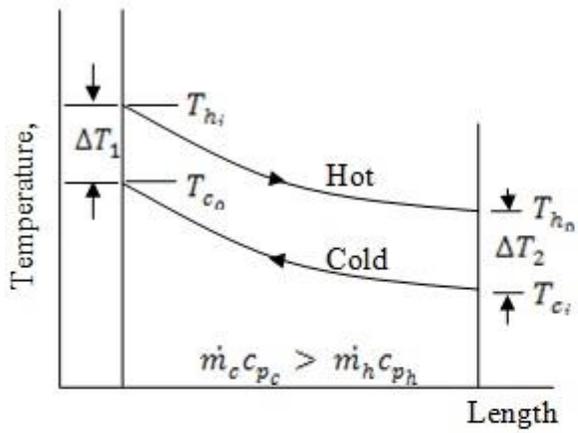
$$T_{ho} \rightarrow T_{ci}$$

$$q_{max} = \dot{m}_h c_{ph} (T_{hi} - T_{ho})$$

$$= \dot{m}_h c_{ph} (T_{hi} - T_{ci})$$



(a)



(b)

Fig.11: Temperature profiles of (a) parallel flow, and (b) counter flow, for different $\dot{m}c_p$ inequalities

For the other case when the limit is determined as the cold fluid is heated to the inlet temperature of the hot fluid:

For,

$$\begin{aligned}\dot{m}_c c_{pc} &< \dot{m}_h c_{ph} : T_{co} \rightarrow T_{ho} \\ q_{max} &= \dot{m}_c c_{pc} (T_{co} - T_{ci}) \\ &= \dot{m}_c c_{pc} (T_{ho} - T_{hi})\end{aligned}$$

Thus for the counter flow exchanger, the above two set equations show that the maximum possible heat exchanger is determined in terms of the inlet parameters. The maximum possible heat exchange may be determined (eq.6) by the fluid stream having low heat capacity rate;

$$q_{max,ccf} = (\dot{m}c_p)_{min} (T_{hi} - T_{ci}) \quad (6)$$

The subscript *ccf* denotes counter current flow. The $(\dot{m}c_p)_{min}$ is for the fluid having lower value of $(\dot{m}c_p)$.

In case of parallel flow, regardless of the relative sizes of the two stream the limiting heat transfer condition is determine by the fact that the two fluid streams approach the same outlet temperature. Thus,

$T_{ho} \rightarrow T_{co}$ condition can be found out by the weighted average of the inlet stream.

$$T_{ho} \rightarrow T_{co} \rightarrow \frac{\dot{m}_c c_{pc} T_{ci} + \dot{m}_h c_{ph} T_{hi}}{\dot{m}_c c_{pc} + \dot{m}_h c_{ph}}$$

Thus, the maximum possible heat transfer may be,

$$\begin{aligned}
 q_{max} &= \dot{m}_c c_{pc} (T_{co} - T_{ci}) \\
 &= \frac{1}{\frac{1}{\dot{m}_c c_{pc}} + \frac{1}{\dot{m}_h c_{ph}}} (T_{hi} - T_{ci})
 \end{aligned}$$

or,

$$q_{max,pf} = \frac{(\dot{m}c_p)_{min} (T_{hi} - T_{ci})}{1 + [(\dot{m}c_p)_{min} / (\dot{m}c_p)_{max}]} \quad (7)$$

The subscript *pf* represents parallel flow (co-current). From the above discussion and q_{max} equations (6 and 7) it can be calculated for a given inlet conditions the counter current flow arrangement always has a better potential for heat transfer as compared to parallel flow arrangement.

1.6 Why multi-pass exchangers?

The simplest type of heat exchangers is double pipe heat exchangers, which is inadequate for flow rates that cannot readily be handled in a few tubes. If several double pipes are used in parallel, the metal weight required for the outer tubes becomes so large that the shell and tube construction, such as 1-1 exchanger will be helpful. In that one shell serves for many tubes, is economical. The heat transfer coefficient of tube side and shell side fluid is very important and the individual heat transfer coefficients must be high enough to attain high overall heat transfer coefficient. As the shell would be quite large as compared to the tubes, the velocity and the turbulence of the shell side fluid is important.

In contrast, the 1-1 exchanger has limitations also. When the tube side flow is divided evenly among all the tubes, the velocity may be quite low, resulting in low heat transfer coefficient. There it may be required to increase the area to have the desired heat exchange for this low heat transfer coefficient. The area may be increased by increasing the length of the tube. However, the tube length requirement may be impractical for a given situation. Thus the number of tubes should be increased without increased the tube length. The increased number of tubes would also provide the increased velocity in the shell side resulting in the higher heat transfer coefficient. Therefore, multi-pass construction is needed, which would permit to use the practical and standard tube lengths. However, the disadvantages are that,

1. The construction of the exchangers become complex.
2. Parallel flow cannot be avoided.
3. Additional friction losses may occur.

It should be noted that generally even number of tube passes are used in multi pass exchanger.

1.7 LMTD correction factor

In the earlier chapter, we have seen for co-current or counter current flow system. The average driving force for heat transfer was defined by log mean temperature difference (LMTD). Thus the LMTD can be used for 1-1 exchangers for co-current and counter current. However, for multi pass exchangers (1-2, 2-4, etc.) the fluids are not always in co-current or counter current flow. The deviation for co-current or counter current flow causes a change in the average driving force. Therefore, in order to use true heat transfer driving force, a correction factor is required into the LMTD. Thus, the heat transfer rate can be written as (eq.8),

$$q = U_d A (F_T \Delta T_m) \quad (8)$$

where,

U_d = overall heat transfer coefficient including fouling/dirt

A = heat transfer area

$F_T \Delta T_m$ = true average temperature difference.

F_T = LMTD correction factor

It is to be noted that the following assumption have been considered for developing LMTD,

1. The overall heat transfer coefficient is constant throughout the exchanger
2. In case any fluid undergoes for phase change (e.g., in condenser), the phase change occurs throughout the heat exchanger and the constant fluid temperature prevails throughout the exchanger.
3. The specific heat and mass flow rate and hence the heat capacity rate, of each fluid is constant.
4. No heat is lost in to the surroundings.
5. There is no conduction in the direction of flow neither in the fluids nor in the tube or shell walls.
6. Each of the fluids may be characterized by a single temperature, at any cross section in the heat exchanger that is ideal transverse mixing in each fluid is presumed.

F_T , the LMTD correction factor can be directly obtained from available charts in the literature. These charts were prepared from the results obtained theoretically by solving the temperature distribution in multi-pass heat exchangers.

Figures 12 and 13 show the two generally used heat exchangers and their corresponding plots for finding F_T . It may be noted that the given figures have the representative plots and any standard book on heat transfer may be consulted for the accurate results.

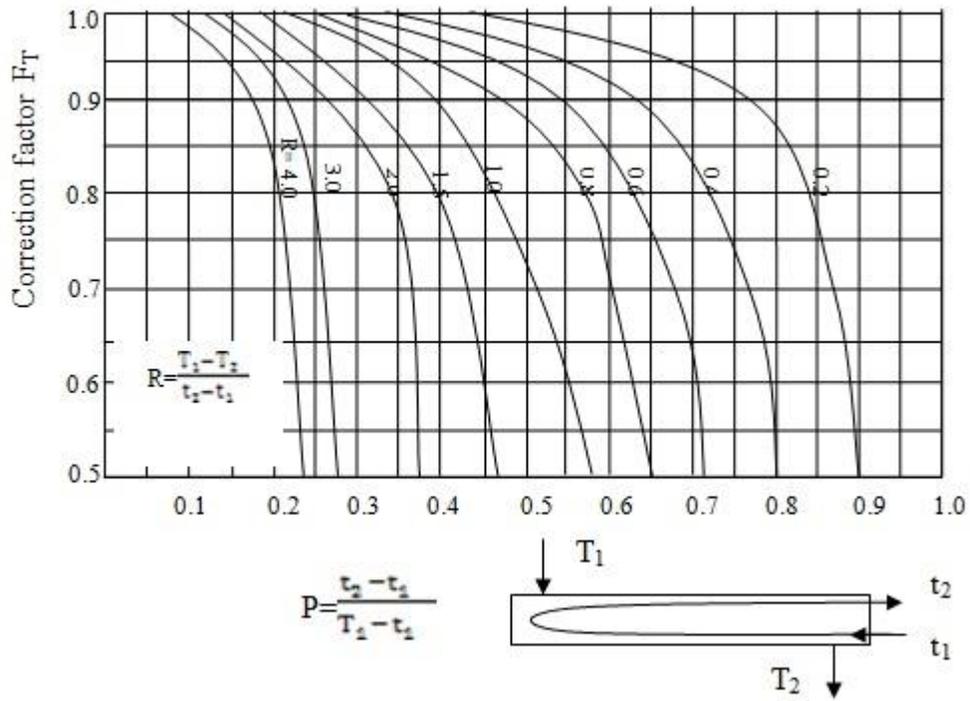


Fig.12: F_T plot for 1-2 exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet

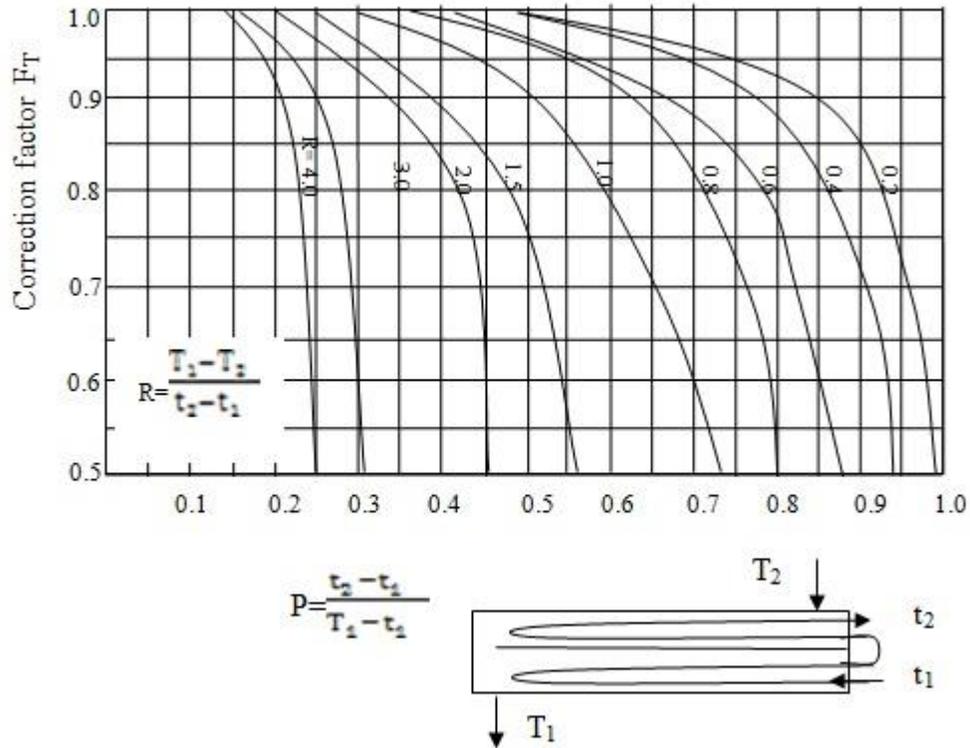


Fig. 13 F_T plot for 2-4 exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet

It should be noted that in case of condensation or evaporation the correction factor becomes unity ($F_T = 1$). While designing a heat exchanger, the rule of thumb is that the F_T should not be less than 0.8.

1.8 Individual heat transfer coefficient

we have seen that the overall heat transfer coefficient can be calculated provide the parameters are known including individual heat transfer coefficients. In this, section we will discuss how to find out the individual heat transfer coefficient, which is basically based on the well-established correlations and discussed earlier also.

The heat transfer coefficient (h_i) for the tube side fluid in a heat exchanger can be calculated either by Sieder-Tate equation or by Colburn equation discussed in earlier chapter.

However, the shell side heat transfer coefficient (h_o) cannot be so easily calculated because of the parallel, counter as well as cross flow patterns of the fluid. Moreover, the fluid mass velocity as well as cross sectional area of the fluid streams vary as the fluid crosses the tube bundle. The leakages between baffles and shell, baffle and tubes, short circuit some of the shell fluid thus reduces the effectiveness of the exchanger.

Generally, modified Donohue equation (eq.9) (suggested by D.Q. Kern) is used to predict the h_o ,

$$\frac{h_o D_H}{k_o} = 0.36 \left(\frac{D_H G_s}{\mu} \right)^{0.55} \left(\frac{\mu}{k_o} \right)^{0.33} \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (9)$$

where,

- h_o = shell side heat transfer coefficient
- D_H = hydraulic diameter of the shell side
- k_o = thermal conductivity of the shell side fluid
- G_s = mass flow rate of the shell side

The D_H and G_s can be easily calculated if the geometry of the tube arrangement in the shell is known. The tubes may be generally arranged as a square or triangular pitch, as shown in figure 14.

