Lecture Notes

EECS 40
Introduction to Microelectronic Circuits

Prof. C. Chang-Hasnain

Spring 2007

EE 40 Course Overview

- EECS 40:
  - One of five EECS core courses (with 20, 61A, 61B, and 61C)
    - introduces "hardware" side of EECS
    - prerequisite for EE105, EE130, EE141, EE150
  - Prerequisites: Math 1B, Physics 7B
  - Course involves three hours of lecture, one hour of discussion and three hours of lab work each week.
- Course content:
  - Fundamental circuit concepts and analysis techniques
  - First and second order circuits, impulse and frequency response
  - Op Amps
  - Diode and FET: Device and Circuits
  - Amplification, Logic, Filter
- Text Book
  - Electrical Engineering: Principles and Applications", third edition, Allan R. Hambley, Pearson Prentice Hall, 2005
  - Supplementary Reader

Important DATES

- Office hours, Discussion and Lab Sessions will start on week 2
  - Stay with ONE Discussion and Lab session you registered.
- Midterm and Final Dates:
  - Midterms: 6-7:40 pm on 2/21 and 4/11(Location TBD)
  - Final: 8-11am on 5/14 (Location TBD)
- Best Final Project Contest
  - 5/4 3-5pm Location TBD
  - Winner projects will be displayed on second floor Cory Hall.

Grading Policy

- Weights:
  - 12%: 12 HW sets
  - 15%: 11 Labs
    - 7 structured experiments (7%)
    - one 4-week final project (8%)
  - 40%: 2 midterm exams
  - 33%: Final exam
- **No** late HW or Lab reports accepted
- **No** make-up exams unless Prof. Chang's approval is obtained at least 24 hours before exam time; proofs of extraneous circumstances are required.
  - If you miss one of the midterms, you lose 20 % of the grade.
- Departmental grading policy:
  - A typical GPA for courses in the lower division is 2.7. This GPA would result, for example, from 17% A's, 50% B's, 20% C's, 10% D's, and 3% F's.

Grading Policy (Cont'd)

- Weekly HW:
  - Assignment on the web by 5 pm Wednesdays, starting 1/24/07.
  - Due 5 pm the following Wednesday in HW box, 240 Cory.
  - On the top page, right top corner, write your name (in the form: Last Name, First Name) with discussion session number.
  - Graded homework will be returned one week later in discussion sessions.
- Labs
  - **Complete the prelab section before going to the lab**, or your points will be taken off.
  - Lab reports are supposed to be turned in at the end of each lab, except for the final project, which is due at the end of the last lab session.
- It is **your** responsibility to check with the head GSI from time to time to make sure all grades are entered correctly.

Classroom Rules

- Please come to class on time. There is no web-cast this semester.
- Turn off cell phones, pagers, radio, CD, DVD, etc.
- No food.
- No pets.
- Do not come in and out of classroom.
- Lectures will be recorded and webcasted.

Chapter 1

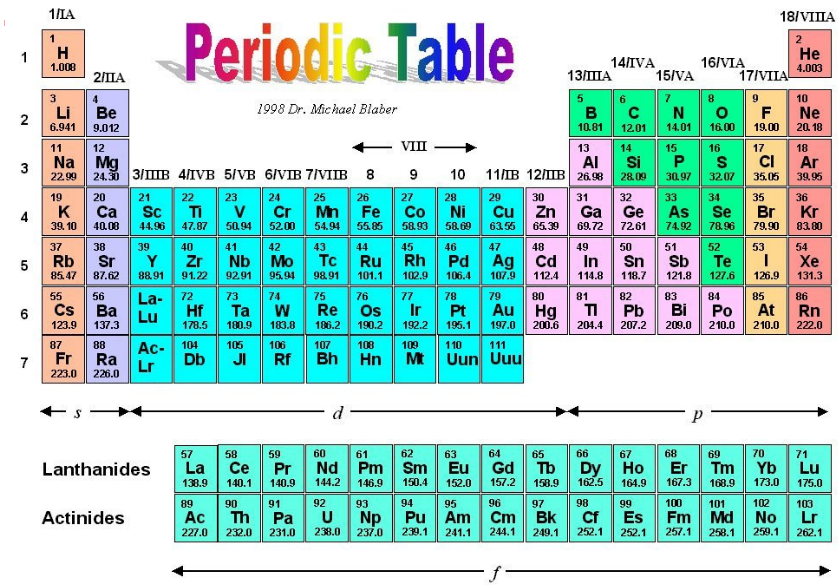
- Outline
  - Electrical quantities
    - Charge, Current, Voltage, Power
  - The ideal basic circuit element
  - Sign conventions
  - Circuit element I-V characteristics
  - Construction of a circuit model
  - Kirchhoff's Current Law
  - Kirchhoff's Voltage Law

Electric Charge

- Electrical effects are due to
  - separation of charge → electric force (voltage)
  - charges in motion → electric flow (current)
- Macroscopically, most matter is electrically neutral most of the time.
  - Exceptions: clouds in a thunderstorm, people on carpets in dry weather, plates of a charged capacitor, etc.
- Microscopically, matter is full of electric charges
  - Electric charge exists in discrete quantities, integral multiples of the electronic charge -1.6×10^{-19} *Coulomb*

Classification of Materials

- Solids in which the outermost atomic electrons are free to move around are metals.
  - Metals typically have ~1 "free electron" per atom
  - Examples:
- Solids in which all electrons are tightly bound to atoms are insulators.
  - Examples:
- Electrons in semiconductors are not tightly bound and can be easily "promoted" to a free state.
  - Examples:



Electric Current

Definition: rate of positive charge flow

Symbol: i

Units: Coulombs per second ≡ Amperes (A)

Note: Current has polarity.