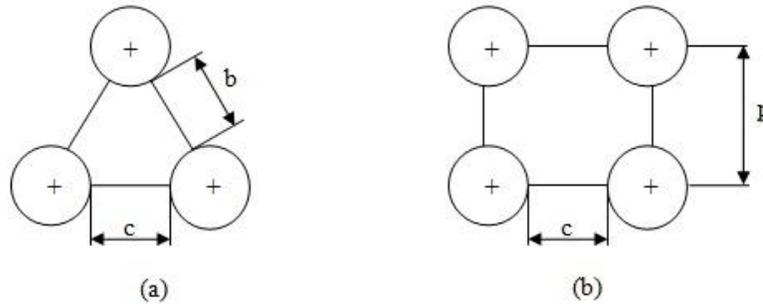


Fig.14: Tube arrangement in the shell (a) triangular pitch (b) square pitch

The hydraulic diameter (D_H) for tubes on square pitch

$$\frac{4 \left(p^2 - \pi \frac{d_o^2}{4} \right)}{\pi d_o}$$

D_H For 60° triangular pitch =

$$\frac{4 \left[(0.5p)(0.86p) - \pi \frac{d_o^2}{8} \right]}{\pi d_o / 2}$$

where,

- d_o = outer diameter of tube
- p = tube pitch

$$G_s = \frac{\dot{v}_s}{a_s}$$

where,

$$\begin{aligned} \dot{v}_s &= \text{flow rate of shell fluid} \\ a_s &= \text{shell side flow area} \end{aligned}$$

Shell side flow area can be calculated using baffle information number of tubes in the shell and tube arrangement. If 25% cut baffles are used, that means the shell side flow will be from this 25% area. However we have to reduce the area of the pipes which are accumulated in this opening. So depending upon the information we may determine the shell side fluid flow area. It may also be found out by the following way,

$$a_s = \frac{CBD_s}{p}$$

where,

$$\begin{aligned} C &= \text{tube clearance} \\ B &= \text{baffle spacing} \\ D_s &= \text{inside diameter of shell} \\ p &= \text{pitch of the tube} \end{aligned}$$

1.9 Pressure drop in the heat exchanger

Pressure drop calculation is an important task in heat exchanger design. The pressure drops in the tube side as well as shell side are very important and quite a few co-relations are available in the literature. One such co-relation is given below in the subsequent subsection.

Correlation for tube side pressure drop (eq. 10)

$$\Delta P_{t,f} = \frac{f G_f L n}{2g\rho_t d_t (\mu/\mu_w)^m} \quad (10)$$

where,

$$\Delta P_{t,f} = \text{total pressure drop in the bundle of tube}$$

f = friction factor (can be found out from Moody's chart)

G_t = mass velocity of the fluid in the tube

L = tube length

n = no of tube passes

g = gravitational acceleration

ρ_t = density of the tube fluid

d_i = inside diameter of the tube

$m=0.14$ for $Re > 2100$

0.25 for $Re < 2100$

The above correlation is for the pressure drop in the tubes owing to the frictional losses. However in case of multi pass flow direction of the flow in the tube changes when flow is from 1-pass to another pass and the pressure losses due to the change in direction is called return-loss. The return-loss ($\Delta P_{t,r}$) is given by eq.11,

$$\Delta p_{t,r} = 4\left(\frac{v_t^2}{2g}\right)\rho_t \times n \quad (11)$$

n = no of tube pass

v_t = velocity of the tube fluid

ρ_t = density of the tube fluid

Therefore, the total tube side pressure drop will be,

$$\Delta p_t = \Delta P_{t,f} + \Delta P_{t,r}$$

Correlation for shell side pressure drop

The following correlation (eq.12) may be used for an unbaffled shell,

$$\Delta p_s = \frac{f_s G_s^2 L n_s}{2g \rho_s D_h (\mu/\mu_w)^{0.14}} \quad (12)$$

The above equation can be modified to the following form (eq.13) for a baffled shell,

$$\Delta p_s = \frac{f_s G_s^2 D_{si} (n_b + 1)}{2g \rho_s D_h (\mu/\mu_w)^{0.14}} \quad (13)$$

where

L = shell length
 n_s = no of shell pass
 n_b = no of baffles
 ρ_s = shell side fluid density
 G_s = shell side mass velocity
 D_h = hydraulic diameter of the shell
 D_{si} = inside diameter of shell
 f_s = shell side friction factor

The hydraulic diameter (D_h) for the shell can be calculated by the following equation (eq.14),

$$D_h = \frac{4 [\pi D_s^2 / 4 - \pi d_o^2 n_t / 4]}{\pi d_o n_t + \pi D_s} \quad (14)$$

where,

n_t = number of tubes in the shell
 d_o = outer diameter of the tube

The friction factor (f_s) can be obtained by the Moody's chart for the corresponding Reynolds number $\left(R_s = \frac{D_h G_s}{\mu} \right)$.

1.10 Heat transfer effectiveness and number of transfer units (NTU)

The LMTD is required to be calculated for the evaluation of heat exchanger performance. However, the LMTD cannot be directly calculated unless all the four terminal temperatures ($T_{c,i}$, $T_{c,o}$, $T_{h,i}$, $T_{h,o}$) of both the fluids are known.

Sometimes the estimation of the exchanger performance (q) is required to be calculated on the given inlet conditions, and the outlet temperature are not known until q is determined. Thus the problem depends on the iterative calculations. This type of problem may be taken care of using performance equivalent in terms of heating effectiveness parameter (η), which is defined as the ratio of the actual heat transfer to the maximum possible heat transfer. Thus,

$$\eta = \frac{q}{q_{max}} \quad (15)$$

For an infinite transfer area the most heat would be transferred in counter-current flow and the q_{max} will be dependent on the lower heat capacity fluid as such,

$$q_{max} = \dot{m}_c c_{pc} (T_{hi} - T_{ci}) \text{ for } \dot{m}_c c_{pc} < \dot{m}_h c_{ph}$$

$$q_{max} = \dot{m}_h c_{ph} (T_{hi} - T_{ci}) \text{ for } \dot{m}_c c_{pc} > \dot{m}_h c_{ph}$$

The actual heat transfer

$$q = \dot{m}_c c_{pc} (T_{co} - T_{ci}) = \dot{m}_h c_{ph} (T_{ho} - T_{hi})$$

$$\eta = \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})} \text{ for } \dot{m}_c c_{pc} < \dot{m}_h c_{ph}$$

$$\eta = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})} \text{ for } \dot{m}_c c_{pc} > \dot{m}_h c_{ph}$$

The capacity ratio, which is the relative thermal size of the two fluid streams, is defined as,

$$C_R = \frac{(\dot{m}c_p)_{min}}{(\dot{m}c_p)_{max}}$$

On careful analysis, we can say that

$U \cdot A$: Heat exchange capacities per unit temperature difference.

This thermal sizing ($U \cdot A$) can be non-dimensionalised by dividing it to the storage capacity of one of the fluid streams. Given $(\dot{m}c_p)_{min}$ limits the maximum heat transfers. The non-dimensional term obtained is known as the number of transfer units (NTU)

$$NTU = \frac{UA}{(\dot{m}c_p)_{min}}$$

It should be noted that

$$\eta = \eta(C_R, NTU)$$

The actual determination of this function may be done using heat balances for the streams. For a parallel flow exchanger the relation is shown below

$$\eta = \frac{1 - \exp[-(C_R + 1) NTU]}{C_R + 1}$$

$$NTU = \frac{-\ln[1 - (C_R + 1)\eta]}{C_R + 1}$$

The above relation is true for both the condition (i) $\dot{m}_c c_{pc} < \dot{m}_h c_{ph}$ (ii) $\dot{m}_c c_{pc} > \dot{m}_h c_{ph}$

Similarly the functional relationship for counter –current exchanger is

$$\eta = \frac{1 - \exp[-(1 - C_R) NTU]}{1 - C_R \exp[-(1 - C_R) NTU]} \quad (16)$$

$$NTU = \frac{\ln[(1 - \eta)/(1 - \eta C_R)]}{C_R - 1} \quad (17)$$

The previous relation (eq.16 and 17) were for 1-1 exchanger. The relation for 1-2 exchanger (counter current) is given by eq.18, 19),

$$\eta = 2 \left[(1 + C_R) + (1 + C_R^2)^{1/2} \frac{1 + \exp[-(1 + C_R^2)^{1/2} NTU]}{1 - \exp[-(1 - C_R^2)^{1/2} NTU]} \right]^{-1} \quad (18)$$

$$NTU = -(1 + C_R^2)^{-1/2} \ln \left[\frac{2/\eta - 1 - C_R - (1 + C_R^2)^{1/2}}{2/\eta - 1 - C_R + (1 + C_R^2)^{1/2}} \right] \quad (19)$$

When the fluid streams are condensing in a 1-1 pass exchanger (fig.15) as shown below,

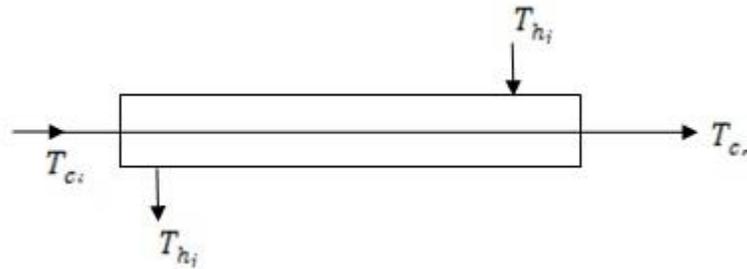


Fig.15: Condenser with the temperature nomenclature

the following relation arrives.

$$\eta = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} = 1 - \exp(-NTU)$$

$$NTU = -\ln(1 - \eta)$$

Problems

Deionized water flows through the inner tube of 30-mm diameter in a thin-walled concentric tube heat exchanger of 0.19-m length. Hot water at 95 °C flows in the annulus formed with the outer tube of 60-mm diameter. The deionized water is to be heated from 40°C to 60°C at a flow rate of 5 kg/s. The thermo physical properties of the fluids are:

	DEIONIZED WATER	PROCESS WATER
kg/m³	982.3	967.1
c_p(J/kg.K)	4181	4197
k(W/m.K)	0.643	0.673
N.s/m²	548	324
pr	3.56	2.02

(a) Considering a parallel-flow configuration of the heat exchanger, determine the minimum flow rate required for the hot process water. (b) Determine the overall heat transfer coefficient required for the conditions of part a. (c) Considering a counter flow configuration, determine the minimum flow rate required for the hot process water. What is the effectiveness of the exchanger for this situation?

Solution

For a fixed term $T_{h,i}$, $(\dot{m})_h$ will be a minimum when $T_{h,o}$ is a minimum. With the parallel flow configuration, this requires that $T_{h,o}=T_{c,o}=60^\circ\text{C}$. Hence,

$$\dot{m}_{h,\min} = \frac{(mc)_c (T_{c,o} - T_{c,i})}{c_h (T_{h,i} - T_{h,o})} = \frac{5\text{kg/s} \times 4181\text{J/kg.K} (60 - 40)^\circ\text{C}}{4197\text{J/kg.K} (95 - 60)^\circ\text{C}} = 2.85\text{kg/s}$$

(b) From the rate equation and the log mean temperature relation,

$$q = UA\Delta T_{lm,PF} \qquad \Delta T_{lm,PF} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

(c) With the CF arrangements \dot{m}_h will be a minimum when $T_{h,o}$ is a minimum. This requires that $T_{h,o}$ is a minimum. This requires that $T_{h,o}$ is a minimum. This requires that $T_{h,o}=T_{c,i}=40^\circ\text{C}$. Hence, from the overall energy balance,

$$\dot{m} = \frac{5\text{kg/s} \times 4181\text{J/kg.K} (60 - 40)\text{K}}{4197\text{J/kg.K} (95 - 40)\text{K}} = 1.81\text{kg/s}$$

For this condition, $C_{\min}=C_h$ which is cooled from $T_{h,i}$ to $T_{c,i}$, hence $\epsilon=1$

Comments: For the counter flow arrangement, the heat exchanger must be infinitely long.

2. Calculate log mean temperature difference for the following shell and tube heat exchanger for two possible arrangements (i) counter-current and (ii) co-current flow patterns. Hot fluid in temperature $TH_1 = 100\text{ }^{\circ}\text{C}$ Hot fluid out temperature $TH_2 = 90\text{ }^{\circ}\text{C}$ Cold fluid out temperature $TC_2 = 50\text{ }^{\circ}\text{C}$ Cold fluid in temperature $TC_1 = 30\text{ }^{\circ}\text{C}$

(LMTD),

$\Delta T_1 \rightarrow$ the temperature difference between hot and cold fluids at one end of the heat exchanger

$\Delta T_2 \rightarrow$ the temperature difference between hot and cold fluids at the other end of the heat exchanger

(i) For counter current heat exchanger,

$\Delta T_1 = TH_1 - TC_2 = 100 - 50 = 50\text{ }^{\circ}\text{C}$ (At one end hot fluid enters and cold fluid exits.)

$\Delta T_2 = TH_2 - TC_1 = 90 - 30 = 60\text{ }^{\circ}\text{C}$

(At the other end cold fluid enters and hot fluid exits.)

by definition given above, LMTD for counter current flow = $(60-50) / \ln(60/50) = 10 / 0.1823 = 54.85\text{ }^{\circ}\text{C}$.

(ii) For co-current heat exchanger,

$\Delta T_1 = TH_1 - TC_1 = 100 - 30 = 70\text{ }^{\circ}\text{C}$ (At first end hot and cold fluids enter the heat exchanger.)

$\Delta T_2 = TH_2 - TC_2 = 90 - 50 = 40\text{ }^{\circ}\text{C}$



SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE

www.sathyabama.ac.in

SCHOOL OF BIO & CHEMICAL ENGINEERING

DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – IV – Heat Transfer – SCHA1301

1.0 Radiation Heat Transfer

It has been observed that the heat transfer studies were based on the fact that the temperature of a body, a portion of a body, which is hotter than its surroundings, tends to decrease with time. The decrease in temperature indicates a flow of energy from the body. In all the previous chapters, limitation was that a physical medium was necessary for the transport of the energy from the high temperature source to the low temperature sink. The heat transport was related to conduction and convection and the rate of heat transport was proportional to the temperature difference between the source and the sink.

Now, if we observe the heat transfer from the Sun to the earth atmosphere, we can understand that there is no medium exists between the source (the Sun) and the sink (earth atmosphere). However, still the heat transfer takes place, which is entirely a different energy transfer mechanism takes place and it is called thermal radiation.