$i = dq/dt$ where

q = charge (Coulombs)

t = time (in seconds)

André-Marie Ampère's

1775-1836

Electric Current Examples

1. 10^5 positively charged particles (each with charge 1.6×10^{-19} C) flow to the right (+x direction) every nanosecond

$$I = \frac{Q}{t} = +\frac{10^5 \times 1.6\times10^{-19}}{10^{-9}} = 1.6\times10^{-5}\ \text{A}$$

2. 10^5 electrons flow to the right (+x direction) every microsecond

$$I = \frac{Q}{t} = -\frac{10^5 \times 1.6\times10^{-19}}{10^{-9}} = -1.6\times10^{-5} A$$

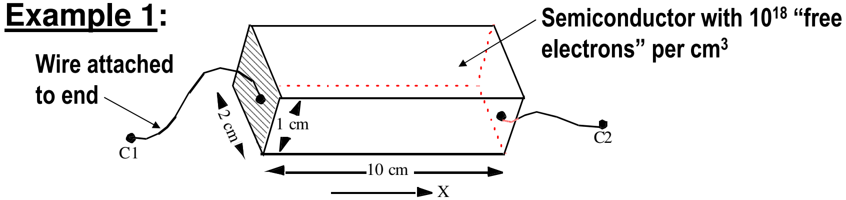
Current Density

Definition: rate of positive charge flow per unit area

Symbol: J

Units: A / cm^2

Example 1:



Suppose we force a current of 1 A to flow from C1 to C2:

- Electron flow is in $-x$ direction:

$$\frac{1C/\sec}{-1.6\times10^{-19}\,C/electron} = -6.25\times10^{18}\,\frac{electrons}{\sec}$$

Current Density Example (cont'd)

- Example 2:

Typical dimensions of integrated circuit components are in the range of 1 μm. What is the current density in a wire with 1 μm² area carrying 5 mA?

Electric Potential (Voltage)

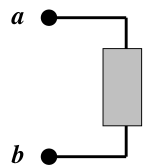
- **Definition:** energy per unit charge
- **Symbol:** v
- **Units:** Joules/Coulomb ≡ Volts (V)

Alessandro Volta
(*1745–1827*)

$$v = dw/dq$$

where w = energy (in Joules), q = charge (in Coulombs)

Note: Potential is always referenced to some point.



Subscript convention:

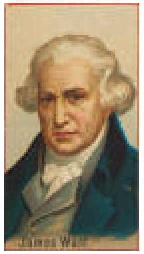
v_{ab} means the potential at a minus the potential at b.

$$v_{ab} \equiv v_a - v_b$$

Electric Power

- **Definition:** transfer of energy per unit time
- **Symbol:** p
- **Units:** Joules per second ≡ Watts (W)

$$p = dw/dt = (dw/dq)(dq/dt) = vi$$

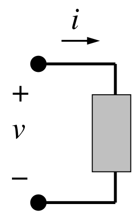


James Watt
1736 - 1819

- **Concept:**

  As a positive charge q moves through a drop in voltage v, it loses energy
  - energy change = qv
  - rate is proportional to # charges/sec

The Ideal Basic Circuit Element



- Polarity reference for voltage can be indicated by plus and minus signs
- Reference direction for the current is indicated by an arrow

Attributes:

- Two terminals (points of connection)
- Mathematically described in terms of current and/or voltage
- Cannot be subdivided into other elements

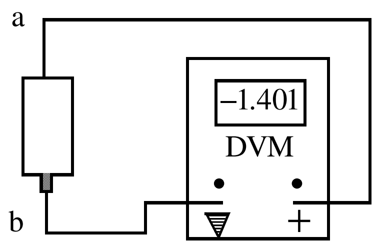
A Note about Reference Directions

- A problem like “Find the current” or “Find the voltage” is always accompanied by a definition of the direction:

- In this case, if the current turns out to be 1 mA flowing to the left, we would say *i* = -1 mA.
- In order to perform circuit analysis to determine the voltages and currents in an electric circuit, you need to specify reference directions.
- There is no need to guess the reference direction so that the answers come out positive.

Sign Convention Example

Suppose you have an unlabelled battery and you measure its voltage with a digital voltmeter (DVM). It will tell you the **magnitude and sign** of the voltage.



With this circuit, you are measuring v_{ab}.

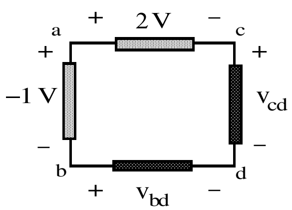
The DVM indicates –1.401, so v_a is lower than v_b by 1.401 V.

Which is the positive battery terminal?

Note that we have used the “ground” symbol (⏚) for the reference node on the DVM. Often it is labeled “C” for “common.”