Thermal radiation is referred when a body is heated or exhibits the loss of energy by radiation. However, more general form “radiation energy” is used to cover all the other forms. The emission of other form of radiant energy may be caused when a body is excited by oscillating electrical current, electronic bombardment, chemical reaction etc. Moreover, when radiation energy strikes a body and is absorbed, it may manifest itself in the form of thermal internal energy, a chemical reaction, an electromotive force, etc. depending on the nature of the incident radiation and the substance of which the body is composed.

In this chapter, we will concentrate on thermal radiation (emission or absorption) that on radiation produced by or while produces thermal excitation of a body.

There are many theories available in literature which explains the transport of energy by radiation. However, a dual theory is generally accepted which enables to explain the radiant energy in the characterisation of a wave motion (electromagnetic wave motion) and discontinuous emission (discrete packets or quanta of energy).

An electromagnetic wave propagates at the speed of light (3×10^8 m/s). It is characterised by its wavelength λ or its frequency ν related by

$$c = \lambda\nu \quad (1)$$

Emission of radiation is not continuous, but occurs only in the form of discrete quanta. Each quantum has energy

$$E = h\nu \quad (2)$$

where, $h = 6.6246 \times 10^{-34}$ J.s, is known as Planck's constant.

Table 1 shows the electromagnetic radiation covering the entire spectrum of wavelength

Table 1: Electromagnetic radiation for entire spectrum of wavelength

Type	Band of wavelength (μm)
Cosmic rays	upto 4×10^{-7}
Gamma rays	4×10^{-7} to 1.4×10^{-4}
X-rays	1×10^{-5} to 2×10^{-2}
Ultraviolet rays	5×10^{-3} to 3.9×10^{-1}
Visible light	3.9×10^{-1} to 7.8×10^{-1}
Infrared rays	7.8×10^{-1} to 1×10^3
Thermal radiation	1×10^{-1} to 1×10^2
Microwave, radar, radio waves	1×10^3 to 5×10^{10}

It is to be noted that the above band is in approximate values and do not have any sharp boundary.

1.1 Basic definition pertaining to radiation

Before we further study about the radiation it would be better to get familiarised with the basic terminology and properties of the radiant energy and how to characterise it.

As observed in the table 1 that the thermal radiation is defined between wavelength of about 1×10^{-1} and 1×10^2 μm of the electromagnetic radiation. If the thermal radiation is emitted by a surface, which

is divided into its spectrum over the wavelength band, it would be found that the radiation is not equally distributed over all wavelength. Similarly, radiation incident on a system, reflected by a system, absorbed by a system, etc. may be wavelength dependent. The dependence on the wavelength is generally different from case to case, system to system, etc. The wavelength dependency of any radiative quantity or surface property will be referred to as a spectral dependency. The radiation quantity may be monochromatic (applicable at a single wavelength) or total (applicable at entire thermal radiation spectrum). It is to be noted that radiation quantity may be dependent on the direction and wavelength both but we will not consider any directional dependency. This chapter will not consider directional effect and the emissive power will always used to be (hemispherical) summed overall direction in the hemisphere above the surface.

1.2 Emissive power

It is the emitted thermal radiation leaving a system per unit time, per unit area of surface. The total emissive power of a surface is all the emitted energy, summed over all the direction and all wavelengths, and is usually denoted as E . The total emissive power is found to be dependent upon the temperature of the emitting surface, the subsystem which this system is composed, and the nature of the surface structure or texture.

The monochromatic emissive power E_λ , is defined as the rate, per unit area, at which the surface emits thermal radiation at a particular wavelength λ . Thus the total and monochromatic hemispherical emissive power are related by

$$E = \int_0^{\infty} E_\lambda d\lambda \quad (3)$$

and the functional dependency of E_λ on λ must be known to evaluate E .

1.3 Radiosity

It is the term used to indicate all the radiation leaving a surface, per unit time and unit area.

$$J = \int_0^{\infty} J_\lambda d\lambda \quad (4)$$

where, J and J_λ are the total and monochromatic radiosity.

The radiosity includes reflected energy as well as original emission whereas emissive power consists

of only original emission leaving the system. The emissive power does not include any energy leaving a system that is the result of the reflection of any incident radiation.

1.4 Irradiation

It is the term used to denote the rate, per unit area, at which thermal radiation is incident upon a surface (from all the directions). The irradiative incident upon a surface is the result of emission and reflection from other surfaces and may thus be spectrally dependent.

$$G = \int_0^{\infty} G_{\lambda} d\lambda \tag{5}$$

where, G and G_{λ} are the total and monochromatic irradiation.

Reflection from a surface may be of two types specular or diffusive as shown in fig.1.

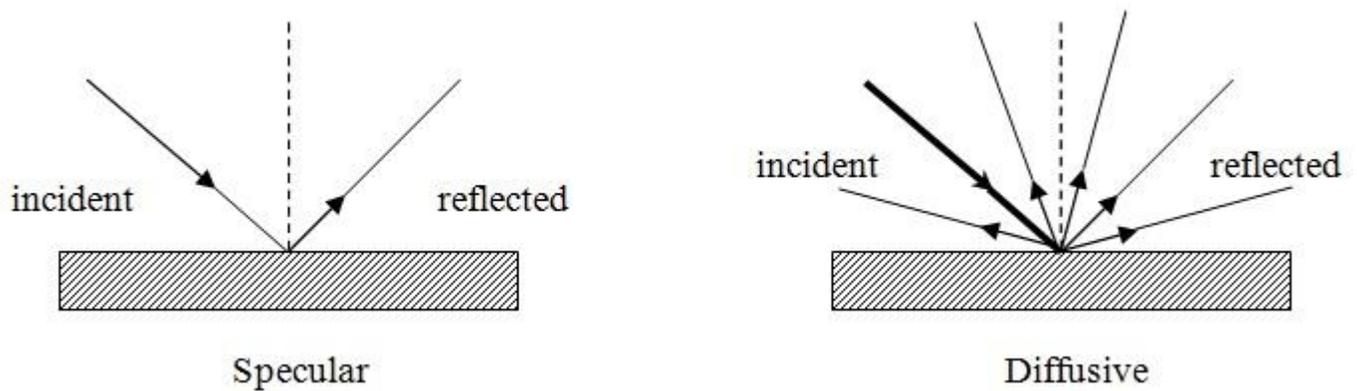


Fig. 1: (a) Specular, and (b) diffusive radiation

Thus,

$$J = E + \rho G \tag{6}$$

1.5 Absorptivity, reflectivity, and transmittivity:

The emissive power, radiosity, and irradiation of a surface are inter-related by the reflective, absorptive, and transmissive properties of the system.

When thermal radiation is incident on a surface, a part of the radiation may be reflected by the surface, a part may be absorbed by the surface and a part may be transmitted through the surface as shown in

fig.2. These fractions of reflected, absorbed, and transmitted energy are interpreted as system properties called reflectivity, absorptivity, and transmissivity, respectively.

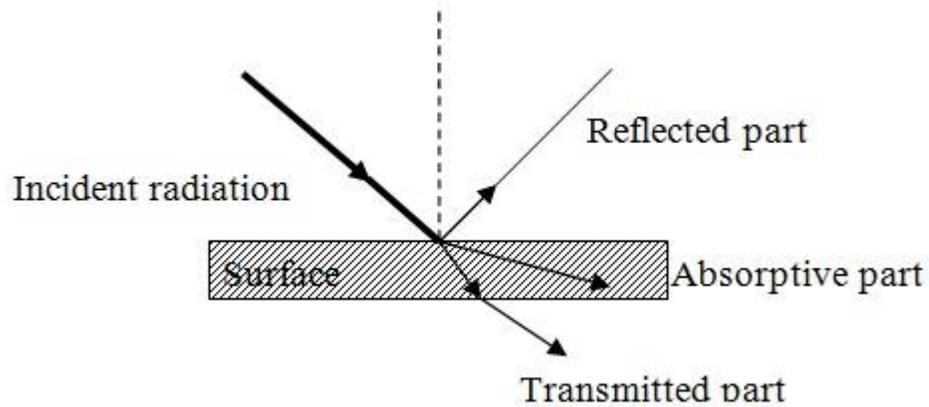


Fig. 2: Reflection, absorption and transmitted energy

Thus using energy conservation,

$$\rho + \alpha + \tau = 1 \tag{7}$$

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \tag{7}$$

where, ρ , α and τ are total reflectivity, total absorptivity, and total transmissivity. The subscript λ indicates the monochromatic property.

In general the monochromatic and total surface properties are dependent on the system composition, its roughness, and on its temperature.

Monochromatic properties are dependent on the wavelength of the incident radiation, and the total properties are dependent on the spectral distribution of the incident energy.

Most gases have high transmissivity, i.e. $\tau \approx 1$ and $\rho = \alpha = 0$ (like air at atmospheric pressure). However, some other gases (water vapour, CO₂ etc.) may be highly absorptive to thermal radiation, at least at certain wavelength.

Most solids encountered in engineering practice are opaque to thermal radiation ($\tau \approx 0$). Thus for thermally opaque solid surfaces,

$$\rho + \alpha = 1 \quad (6)$$

Another important property of the surface of a substance is its ability to emit radiation. Emission and radiation have different concept. Reflection may occur only when the surface receives radiation whereas emission always occurs if the temperature of the surface is above the absolute zero. Emissivity of the surface is a measure of how good it is an emitter.

1.6 Blackbody radiation

In order to evaluate the radiation characteristics and properties of a real surface it is useful to define an ideal surface such as the perfect blackbody. The perfect blackbody is defined as one which absorbs all incident radiation regardless of the spectral distribution or directional characteristic of the incident radiation.

$$\alpha = \alpha_\lambda = 1$$

$$\rho = \rho_\lambda = 0$$

A blackbody is black because it does not reflect any radiation. The only radiation leaving a blackbody surface is original emission since a blackbody absorbs all incident radiation. The emissive power of a blackbody is represented by E_b , and depends on the surface temperature only.

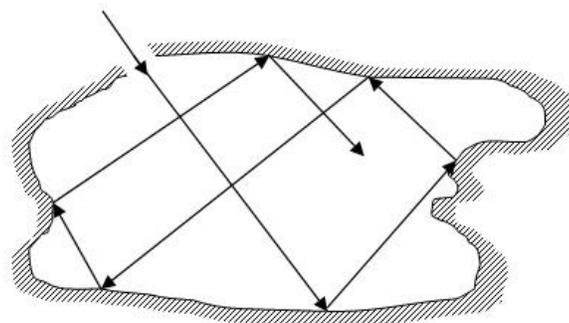


Fig. 3: Example of a near perfect blackbody

It is possible to produce a near perfect blackbody as shown in fig.3.

Figure.2 shows a cavity with a small opening. The body is at isothermal state, where a ray of incident radiation enters through the opening will undergo a number of internal reflections. A portion of the radiation absorbed at each internal reflection and a very little of the incident beam ever find the way out through the small hole. Thus, the radiation found to be evacuating from the hole will appear to that coming from a nearly perfect blackbody.

1.6.1 Planck's law

A surface emits radiation of different wavelengths at a given temperature (theoretically zero to infinite wavelengths). At a fixed wavelength, the surface radiates more energy as the temperature increases.

Monochromatic emissive power of a blackbody is given by eq.10.

$$E_{b,\lambda} = \frac{2\pi hc^2 \lambda^{-5}}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \quad (7)$$

where, $h = 6.6256 \times 10^{-34}$ JS; Planck's constant

$c = 3 \times 10^8$ m/s; speed of light

T = absolute temperature of the blackbody

λ = wavelength of the monochromatic radiation emitted

k = Boltzmann constant

Equation.10 is known as Planck's law. Figure.4 shows the representative plot for Planck's distribution.

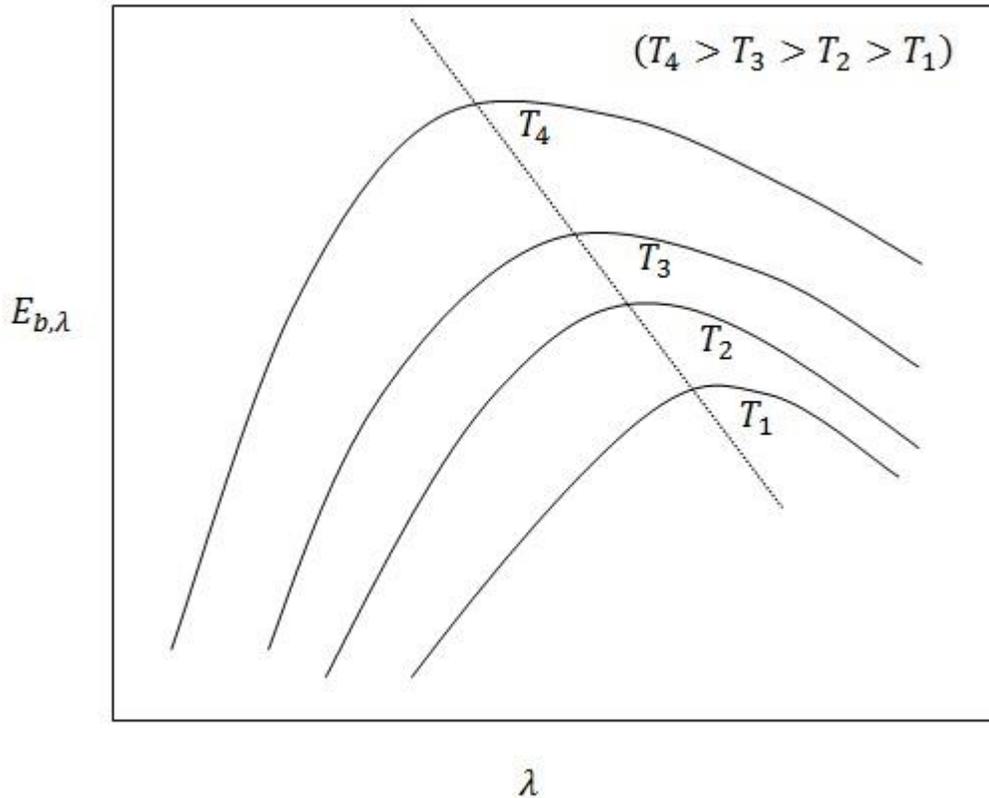


Fig. 4: Representative plot for Planck's distribution

1.6.2 Wien's law

Figure.4 shows that as the temperature increases the peaks of the curve also increases and it shift towards the shorter wavelength. It can be easily found out that the wavelength corresponding to the peak of the plot (λ_{max}) is inversely proportional to the temperature of the blackbody (Wein's law) as shown in eq.11.

$$\lambda_{max} T = 2898 \quad (11)$$

Now with the Wien's law or Wien's displacement law, it can be understood if we heat a body, initially the emitted radiation does not have any colour. As the temperature rises the λ of the radiation reach the visible spectrum and we can able to see the red colour being height λ (for red colour). Further increase in temperature shows the white colour indicating all the colours in the light.