Another Example

Find v_{ab}, v_{ca}, v_{cb}

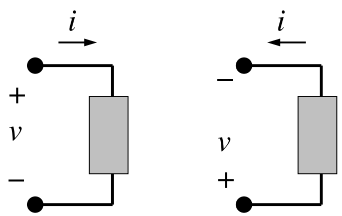


Note that the labeling convention has nothing to do with whether or not v is positive or negative.

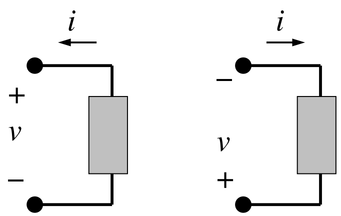
Sign Convention for Power

Passive sign convention

$$p = vi$$



$$p = -vi$$

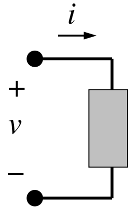


- **If $p > 0$, power is being delivered to the box.**
- **If $p < 0$, power is being extracted from the box.**

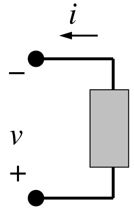
Power

If an element is absorbing power (*i.e.* if $p > 0$), positive charge is flowing from higher potential to lower potential.

$p = vi$ if the "passive sign convention" is used:



or

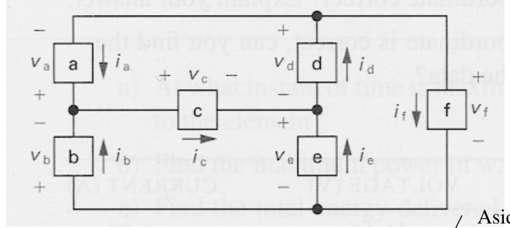


How can a circuit element absorb power?

By converting electrical energy into heat (resistors in toasters), light (light bulbs), or acoustic energy (speakers); by storing energy (charging a battery).

Power Calculation Example

Find the power absorbed by each element:



Conservation of energy
→ total power delivered equals total power absorbed

Aside: For electronics these are unrealistically large currents – milliamperes or smaller is more typical

ELEMENT	VOLTAGE (V)	CURRENT (A)	vi (W)	p (W)
a	−18	−51	918	
b	−18	45	- 810	
c	2	−6	- 12	
d	20	−20	- 400	
e	16	−14	- 224	
f	36	31	1116	

Circuit Elements

- 5 ideal basic circuit elements:
  - voltage source
  - current source
  - resistor
  - inductor
  - capacitor

(voltage source, current source:) ***active elements***, capable of generating electric energy

(resistor, inductor, capacitor:) ***passive elements***, incapable of generating electric energy

- Many practical systems can be modeled with just sources and resistors
- The basic analytical techniques for solving circuits with inductors and capacitors are similar to those for resistive circuits

Electrical Sources

- An ***electrical source*** is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

Examples:

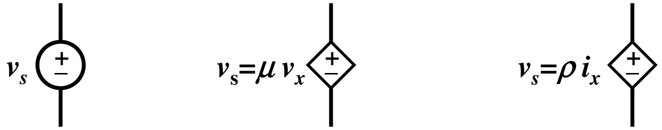
- battery: chemical ⟺ electric
- dynamo (generator/motor): mechanical ⟺ electric
  (Ex. gasoline-powered generator, Bonneville dam)

→ Electrical sources can either deliver or absorb power

Ideal Voltage Source

- Circuit element that maintains a prescribed voltage across its terminals, **regardless of the current flowing in those terminals**.
  - Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either **independent** or **dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

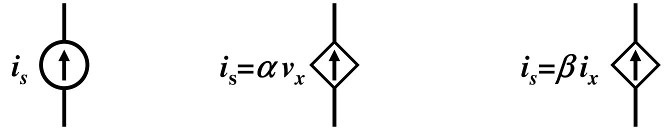
Device symbols:



Ideal Current Source

- Circuit element that maintains a prescribed current through its terminals, **regardless of the voltage across those terminals**.
  - Current is known, but voltage is determined by the circuit to which the source is connected.
- The current can be either **independent or dependent** on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Device symbols:



Electrical Resistance

- ***Resistance:*** the ratio of voltage drop and current. The circuit element used to model this behavior is the ***resistor***.

Circuit symbol:

Units: Volts per Ampere ≡ ohms (Ω)

Georg Simon Ohm
1789-1854

- The current flowing in the resistor is proportional to the voltage across the resistor:

$v = i\,R$ (Ohm's Law)

where v = voltage (V), i = current (A), and R = resistance (Ω)

Electrical Conductance

- **Conductance** is the reciprocal of resistance.