1.6.3 The Stefan-Boltzmann law for blackbody

Josef Stefan based on experimental facts suggested that the total emissive power of a blackbody is proportional to the fourth power of the absolute temperature. Later, Ludwig Boltzmann derived the same using classical thermodynamics. Thus the eq.12 is known as Stefan-Boltzmann law,

$$E_b = \int_0^{\infty} E_{b\lambda}(\lambda, T) d\lambda$$

$$E_b = \sigma T^4 \quad (12)$$

where, E_b is the emissive power of a blackbody, T is absolute temperature, and $\sigma (= 5.67 \times 10^{-8} \text{W/m}^2/\text{K}^4)$ is the Stefan-Boltzmann constant.

The Stefan-Boltzmann law for the emissive power gives the total energy emitted by a blackbody defined by eq.3.

Special characteristic of blackbody radiation

It has been shown that the irradiation field in an isothermal cavity is equal to E_b . Moreover, the irradiation was same for all planes of any orientation within the cavity. It may then be shown that the intensity of the blackbody radiation, I_b , is uniform. Thus, blackbody radiation is defined as,

$$E_b = \pi I_b \quad (13)$$

where, $I_b = \int_0^{\infty} I_{b\lambda} d\lambda$ is the total intensity of the radiation and is called the spectral radiation intensity of the blackbody.

1.6.4 Kirchhoff's law

Consider an enclosure as shown in fig.2 and a body is placed inside the enclosure. The radiant heat flux (q) is incident onto the body and allowed to come into temperature equilibrium. The rate of energy absorbed at equilibrium by the body must be equal to the energy emitted.

$$EA = \alpha qA$$

$$E = \alpha q \quad (7.14)$$

where, E is the emissive power of the body, α is absorptivity of the of the body at equilibrium temperature, and A is the area of the body.

Now consider the body is replaced by a blackbody i.e. $E \rightarrow E_b$ and $\alpha = 1$, the equation.14 becomes

$$E_b = q \quad (15)$$

Dividing eq.14 by eq.15,

$$\frac{E}{E_b} = \alpha \quad (16)$$

At this point we may define emissivity, which is a measure of how good the body is an emitter as compared to blackbody. Thus the emissivity can be written as the ratio of the emissive power to that of a blackbody,

$$\frac{E}{E_b} = \epsilon \quad (17)$$

On comparing eq.16 and eq.17, we get

$$\epsilon = \alpha \quad (18)$$

Equation 18 is the Kirchoff's law, which states that the emissivity of a body which is in thermal equilibrium with its surrounding is equal to its absorptivity of the body. It should be noted that the source temperature is equal to the temperature of the irradiated surface. However, in practical purposes it is assumed that emissivity and absorptivity of a system are equal even if it is not in thermal equilibrium with the surrounding. The reason being the absorptivity of most real surfaces is relatively insensitive to temperature and wavelength. This particular assumption leads to the concept of grey body. The emissivity is considered to be independent of the wavelength of radiation for grey body.

1.7 Grey body

If grey body is defined as a substance whose monochromatic emissivity and absorptivity are independent of wavelength. A comparative study of grey body and blackbody is shown in the table.2.

Table-2: Comparison of grey and blackbody

Blackbody	Grey body
Ideal body	Ideal body
Emissivity independent of wavelength	Emissivity is independent of wavelength
Absorptivity is independent of wavelength	Absorptivity (α) is independent of wavelength
$\epsilon = 1$	$\epsilon < 1$
$\alpha = 1$	$\alpha < 1$

1.8 Radiative heat exchanger between surfaces

Till now we have discussed fundamental aspects of various definitions and laws. Now we will study the heat exchange between two or more surfaces which is of practical importance. The two surfaces which are not in direct contact, exchanges the heat due to radiation phenomena. The factors those determine the rate of heat exchange between two bodies are the temperature of the individual surfaces, their emissivities, as well as how well one surface can see the other surface. The last factor is known as view factor, shape factor, angle factor or configuration factor.

1.8.1 View factor

In this section we would like to find the energy exchange between two black surfaces having area A_1 and A_2 , respectively, and they are at different temperature and have arbitrary shape and orientation with respect to each other. In order to find the radiative heat exchange between the bodies we have to first define the view factor as

F_{12} = fraction of the energy leaving surface 1 which reaches surface 2

F_{21} = fraction of the energy leaving surface 2 which reaches surface 1 or in general,

F_{mn} = fraction of the energy leaving surface m which reaches surface n

Thus the energy leaving surface 1 and arriving at surface 2 is $E_{b1}A_1F_{12}$ and the energy leaving surface 2 and arriving at surface 1 is $E_{b2}A_2F_{21}$. All the incident radiation will be absorbed by the blackbody and the net energy exchange will be,

$$Q = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$$

At thermal equilibrium between the surfaces $Q_{12} = 0$ and $E_{b1} = E_{b2}$, thus

$$0 = E_{b1} (A_1F_{12} - A_2F_{21})$$

$$A_1F_{12} - A_2F_{21} \tag{19}$$

Equation.19 is known as reciprocating relation, and it can be applied in general way for any blackbody surfaces.

$$A_iF_{ij} - A_jF_{ji} \tag{20}$$

Though the relation is valid for blackbody it may be applied to any surface as long as diffuse radiation is involved.

Relation between view factors

In this section we will develop some useful relation of view factor considering fig..5

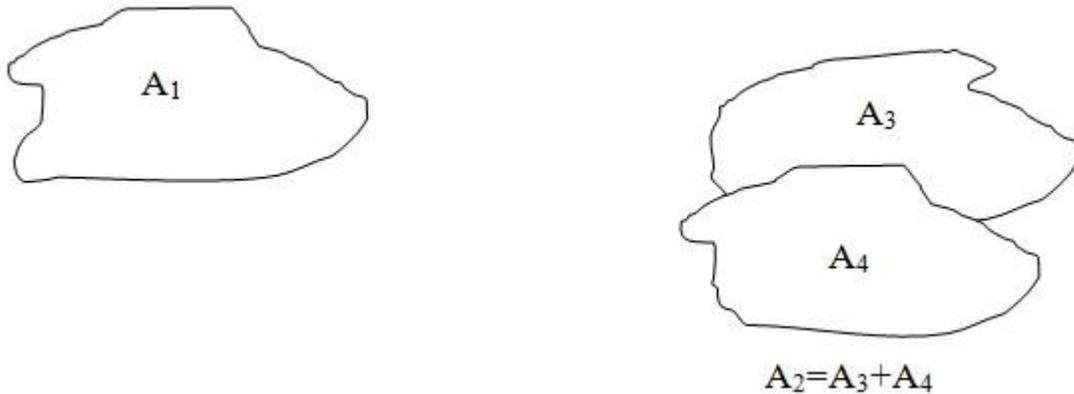


Fig. 5: Exchange of energy between area A_1 and A_2 (A is area of blackbody)

View factor for radiation from A_1 to the combined area A_2 ,

$$F_{12} = F_{13} + F_{14} \tag{21}$$

and using the reciprocating relations for surface 1 and 4,

$$A_1 F_{14} = A_4 F_{41} \quad (22)$$

Using eq..21 and22,

$$F_{41} = \frac{A_1}{A_4} F_{14}$$

$$F_{41} = \frac{A_1}{A_4} (F_{12} - F_{13})$$

Thus the unknown view factor F_{14} can be estimated if the view factors F_{12} and F_{13} , as well as their areas are (A_1, A_2) known.

Now, consider a flat plate (for eg.) which is emitting the radiation, it can be understood that the radiation of the flat plat cannot fall on its own surface (partly or fully). Such kind or surfaces are termed as “not able to see itself”. In such situations,

$$F_{11} = F_{22} = F_{33} = F_{44} = 0$$

However, if the surface can see itself like concave curved surfaces, which may thus see themselves and then the shape factor will not be zero in those cases.

Another property of the shape factor is that when the surface is enclosed, then the following relation holds,

$$\sum_{j=1}^n F_{ij} = 1 \quad (23)$$

where, F_{ij} is the fraction of the total energy leaving surface i which arrives at surface j.

In case of N-walled enclosure, some of the view factors may be evaluated from the knowledge of the rest and the total N^2 view factors may be represented in square matrix form shown below,

$$\begin{bmatrix} F_{11} & F_{12} & \dots & F_{1N} \\ F_{21} & F_{22} & \dots & F_{2N} \\ \dots & \dots & \dots & \dots \\ F_{N1} & F_{N2} & \dots & F_{NN} \end{bmatrix}$$

1.9 Heat exchange between non blackbodies

Evaluation of radiative heat transfer between black surfaces is relatively easy because in case of blackbody all the radiant energy which strikes the surface is absorbed. However, finding view factor is slightly complex, but once it can be done, finding heat exchange between the black bodies is quite easy.

When non blackbodies are involved the heat transfer process becomes very complex because all the energy striking on to the surface does not get absorbed. A part of this striking energy reflected back to another heat transfer surface, and part may be reflected out from the system entirely. Now, one can imagine that this radiant energy can be reflected back and forth between the heat transfer surfaces many times.

In this section, we will assume that all surfaces are in the analysis are diffuse and uniform in temperature and that the reflective and emissive properties are constant over all surfaces.

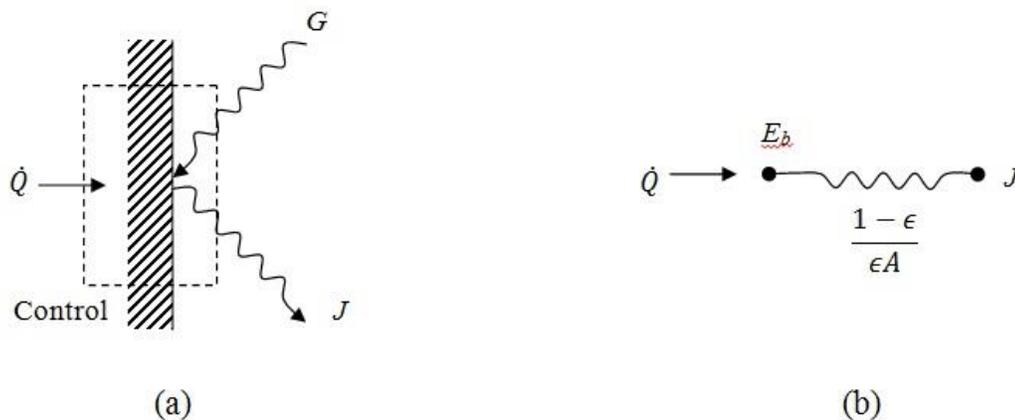


Fig. 6: (a) Surface energy balance for opaque surface (b) equivalent electrical circuit

It is also assumed that the radiosity and irradiation are uniform over each surface. As we have already discussed that the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted (as opaque body), or

$$J = \epsilon E_b + \rho G \quad (24)$$

where, ϵ is the emissivity and E_b is the blackbody emissive power. Because the transmissivity is zero due to opaque surface and absorptivity of the body (grey) will be equal to its emissivity by Kirchhoff's law.

$$\rho = 1 - \alpha = 1 - \epsilon$$

Thus, eq.24 becomes

$$J = \epsilon E_b + (1 - \epsilon)G \quad (25)$$

The net energy leaving the surface is the difference between the radiosity and the irradiance (fig.6a),

$$\frac{\dot{Q}}{A} = J - G = \epsilon E_b + (1 - \epsilon)G - G$$

$$\frac{\dot{Q}}{A} = \frac{\epsilon A}{1 - \epsilon} (E_b - J)$$

$$\dot{Q} = \frac{E_b - J}{(1 - \epsilon)/\epsilon A} \quad (26)$$

The eq.26 can be analogous to the electrical circuit as shown in fig.6(b). The numerator of the eq.26 is equivalent to the potential difference, denominator is equivalent to the surface resistance to radiative heat, and left part is equivalent to the current in the circuit.

In the above discussion we have considered only one surface. Now we will analyse the exchange of radiant energy by two surfaces, A_1 and A_2 , as shown in the fig.7a.