Symbol: G

Units: **siemens (S) or mhos (℧)**

Example:

Consider an 8 Ω resistor. *What is its conductance?*

Werner von Siemens
1816-1892

Short Circuit and Open Circuit

- Short circuit
  - R = 0 → no voltage difference exists
  - all points on the wire are at the same potential.
  - Current can flow, as determined by the circuit
- Open circuit
  - R = ∞ → no current flows
  - Voltage difference can exist, as determined by the circuit

Example: Power Absorbed by a Resistor

$$p = vi = (iR)i = i^2R$$

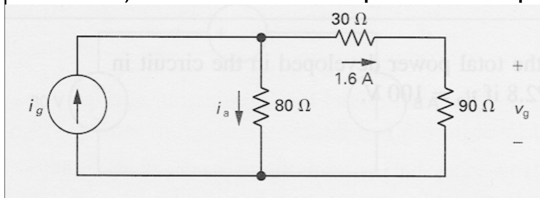
$$p = vi = v (v/R) = v^2/R$$

Note that $p > 0$ always, for a resistor → a resistor dissipates electric energy

Example:

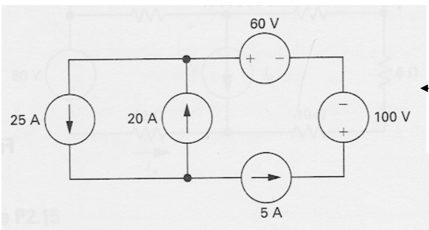
a) Calculate the voltage v_g and current i_a.

b) Determine the power dissipated in the 80Ω resistor.

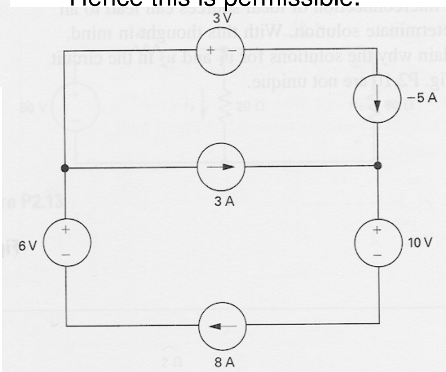


More Examples

- Are these interconnections permissible?



This circuit connection is permissible. This is because the current sources can sustain any voltage across; Hence this is permissible.



This circuit connection is NOT permissible. It violates the KCL.

Summary

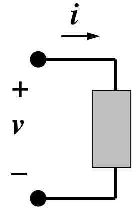
- ***Current*** = rate of charge flow $i = dq/dt$
- ***Voltage*** = energy per unit charge created by charge separation
- ***Power*** = energy per unit time
- ***Ideal Basic Circuit Elements***
  - two-terminal component that cannot be sub-divided
  - described mathematically in terms of its terminal voltage and current
  - An ***ideal voltage source*** maintains a prescribed voltage regardless of the current in the device.
  - An ***ideal current source*** maintains a prescribed current regardless of the voltage across the device.
  - A ***resistor*** constrains its voltage and current to be proportional to each other: $v = iR$ (Ohm's law)

Summary (cont'd)

- ***Passive sign convention***
  - For a passive device, the reference direction for current through the element is in the direction of the reference voltage drop across the element

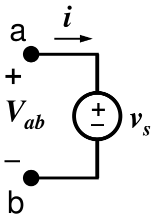
Current *vs.* Voltage (*I-V*) Characteristic

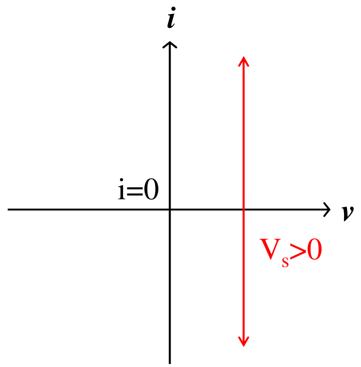
- Voltage sources, current sources, and resistors can be described by plotting the current (i) as a function of the voltage (v)



Passive? Active?

I-V Characteristic of Ideal Voltage Source

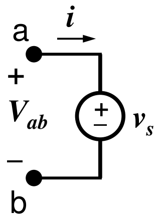


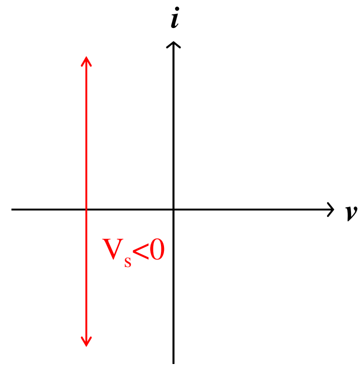


1. **Plot the *I-V* characteristic for $v_s > 0$. For what values of i does the source absorb power? For what values of i does the source release power?**

$V_s>0 \rightarrow$ i<0 release power; i>0 absorb power

I-V Characteristic of Ideal Voltage Source

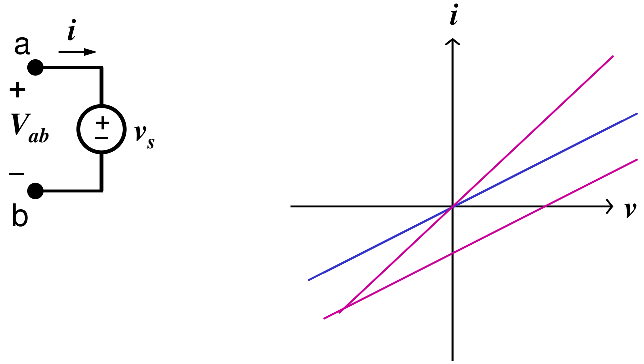




2. **Plot the *I-V* characteristic for $v_s < 0$. For what values of i does the source absorb power? For what values of i does the source release power?**

$V_s<0 \rightarrow$ i>0 release power; i<0 absorb power

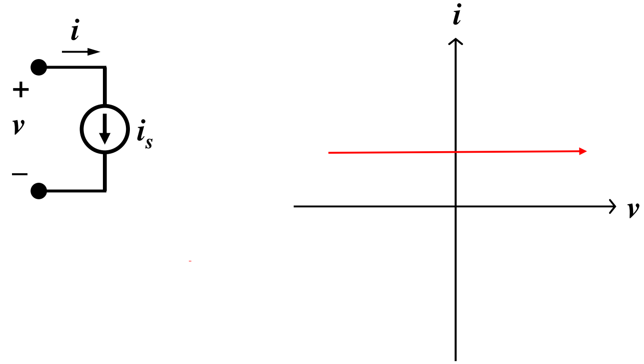
I-V Characteristic of Ideal Voltage Source



3. What is the *I-V* characteristic for an ideal wire?

Do not forget $V_{ab}=-V_{ba}$

I-V Characteristic of Ideal Current Source



1. **Plot the *I-V* characteristic for $i_s > 0$. For what values of v does the source absorb power? For what values of v does the source release power?**

   V>0 absorb power; V<0 release power

Short Circuit and Open Circuit

Wire ("short circuit"):

- $R = 0$ → **no voltage difference exists**
  (all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

Air ("open circuit"):

- $R = \infty$ → **no current flows**
- Voltage difference can exist, as determined by the circuit

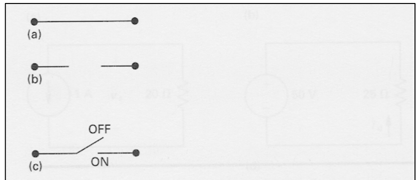
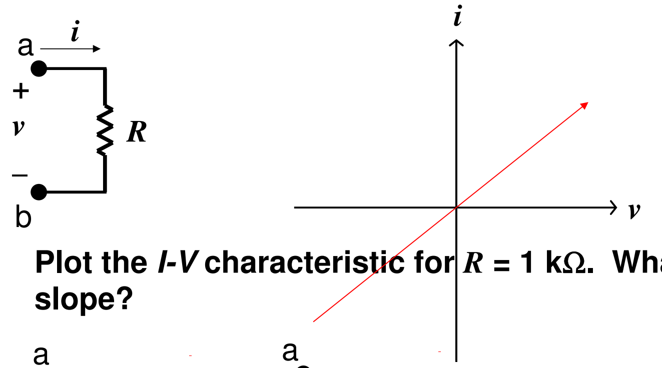
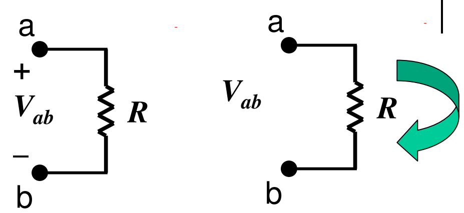


Figure 2.10 Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.

I-V Characteristic of Ideal Resistor

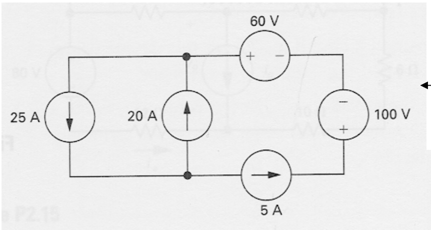


1. **Plot the *I-V* characteristic for R = 1 kΩ. What is the slope?**

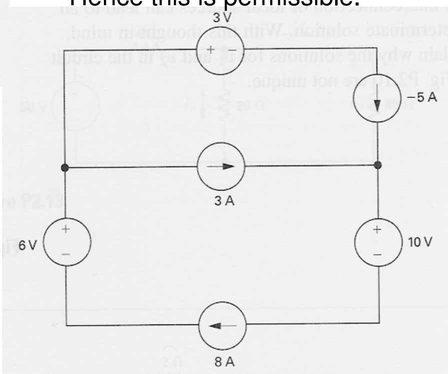


More Examples: Correction from last Lec.

- Are these interconnections permissible?



This circuit connection is permissible. This is because the current sources can sustain any voltage across; Hence this is permissible.



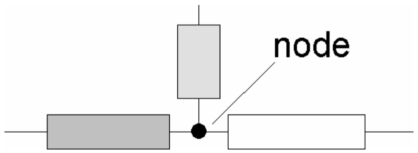
This circuit connection is NOT permissible. It violates the KCL.

Construction of a Circuit Model

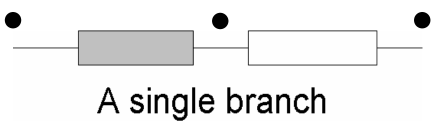
- The electrical behavior of each physical component is of primary interest.
- We need to account for undesired as well as desired electrical effects.
- Simplifying assumptions should be made wherever reasonable.

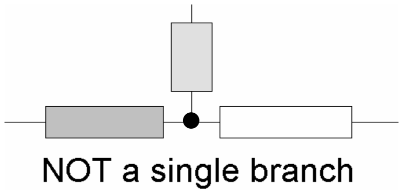
Terminology: Nodes and Branches

Node: A point where two or more circuit elements are connected



Branch: A path that connects two nodes





Circuit Nodes and Loops

- A ***node*** is a point where two or more circuit elements are connected.
- A ***loop*** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once

Kirchhoff's Laws

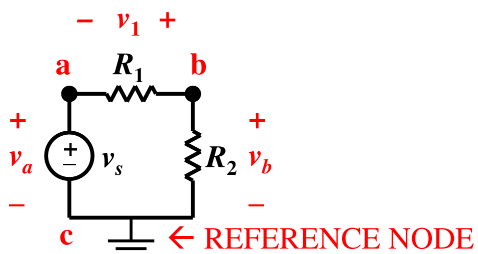
- **Kirchhoff's Current Law (KCL):**
  - The algebraic sum of all the **currents** entering any **node** in a circuit equals zero.
- **Kirchhoff's Voltage Law (KVL):**
  - The algebraic sum of all the **voltages** around any **loop** in a circuit equals zero.