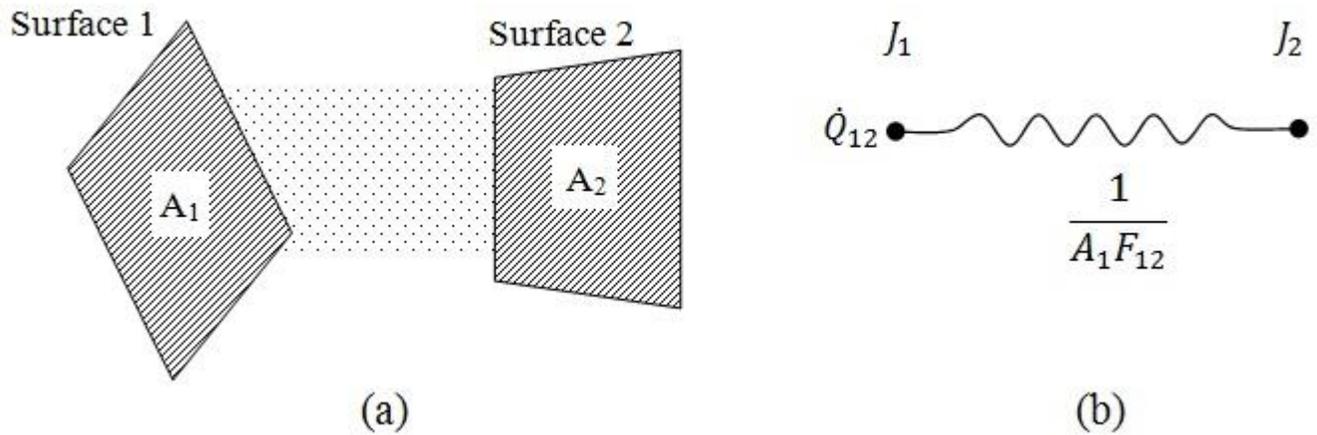


Fig. 7: (a) Energy exchange between two surfaces, (b) equivalent circuit diagram

The radiation which leaves surface 1, the amount that reaches surface 2 is

$$J_1 A_1 F_{12}$$

Similarly, the radiation which leaves system 2, the amount that reaches surface 1 is

$$J_2 A_2 F_{21}$$

The net energy transfer between the surfaces,

$$\dot{Q}_{12} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

Reciprocity theorem states that

$$A_1 F_{12} = A_2 F_{21}$$

$$\Rightarrow \dot{Q}_{12} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21}$$

$$\Rightarrow \dot{Q}_{12} = \frac{(J_1 - J_2)}{1/A_1 F_{12}}$$

(27)

It also resembles an electrical circuit shown in fig.7b. The difference between eq.26 and 27 is that in eq.27 the denominator term is space resistance instead of surface resistance.

Now, to know, the net energy exchange between the two surfaces we need to add both the surface resistances along with the overall potential as shown in the fig.8. Here the surfaces see each other and nothing else.

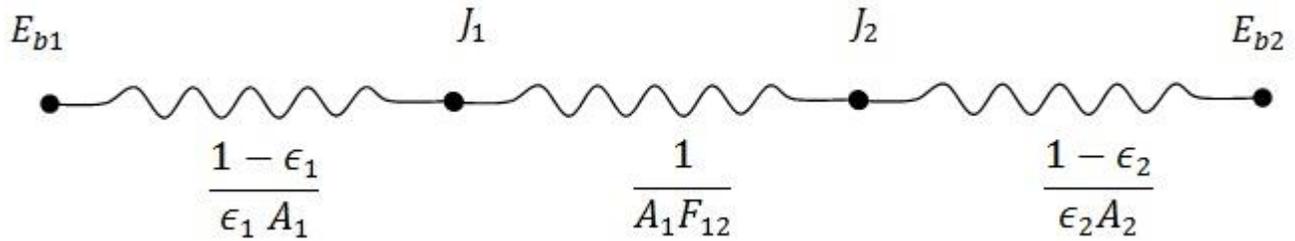


Fig.8: Radiative nature for two surfaces which can see each other nothing else

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{\frac{1 - \epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{\epsilon_2 A_2}} \quad (28)$$

Radiation

shield

Till now we have discussed about the radiative heat transfer from one surface to another without any interfering surface in between. Here we will discuss about an interfering shield in between, which is termed as radiation shield. A radiation shield is a barrier wall of low emissivity placed between two surfaces which reduce the radiation between the bodies. In fact, the radiation shield will put additional resistance to the radiative heat transfer between the surfaces as shown in fig.9.

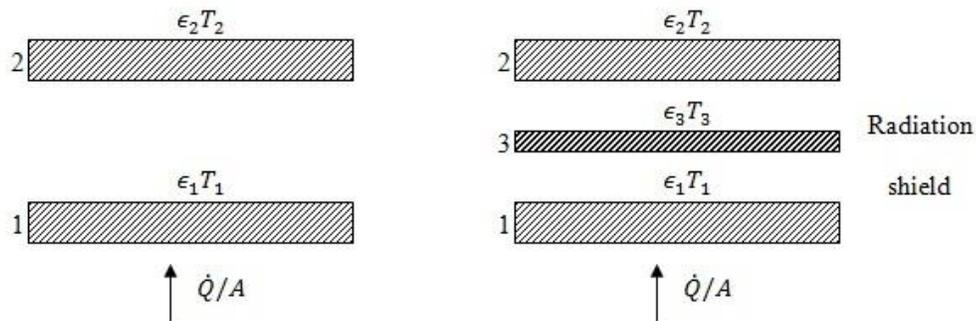


Fig. 9: Radiation between two large infinite plates (a) without and (b) with radiation shield

Considering fig.9(b) and the system is at steady state, and the surfaces are flat (F_{ij} because each plate is in full view of the other). Moreover, the surface are large enough and $\frac{A_1}{A_2} \approx 1$ may be considered and the equivalent blackbody radiation energy may be written as $E_b = \sigma T^4$.

Thus, eq.28 becomes

$$\left. \frac{\dot{Q}}{A} \right|_{net} = \left. \frac{\dot{Q}_{13}}{A_1} \right|_{net} = \left. \frac{\dot{Q}_{32}}{A_2} \right|_{net} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{\sigma(T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1} \quad (29)$$

In order to have a feel of the role of the radiation shield, consider that the emissivities of all the three surfaces are equal.

$$\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$$

Then it can be seen that the heat flux is just one half of that which would be experienced if there were no shield present.

In similar line we can deduce that when n-shields are arranged between the two surfaces then,

$$\left(\frac{\dot{Q}}{A} \right)_{net_{with\ shield}} = \frac{1}{n+1} \left(\frac{\dot{Q}}{A} \right)_{without\ shield} \quad (30)$$

1.10 Electrical network for radiation through absorbing and transmitting medium

The previous discussions were based on the consideration that the heat transfer surfaces were separated by a completely transparent medium. However, in real situations the heat transfer medium absorbs as well as transmits. The examples of such medium are glass, plastic film, and various gases.

Consider two non-transmitting surfaces (same as in fig.8) are separated by a transmitting and absorbing medium. The medium may be considered as a radiation shield which see themselves and others. If we distinguish the transparent medium by m and if the medium is non-reflective (say gas) then using Kirchhoff's law,

$$\alpha_m + \tau_m = 1 = \epsilon_m + \tau_m \quad (31)$$

The energy leaving surface 1 which is transmitted through the medium and reaches the surface 2 is,

$$J_1 A_1 F_{12} \tau_m$$

and that which leaves surface 2 and arrives at surface 1 is,

$$J_2 A_2 F_{21} \tau_m$$

Therefore, the net exchange in the transmission process is therefore,

$$\dot{Q}_{12} = A_1 F_{12} \tau_m (J_1 - J_2)$$

Using eq.31,

$$\dot{Q}_{12} = \frac{(J_1 - J_2)}{\left(\frac{1}{A_1 F_{12} (1 - \epsilon_m)}\right)}$$

Thus the equivalent circuit diagram is shown in fig.9

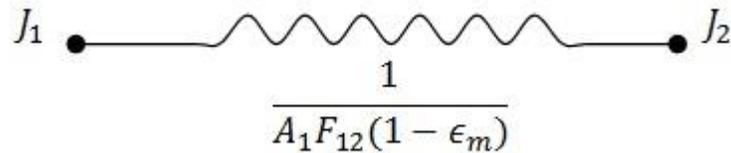


Fig.10. Equivalent electrical circuit for radiation through gas

1.11 Radiation combined with conduction and convection

In industrial processes, in general, the heat transfer at higher temperature has significant portion of radiation along with conduction and convection. For example, a heated surface is shown in the fig.10 with all the three mechanism of heat transfer.

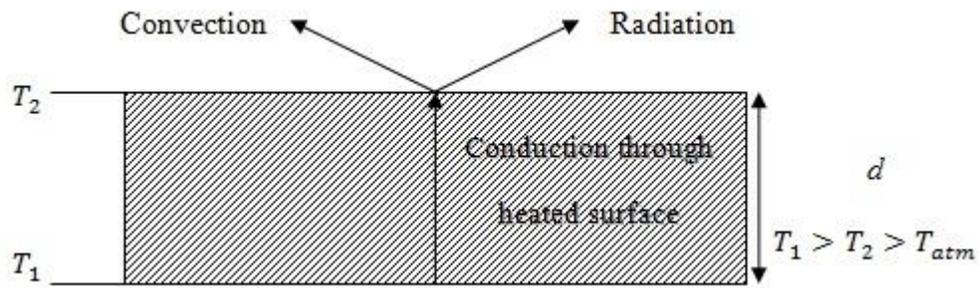


Fig.11: Radiation combined with conduction and convection

At steady state

Heat flux by conduction = heat flux by convection + heat flux by radiation

$$\frac{K}{d}(T_1 - T_2) = h(T_2 - T_{atm}) + \epsilon\sigma(T_2^4 - T_{atm}^4)$$

where, h is the heat transfer coefficient at the surface in contact (outer surface) with atmosphere due to natural and forced convection combined together, ϵ is the emissivity of the outer surface, and T_{atm} is the atmospheric temperature.



SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE

www.sathyabama.ac.in

SCHOOL OF BIO & CHEMICAL ENGINEERING

DEPARTMENT OF CHEMICAL ENGINEERING

UNIT – V – Heat Transfer – SCHA1301

1.0 Evaporation

Evaporation is the vaporization of a liquid. Chemical process industries, in general, use evaporator for the vaporization of a solvent from a solution. We have already discussed the heat transfer for boiling liquids in early chapter. However the evaporation is so important operation in chemical process industry that it is considered an individual operation. In this chapter we will focus on the evaporation with an objective to concentrate a solution consisting of a non-volatile solute and a volatile solvent. If we continue the evaporation process, the residual mater will be solid, which is known as drying. However, our aim is not to dry but to concentrate the solution, moreover, we will also not deal with the crystallization, in which the evaporation leads to formation of crystal in the solution. It is suggested that reader should learn the difference between evaporator, drying and crystallization.

As we will deal with the solution for the evaporation process, a few of the facts must be known about the solution properties.

Solution properties

Knowledge of solution properties is important for the design of the equipment for evaporation. Some of the important properties of the solution are given below,

Concentration

Initially, the solution may be quite dilute and the properties of the solution may be taken as the properties of solvent. As the concentration increases, the solution becomes viscous and heat transfer resistance increases. The crystal may grow on the heating coil or on the heating surface. The boiling points of the solution also rise considerably. Solid or solute contact increases and the boiling temperature of the concentrated solution became higher than that of the solvent as the same pressure (i.e. elevation in boiling point).

Foaming

Many of the materials like organic substance may foam during vaporization. If the foam is stable, it may come out along the vapor known as entrainment. Heat transfer coefficient changes abruptly for such systems.

Degradation due to high temperature

The products of many chemical, food, pharmaceutical industries etc. are very temperature sensitive

and they may get damaged during evaporation. Thus special case or technique is required for concentrating such solution.

Scaling

Many solution have tendency to deposit the scale on the heating surface, which may increase the heat transfer resistance. These scales produce extra thermal resistance of significant value. Therefore, scaling in the equipment should not be ignored thus de-scaling becomes an important and routine matter.

Equipment material

The material of the equipment must be chosen considering the solution properties so that the solution should neither be contaminated nor react with the equipment material.

Evaporator

Equipment, in which evaporation is performed, is known as evaporator. The evaporators used in chemical process industries are heated by steam and have tubular surface. The solution is circulated in the tube and the tubes are heated by steam. In general the steam is the saturated steam and thus it condenses on the outer tube surface in order to heat the tube. The circulation of the solution in the tube have reasonable velocity in order to increase the heat transfer coefficient and remove of scales on the inner surface of the tube. The steam heated tubular evaporators may be classified as natural and forced circulation evaporators.

1.1 Types of evaporator

Natural circulation evaporator

In this category the main evaporators are,

1. Calandria type or short tube evaporator
2. Long tube vertical evaporator

As the name indicates, the circulation of the solution is natural and the density difference derives it. The solution gets heat up and partially vaporized as it flows up the tubes. The heated liquid flows up because of the density difference. Vapor-liquid disengagement occurs above the tube. Thick liquor comes down from this down comer and withdrawn from the bottom. The natural-circulation evaporators may be used if the solution is quite dilute. In the dilute solution the natural circulation will

be at sufficient speed. It may also be used when the solution does not have suspended solid particles. As the solution stays in the tube for larger time, the solution should not be heat sensitive.

The Calandria type or short-tube evaporators have short tubes as compared to the long tube evaporators. The short-tube evaporation uses circulation and solution flows many times in the evaporators. However, in case of the long tube evaporator the flow is once through.

Forced circulation evaporator

Natural circulation evaporators have many limitations (as mentioned earlier) through they are economical as compared to forced circulation evaporator. A forced circulation evaporator has a tubular exchanger for heating the solution without boiling. The superheated solution flashes in the chamber, where the solution gets concentrated. In forced circulation evaporator horizontal or vertical both type of design is in- practice. The forced circulation evaporators are used for handling viscous or heat sensitive solution.

Falling film evaporator

Highly heat sensitive materials are processed in falling film evaporators. They are generally once-through evaporator, in which the liquid enters at the top, flows downstream inside the heater tubes as a film and leaves from the bottom. The tubes are heated by condensing steam over the tube. As the liquid flows down, the water evaporates and the liquid gets concentrated. To have a film inside of the tube, the tube diameter is kept high whereas the height low to keep the residence time low for the flowing liquid. Therefore, these evaporators, with non-circulation and short resistance time, handle heat sensitive material, which are very difficult to process by other method. The main problem in falling film evaporator is the distribution of the liquid uniformly as a thin film inside the tube.

1.2 Performance of steam heated tubular evaporators

The performance of a steam heated tubular evaporator is evaluated by the capacity and the economy.

Capacity and economy

Capacity is defined as the no of kilograms of water vaporized per hour. Economy is the number of kg of water vaporized per kg of steam fed to the unit. Steam consumption is very important to know, and can be estimated by the ratio of capacity divided by the economy. That is the steam consumption (in kg/h) is

$$\text{Steam Consumption} = \text{Capacity} / \text{Economy}$$

1.3 Single and multiple effect evaporators

In single effect evaporator, as shown in fig.1, the steam is fed to the evaporator which condenses on the tube surface and the heat is transferred to the solution. The saturated vapor comes out from the evaporator and this vapor either may be vented out or condensed. The concentrated solution is taken out from the evaporator.

Now we can see if we want the further concentrate, the solution has to be sent into another similar evaporator which will have the fresh steam to provide the necessary heat.

It may be noted that in this process the fresh steam is required for the second evaporator and at the same time the vapor is not utilized. Therefore it can be said the single effect evaporator does not utilize the steam efficiently. The economy of the single effect evaporator is thus less than one. Moreover, the other reason for low economy is that in many of the cases the feed temperature remains below the boiling temperature of the solution. Therefore, a part of the heat is utilized to raise the feed temperature to its boiling point.

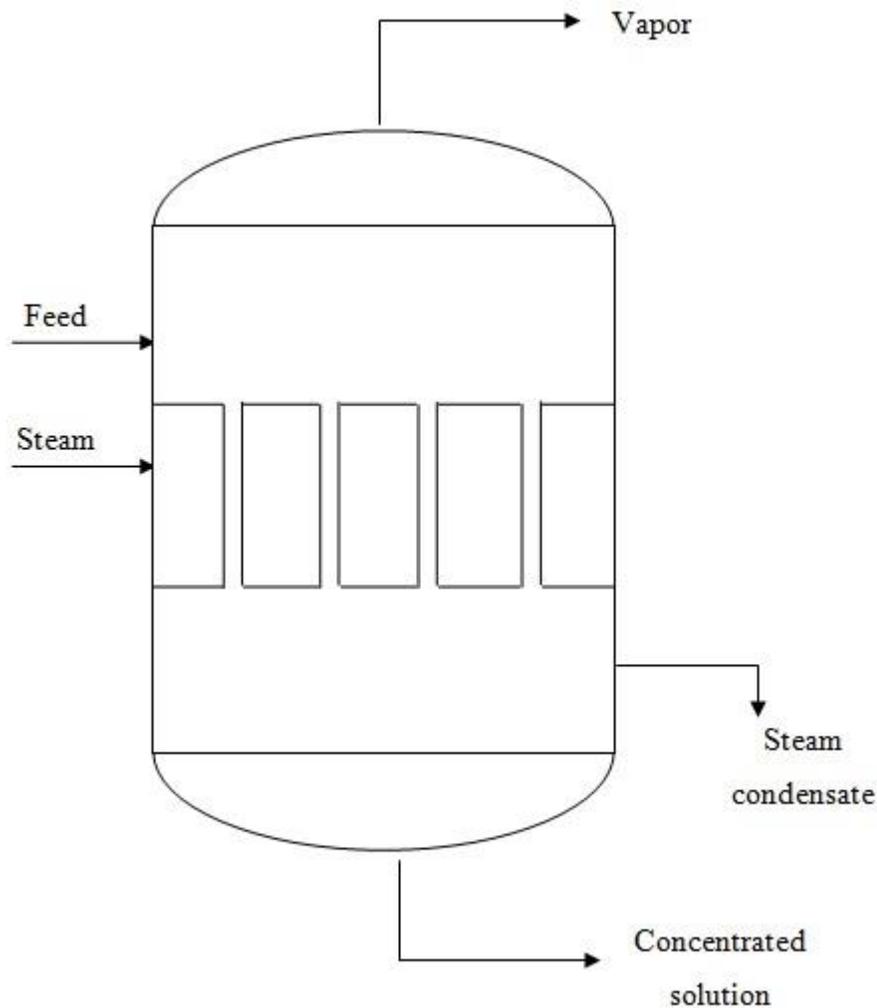


Fig.1: Single effect evaporator

In order to increase the economy we may consider the arrangement of the two evaporators as shown in the fig.2.

The figure 2 shows that the two evaporators are connected in series. The saturated vapor coming out from the evaporator-1 is used as steam in the second evaporator. Partially concentrated solution works as a feed to the second evaporator. This arrangement is known as double effect evaporator in forward feed scheme. A few of the important point that we have to note for this scheme is that the vapour leaving evaporator-2 is at the boiling temperature of the liquid leaving the first effect. In order to transfer this heat from the condensing vapor from the evaporator-1 to the boiling liquid in evaporator-2, the liquid in evaporator-2 must boil at a temperature considerable less than the condensation

temperature of the vaporization, in order to ensure reasonable driving force for heat transfer. A method of achieving this is to maintain a suitable lower pressure in the second effect so that the liquid boils at a lower temperature. Therefore, if the evaporator-1 operates at atmospheric pressure, the evaporator-2 should be operated at same suitable vacuum.

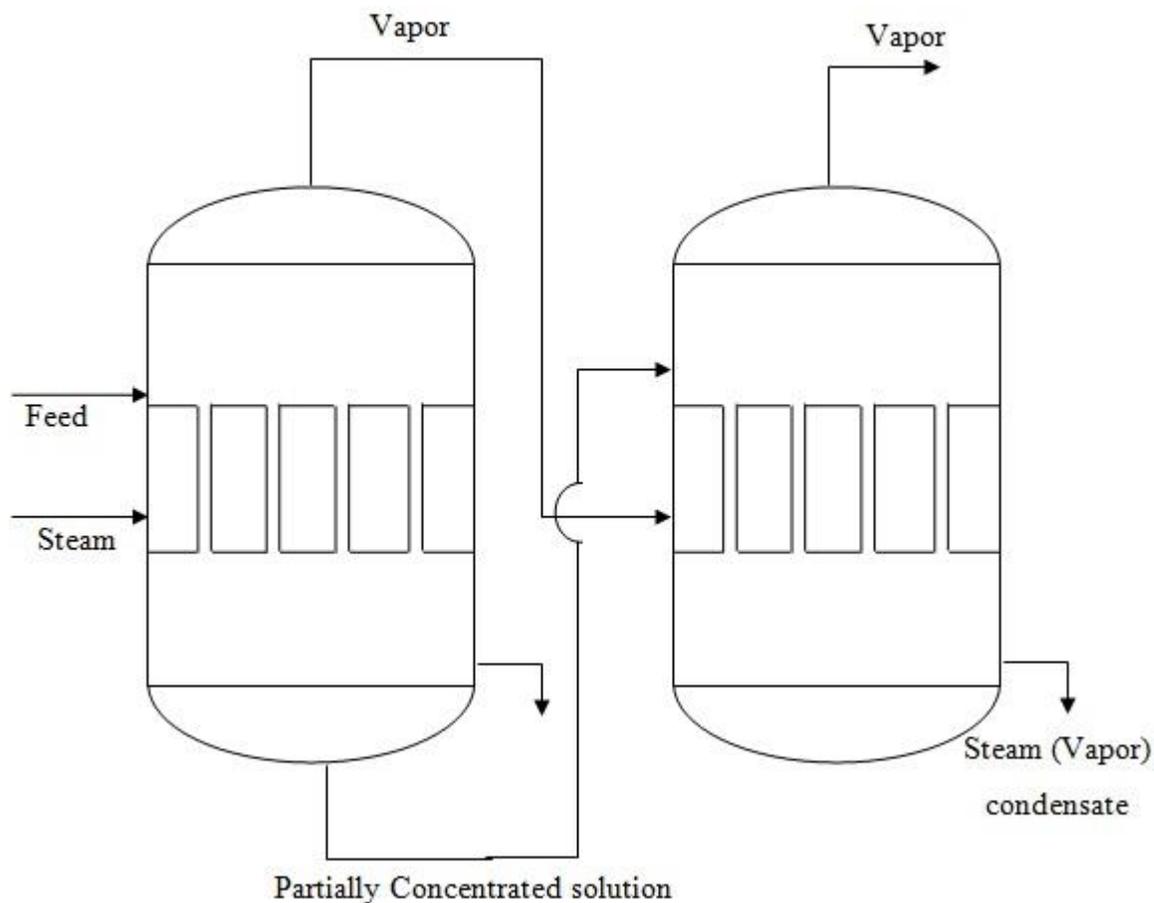


Fig.2: Double effect evaporator with forward feed scheme

The benefit of the use of multiple effect evaporators is that in this arrangement multiple reuse of heat supplied to the first effect is possible and results in improved steam economy.

1.3.1 Boiling point elevation

The evaporators produce concentrated solution having substantially higher boiling point than that of the solvent (of the solution) at the prevailing pressure. The increase in boiling point over that of water is known as boiling point elevation (BPE) of the solution. As the concentration increases the boiling point of the solution also increases. Therefore, in order to get the real temperature difference (or

driving force) between the steam temperature and the solution temperature, the BPE must be subtracted from the temperature drop. The BPE may be predicted from the steam table (in case water is a solvent).

An empirical rule known as Dühring rule is suitable for estimating the BPE of strong solution. The Dühring rule states that the boiling point of a given solution is a linear function of the boiling point of the pure water at the same pressure. Therefore, if the boiling point of the solution is plotted against that of the water at the same pressure, a straight line results. Different lines are obtained at different concentrations. The fig.3 shows representative Dühring plots for a solution (non-volatile solute in water).

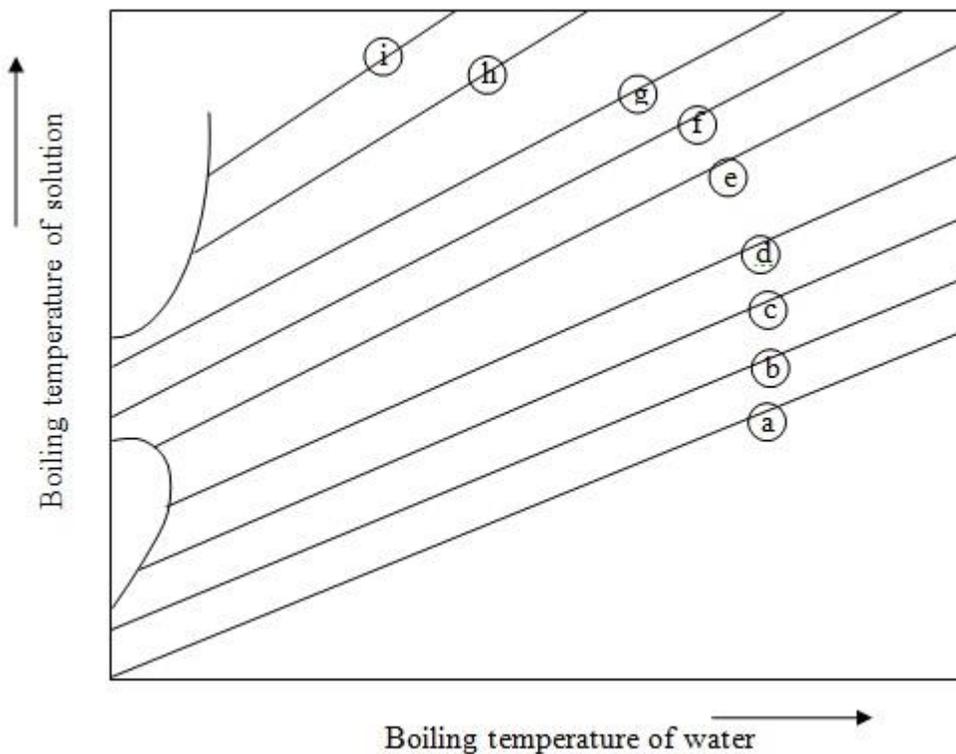


Fig.3: Representative Dühring lines for a system (non-volatile solute in water) mole fraction of solute in the solution (a) 0.1 (b) 0.2 (c) 0.25 (d) 0.39 (e) 0.35 (f) 0.45 (g) 0.5 (h) 0.6 (i) 0.7

The fig.3 helps to find out the boiling point of solution at moderate pressure. For example if a solution having 'x' mole fractions of solute have a pressure over it such that water boils at T° C, by reading up

from the x-axis at T °C to the line for the x mole fraction solution and then moving horizontally to the y-axis, the boiling point of the solution can be found at that pressure.