Gustav Robert Kirchhoff
1824-1887

Notation: Node and Branch Voltages

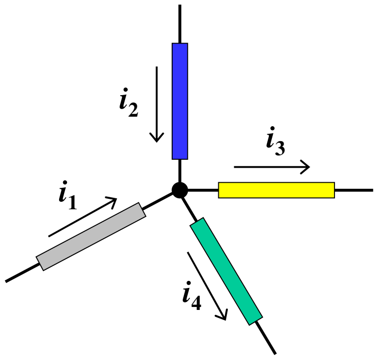
- Use one node as the reference (the "common" or "ground" node) – label it with a symbol
- The voltage drop from node **x** to the reference node is called the **node voltage v_x**.
- The voltage across a circuit element is defined as the difference between the node voltages at its terminals

Example:



Using Kirchhoff's Current Law (KCL)

Consider a node connecting several branches:



- Use **reference directions** to determine whether currents are "entering" or "leaving" the node – with no concern about actual current directions

Formulations of Kirchhoff's Current Law

(Charge stored **in node** is zero.)

Formulation 1:

Sum of currents entering node
= sum of currents leaving node

Formulation 2:

Algebraic sum of currents entering node = 0

- Currents leaving are included with a minus sign.

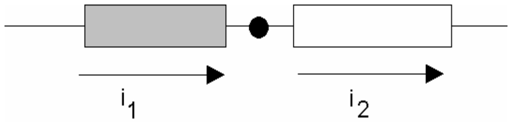
Formulation 3:

Algebraic sum of currents leaving node = 0

- Currents entering are included with a minus sign.

A Major Implication of KCL

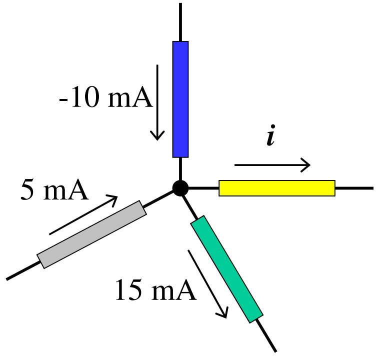
- KCL tells us that **all of the elements in a single branch carry the same current**.
- We say these elements are connected ***in series***.



Current entering node = Current leaving node

$i_1 = i_2$

KCL Example



Currents entering the node:

Currents leaving the node:

<u>3 formulations of KCL</u>:

1.

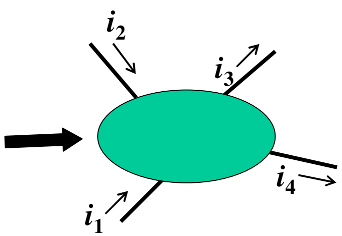
2.

3.

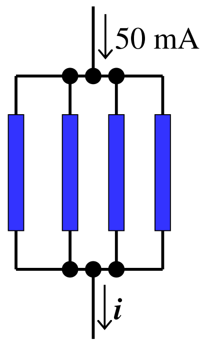
Generalization of KCL

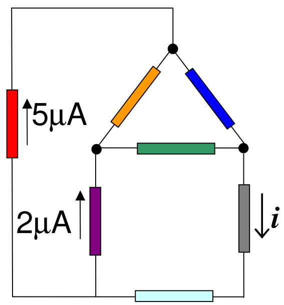
- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a "black box"



Generalized KCL Examples





Using Kirchhoff's Voltage Law (KVL)

Consider a branch which forms part of a loop:

Moving from + to -
We add V_1

Moving from - to +
We subtract V_1

- Use **reference polarities** to determine whether a voltage is dropped
- No concern about actual voltage polarities

Formulations of Kirchhoff's Voltage Law

(Conservation of energy)

Formulation 1:

Sum of voltage drops around loop
= sum of voltage rises around loop

Formulation 2:

Algebraic sum of voltage drops around loop = 0

- Voltage rises are included with a minus sign.

(Handy trick: Look at the first sign you encounter on each element when tracing the loop.)

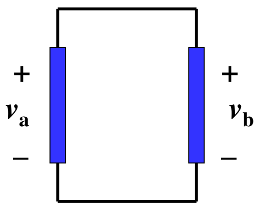
Formulation 3:

Algebraic sum of voltage rises around loop = 0

- Voltage drops are included with a minus sign.

A Major Implication of KVL

- KVL tells us that **any set of elements which are connected at both ends carry the same voltage.**
- We say these elements are connected **in parallel**.