1.3.2 Temperature profile in an evaporator

Let us consider the case of long-tube vertical evaporator heated by steam. After boiling and flashing of the superheated liquid, the disengagement of the vapor and liquid occur in vapor space of the evaporator and the recycled liquid flows down the external pipe. A part of this concentrated liquid is withdrawn as a product and the remaining part get mixed with a feed and again enter the evaporator tube. If T_{BP} is the boiling of the liquid in the evaporator as the prevailing pressure, then the temperature of the liquid in the tube will be T_{BP} . The temperature of the recycled stream entering the tubes will then also be T_{BP} , if the feed is sufficiently hot. Now, we will imagine how the temperature is changing in the tube. Let us see that when the liquid flows up in the tube, its temperature rises because at the bottom of the tube the pressure is higher (vapor chamber pressure + hydrostatic pressure + frictional loss) as compared to the top of the tube. Therefore, a liquid starts boiling at a level when its temperature rises to its saturation temperature at the pressure at that point. After the boiling in between the tube, as liquid goes up in the tube, the local temperature drops because of the reduction in the local pressure. It may also be mentioned that as the liquid moves up it gets concentrated and thus the boiling point of the solution also increases as the liquid traversed up in the tube. The liquid temperature profile in the tube is shown in the fig.4 for low (plot i) and high (plot ii) liquid velocity. The liquid temperature in the tubes increases up to certain height and then the temperature decreases due to the loss of superheat. At higher velocity the temperature raise is less and the liquid boils near the top of the tube. The plot (iii) shows the shell side temperature profile where steam is heating the tube. As can be seen, the slightly superheated steam enters the shell and soon the temperature of the steam losses its sensible heat and then condenses on the tubes and provide the latent heat of condensation (at temperature T_{steam}) to the tube and before boiling from the shell may get slightly sub-cooled. The plot (iv) is the boiling temperature of the water (T_w) at the pressure in the vapor chamber. Thus, the $BPE = T_{BP} - T_w$ and the true temperature driving force is the difference between the plot (iii) and the plot (i) or (ii).

It can be understood with the help of the discussion and fig.4 that the temperature changes all along the length of the tube. Thus, the real temperature driving force will be the difference in steam temperature and liquid temperature always the high. However, it is practically not easy to determine

the temperature profile in the tube. Therefore, the driving force can be taken as $(T_{steam} - T_{BP})$ for the design purpose.

1.4 Heat transfer coefficient

The correlation used in the boiling and condensation may be used here. If the evaporator operates at very high liquid velocity so that the boiling occurs at the top end of the tube, the following correlation (eq. 9.1) may be used,

$$\frac{h_i D}{k} = 0.0278 \text{Re}^{0.8} \text{Pr}^{0.4}$$

where, D is the inner diameter of the tube, k is the thermal conductivity of the liquid or solution.

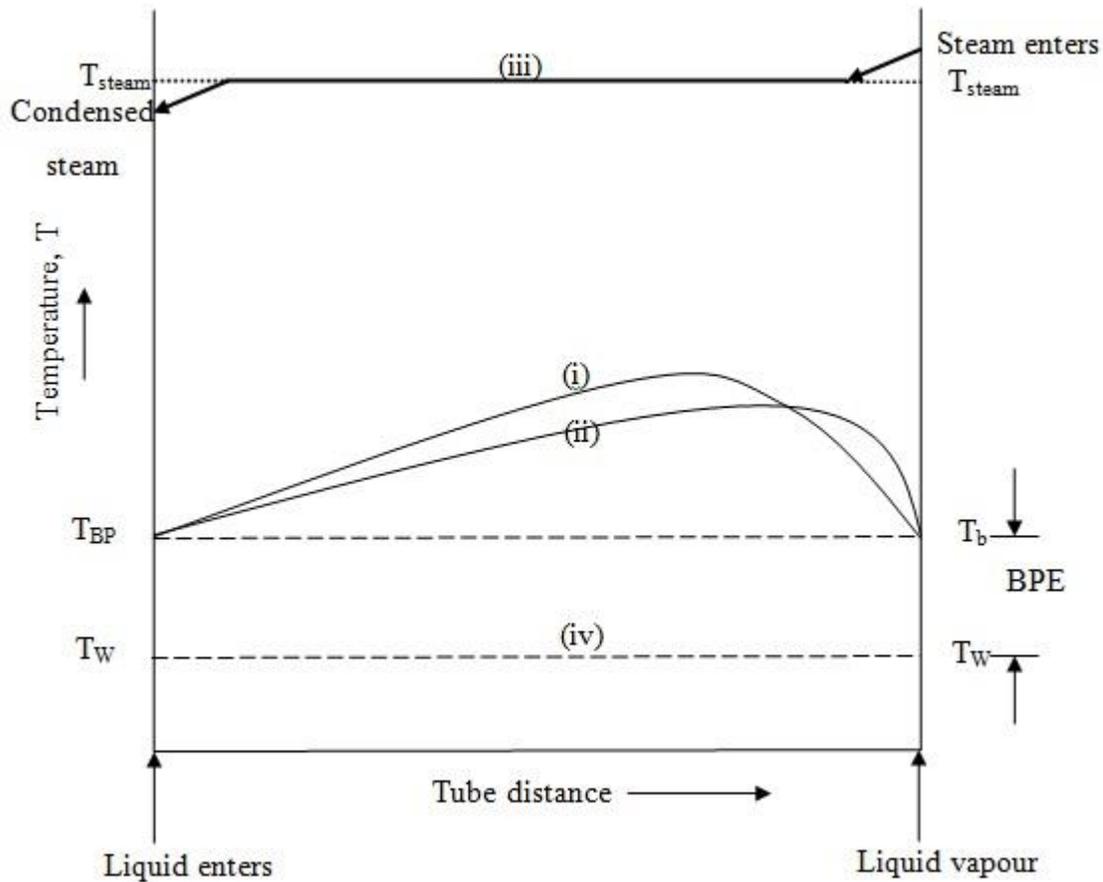


Fig.4: Temperature profiles in an evaporator

Fouling is a concern in the evaporator; therefore the following equation (eq.9.2) may be used for the overall heat transfer coefficient with time,

$$\frac{1}{U_{dirty}^2} = \frac{1}{U_{clean}^2} + \alpha t \quad (9.2)$$

where, t is the time for where the evaporator is the operation, α is a constant for a particular liquid, U_{dirty} and U_{clean} all the overall heat transfer coefficient of the dirty and clean evaporator.

1.5 Method of feeding: Multiple effect evaporators

The fig.5, 6, 7, and 8 show the four different feeding arrangement of feed to the evaporators. In the fig.5 the liquid feed is pumped into the first effect and the partially concentrated solution is sent to the second effect and so on. The heating steam is also sent through the first effect to another effect. This particular strategy is known as forward feed. In the forward feed the concentration of the liquid increases from first effect to the subsequent effects till the last effect. It may be noted that the first effect is that in which the fresh steam is fed, whereas the vapour generated in the first effect is fed to the next evaporator (connected in series with the first effect) is known as second effect and so on.

The forward feed requires a pump for feeding dilute solution to the first effect. The first effect is generally at atmospheric pressure and the subsequent effects are in decreasing pressure. Thus, the liquid may move without the pump from one effect to another effect in the direction of decreasing pressure. However, to take out the concentrated liquid from the last effect may need a pump.

The backward feed arrangement is very common arrangement. A triple-effect evaporator in backward arrangement is shown in the fig.6. In this arrangement the dilute liquid is fed to the last effect and then pumped through the successive effects to the first effect. The method requires additional pumps (generally one pump in between two effects) as shown in the fig.6. Backward feed is advantageous and gives higher capacity than the forward feed when the concentrated liquid is viscous, because the viscous fluid is at higher temperature being in the first effect. However, this arrangement provides lower economy as compared to forward feed arrangement.

The combination of forward-feed and backward-feed is known as mixed feed arrangement. In mixed feed the dilute liquid enters in between effects, flows in forward feed to the end of the effect and then pumped back to the first effect for final concentration. Figure 7 shows triple effect mixed feed arrangement. This mixed feed arrangement eliminates the need of a few of the pumps. Moreover, it still passes the most concentrated liquid through the first effect, which is having higher temperature among all the effect (being at highest pressure compared to other effects).

Another common evaporator arrangements, which is more common in crystallization is parallel feed where feed is admitted individually to all the effects. Figure 7 shows such arrangement.

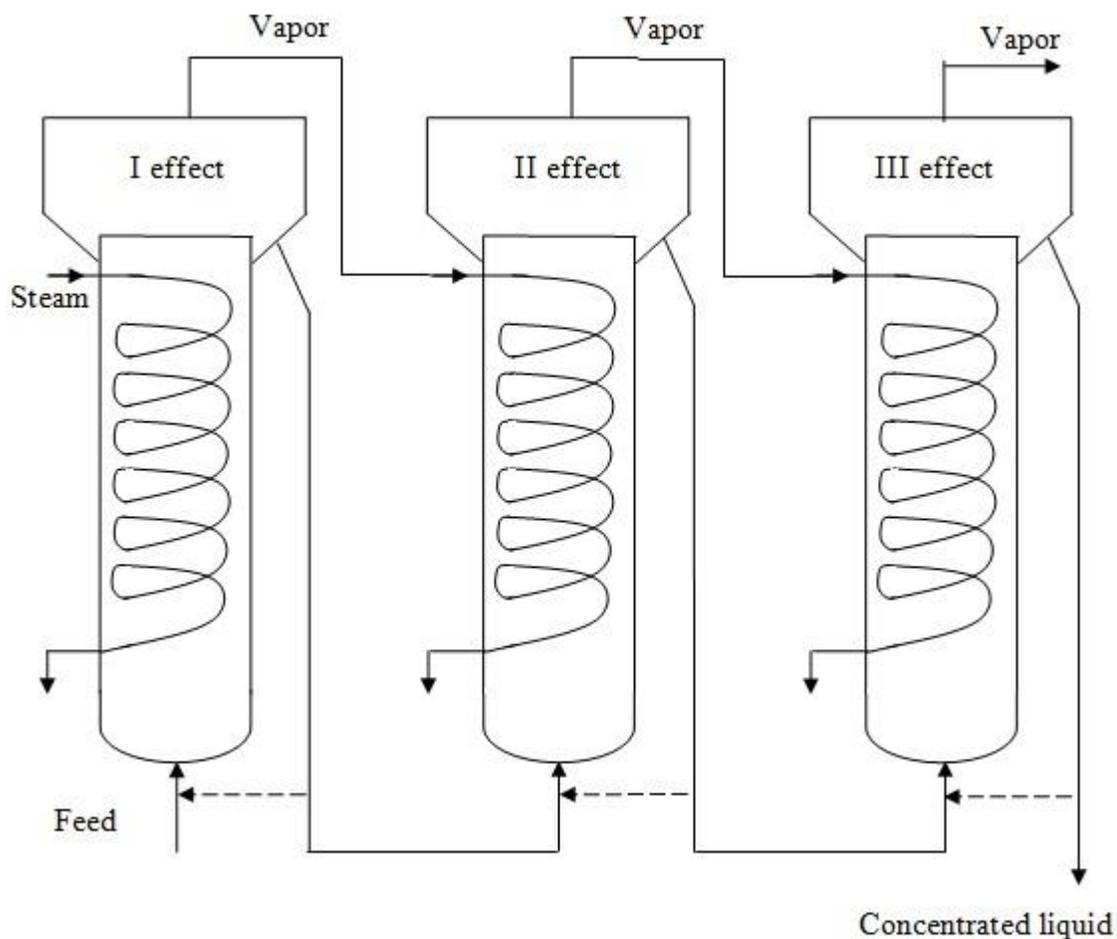


Fig.5: Forward feed arrangement in triple-effect evaporator (dotted line: recycle stream)

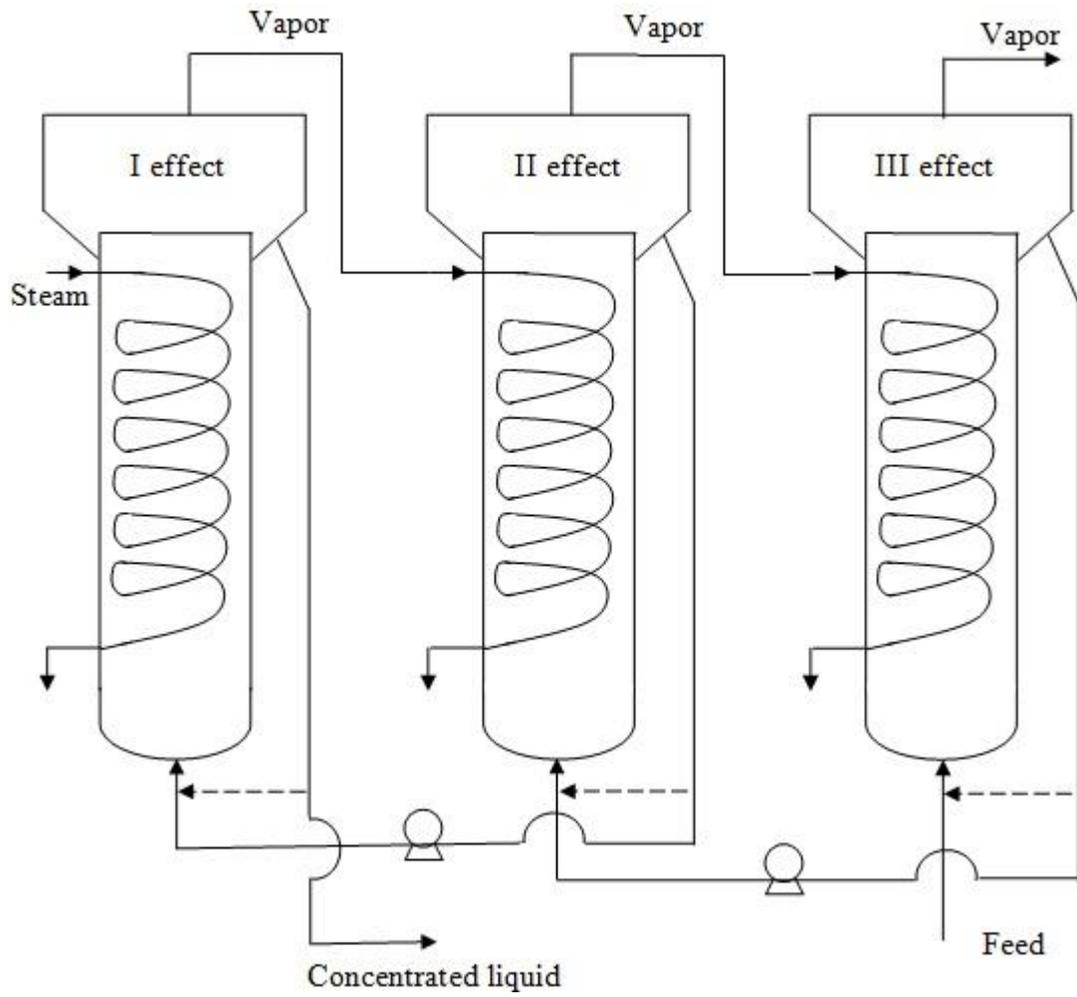


Fig.6: Backward feed arrangement in triple-effect evaporator (dotted line: recycle stream)

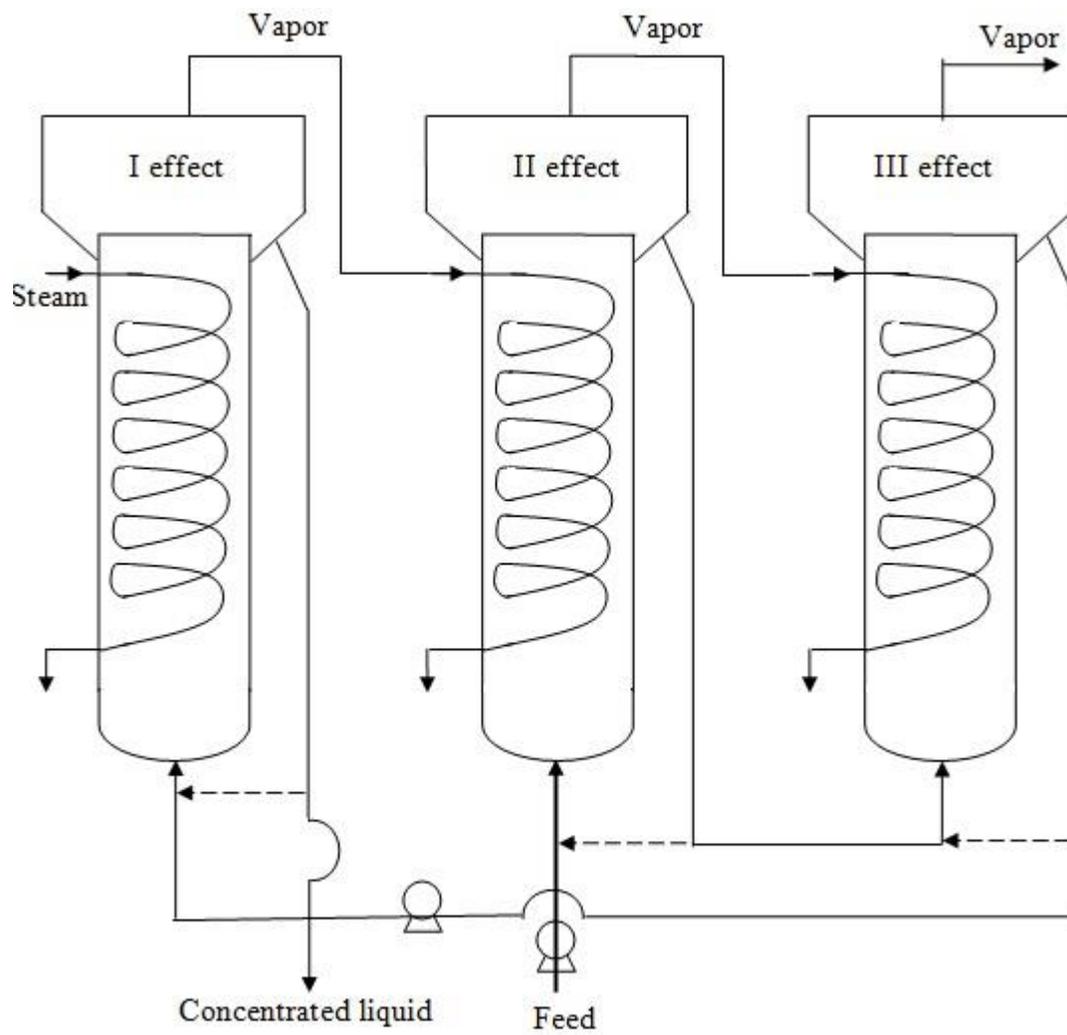


Fig.7: Mixed feed arrangement in triple-effect evaporator (dotted line: recycle stream)

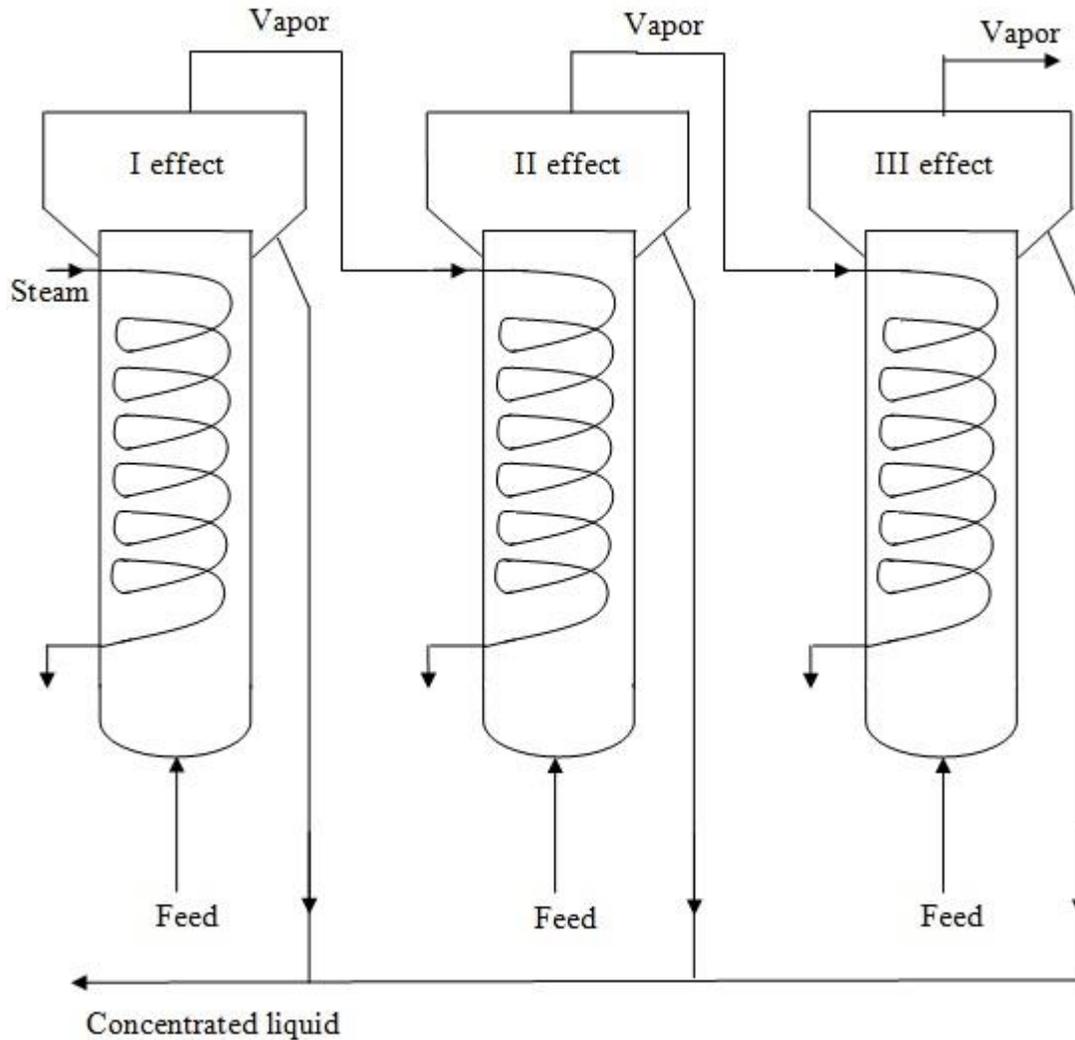


Fig.8: Parallel feed arrangement in triple-effect evaporator

1.6 Enthalpy Balance

Single effect evaporator

The latent heat of condensation of the steam is transferred to the boiling solution through the heating surface in order to vaporize the water. Thus, two enthalpy balances are required one for the liquid and another for the steam.

The following assumptions are required, in order to make the enthalpy balance,

1. Flow of non-condensable is negligible
2. The superheat and sub-cooling (see fig.4) of the condensable steam is negligible
3. No solid precipitates out from the concentrating solution

The enthalpy balance for the steam side is,

$$\dot{q}_s = \dot{m}_s(h_s - h_c) = \dot{m}_s\lambda_s$$

Where,

\dot{q}_s = rate of heat transfer through heating surface from steam

\dot{m}_s = flow rate of steam

λ_s = latent heat of condensation of steam

h_s = specific enthalpy of steam

h_c = specific enthalpy of condensate

Enthalpy balance for the liquid side is (eq.9.3),

$$\dot{q}_l = (\dot{m}_{fl} - \dot{m}_{cl})h_v - \dot{m}_{fl}h_{fl} + \dot{m}_{cl}h_{cl} \quad (9.3)$$

Where,

\dot{q}_l = rate of heat transfer from heating surface to the liquid

h_v = specific enthalpy of vapour

h_{cl} = specific enthalpy of concentrated liquid

h_{fl} = specific enthalpy of liquid feed

\dot{m}_{fl} = flow rate of liquid feed

\dot{m}_{cl} = flow rate of concentrated liquid

The enthalpy balance at steam side and liquid side will be same in the absence of any heat loss (eq.9.4). Thus,

$$(\dot{m}_{fl} - \dot{m}_{cl})h_v - \dot{m}_{fl}h_{fl} + \dot{m}_{cl}h_{cl} = \dot{m}_s\lambda_s \quad (9.4)$$

The area of heat transfer A can be calculated from

$$\dot{q}_l = \dot{q}_s = U_D A \Delta T$$

When $\Delta T = (T_b - T_c)$;

T_b = Saturated temperature of steam in the shell

T_s = Boiling point of the solution at the prevailing pressure

U_D = Overall coefficient (dirty)

1.6.1 Effect of heat of dilution

Most of the solutions when mixed or diluted at constant temperature do not give significant heat effect. It is generally true for organic solutions (like sugar). However, many of the inorganic solutions (like sulfuric acid, potassium hydroxide, calcium carbonate etc.) evolve significant heat on dilution. Therefore, an equivalent amount of heat is required (in addition to the latent heat of vaporization), when dilute solutions of these inorganic chemicals are concentrated. Enthalpy-concentration diagrams are helpful in order to find the enthalpy of the solution at different concentrations of these chemicals in the solution.

1.7 Multiple effect evaporators

The steam goes into I-effect and heats the solution by the latent heat of condensation. If the heat required to boil the feed is negligible, it follows that practically all this heat

$$\dot{q}_s = A_1 U_1 \Delta T_1$$

must appear as latent heat in the vapor that leaves the I-effect and enters into II-effect as steam. The temperature of the condensate leaving the II-effect will be very near the temperature T_1 of the vapors from the boiling liquid in the I-effect. Thus, in steady state operation all the heat that was expended in creating vapor in the I-effect must be given by when this same vapor condenses in the II-effect and so on.

The heat delivered into the II-effect will be,

$$\dot{q}_{v1} = A_2 U_2 \Delta T_2$$

The $\dot{q}_{v1} = \dot{q}_s$

Similarly, for III-effect

$$A_1 U_1 \Delta T_1 = A_2 U_2 \Delta T_2 = A_3 U_3 \Delta T_3 = \dot{q} \quad (9.5)$$

It can be seen (eq. 9.5) that the temperature drops in a multiple effect evaporator is approximately inversely proportional to the heat-transfer coefficient.

The total available temperature drop will be given by eq.9.6,

$$\begin{aligned} \sum \Delta T &= \Delta T_1 + \Delta T_2 + \Delta T_3 \\ &= (T_s + T_{v3}) - \sum BPE \end{aligned} \quad (9.6)$$

where,

T_s : Steam temp. (I-effect);

T_{v3} : Vapor temperature leaving III-effect

BPE : boiling point elevation in the solution in various effects

1.8 Problems:

1. A triple effect forward feed evaporator is used to concentrate a liquid which has marginal elevation in boiling point. The temperature of the stream to the first effect is 105°C, and the boiling point of the solution within third effect is 45°C. The overall heat transfer coefficients are, 2,200 W/m²: in the I-effect, 1,800 W/m²: in the II-effect and 1,500 W/m²: in the III-effect.

Find out at what temperatures the fluid boils in the I and II effects.

Solution:

Assumptions

1. We may assume that there is no elevation in boiling point in the evaporators.
2. Area of all the three evaporators are same ($A_I = A_{II} = A_{III} = A$)

Total temperature drop=(105-45)°C = 60 °C

the temperature drop across I-effect,

$$\Delta T_I = \frac{\frac{1}{2200}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 15.2 \text{ } ^\circ\text{C}$$

Similarly, the temperature drop across II-effect,

$$\Delta T_{II} = \frac{\frac{1}{1800}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 18.6 \text{ } ^\circ\text{C}$$

And the temperature drop across III-effect,

$$\Delta T_{III} = \frac{\frac{1}{1500}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 22.3 \text{ } ^\circ\text{C}$$

Therefore, the boiling point in the first effect will be = $(105 - 15.2) \text{ } ^\circ\text{C} = 89.8 \text{ } ^\circ\text{C}$

Similarly, the boiling point in the second effect will be = $(89.8 - 18.6) \text{ } ^\circ\text{C} = 71.2 \text{ } ^\circ\text{C}$.