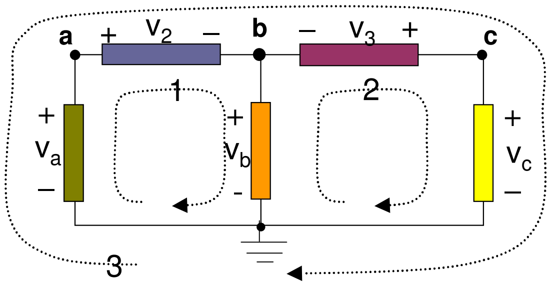


Applying KVL in the clockwise direction, starting at the top:

$$v_b - v_a = 0 \;\Rightarrow\; v_b = v_a$$

KVL Example

Three closed paths:



Path 1:

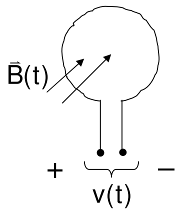
Path 2:

Path 3:

An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop
  Otherwise, there would be an induced voltage (Faraday's Law)

- Note: Antennas are designed to "pick up" electromagnetic waves; "regular circuits" often do so undesirably.

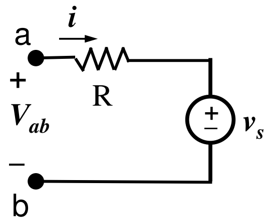


Avoid these loops!

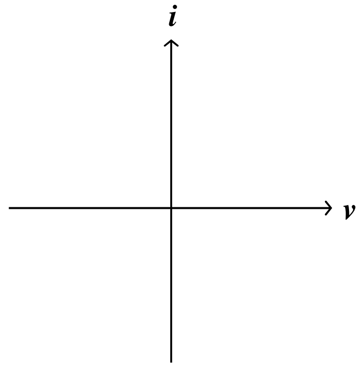
How do we deal with antennas (EECS 117A)?

Include a voltage source as the circuit representation of the induced voltage or "noise".
(Use a lumped model rather than a distributed (wave) model.)

I-V Characteristic of Elements



Find the *I-V* characteristic.



Summary

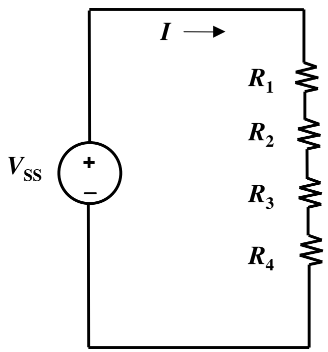
- An electrical system can be modeled by an **electric circuit** (***combination of paths***, each containing 1 or more ***circuit elements***)
  - Lumped model
- The ***Current versus voltage characteristics (I-V plot)*** is a universal means of describing a circuit element.
- ***Kirchhoff's current law (KCL)*** states that the algebraic sum of all currents at any node in a circuit equals zero.
  - Comes from conservation of charge
- ***Kirchhoff's voltage law (KVL)*** states that the algebraic sum of all voltages around any closed path in a circuit equals zero.
  - Comes from conservation of potential energy

Chapter 2

- Outline
  - Resistors in Series – Voltage Divider
  - Conductances in Parallel – Current Divider
  - Node-Voltage Analysis
  - Mesh-Current Analysis
  - Superposition
  - Thévenin equivalent circuits
  - Norton equivalent circuits
  - Maximum Power Transfer

Resistors in Series

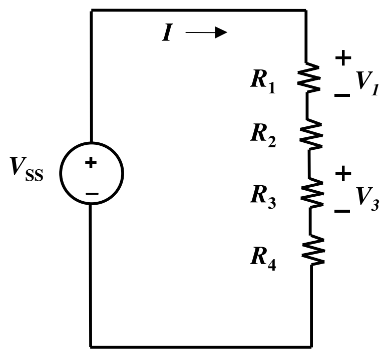
Consider a circuit with multiple resistors connected in series. Find their "equivalent resistance".



- KCL tells us that the same current (I) flows through every resistor
- KVL tells us

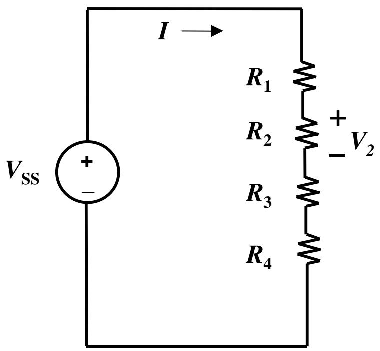
Equivalent resistance of resistors in series is the sum

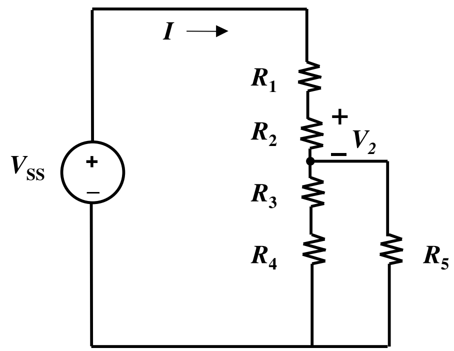
Voltage Divider



$$I = V_{SS} / (R_1 + R_2 + R_3 + R_4)$$

When can the Voltage Divider Formula be Used?





$$V_2 = \frac{R_2}{R_1+R_2+R_3+R_4} \cdot V_{SS}$$

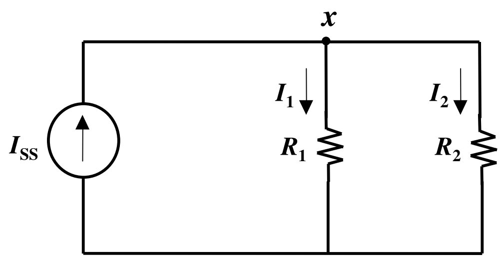
Correct, if nothing else is connected to nodes

$$V_2 \neq \frac{R_2}{R_1+R_2+R_3+R_4} \cdot V_{SS}$$

Why? What is V_2?

Resistors in Parallel

Consider a circuit with two resistors connected in parallel. Find their "equivalent resistance".

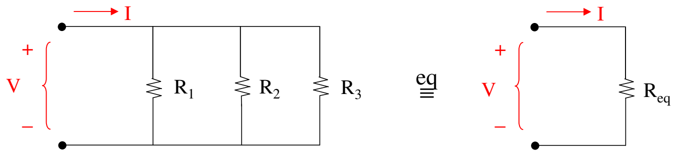


- KVL tells us that the same voltage is dropped across each resistor

  $V_x = I_1 R_1 = I_2 R_2$
- KCL tells us

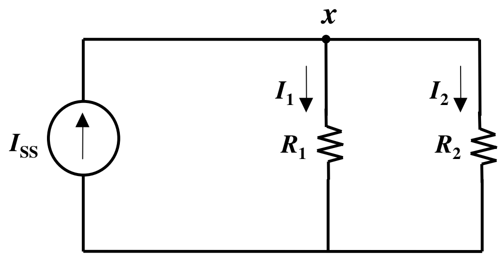
General Formula for Parallel Resistors

What single resistance R_{eq} is equivalent to three resistors in parallel?



Equivalent conductance of resistors in parallel is the sum

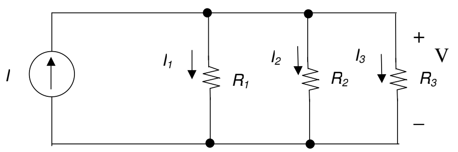
Current Divider



$$V_x = I_1 R_1 = I_{SS} R_{eq}$$

Generalized Current Divider Formula

Consider a current divider circuit with >2 resistors in parallel:



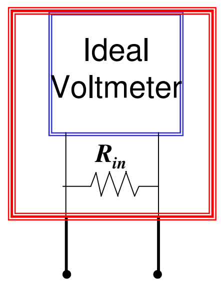
$$V = \frac{I}{\left(\frac{1}{R_1}\right)+\left(\frac{1}{R_2}\right)+\left(\frac{1}{R_3}\right)}$$

$$I_3 = \frac{V}{R_3} = I\left[\frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3}\right]$$

Measuring Voltage

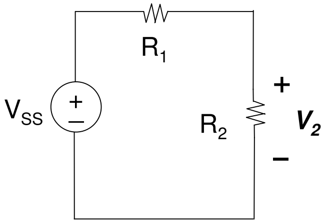
To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.

Voltmeters are characterized by their "voltmeter input resistance" (***R**$_{in}$*). Ideally, this should be very high (typical value 10 MΩ)

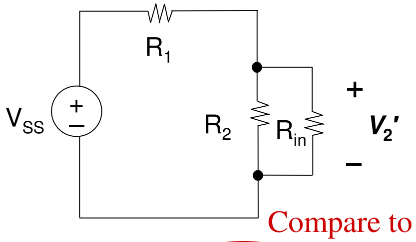


Effect of Voltmeter

undisturbed circuit



circuit with voltmeter inserted



Compare to R_2

$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right] \qquad V_2' = V_{SS}\left[\frac{R_2 \| R_{in}}{R_2 \| R_{in} + R_1}\right]$$

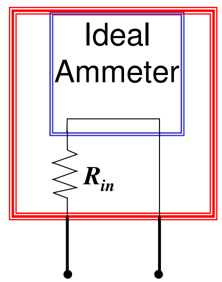
Example: $V_{SS} = 10\,V, R_2 = 100\,K, R_1 = 900\,K \Rightarrow V_2 = 1V$

$R_{in} = 10M, \; V_2' = ?$

Measuring Current

To measure the current flowing through an element in a real circuit, insert an ammeter (digital multimeter in current mode) **in series** with the element.

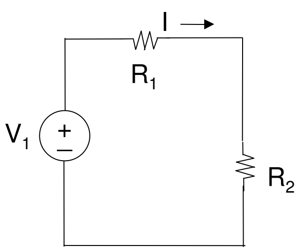
Ammeters are characterized by their "ammeter input resistance" (***R**_in_*). Ideally, this should be very low (typical value 1Ω).



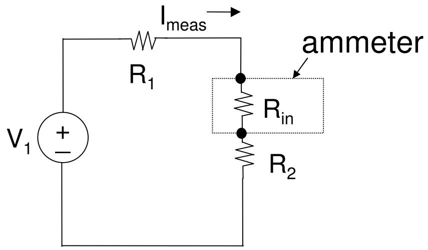
Effect of Ammeter

Measurement error due to non-zero input resistance:

undisturbed circuit



circuit with ammeter inserted



$$I = \frac{V_1}{R_1 + R_2} \qquad I_{meas} = \frac{V_1}{R_1 + R_2 + R_{in}}$$

Example: V_1 = 1 V, R_1= R_2 = 500 Ω, R_{in} = 1Ω

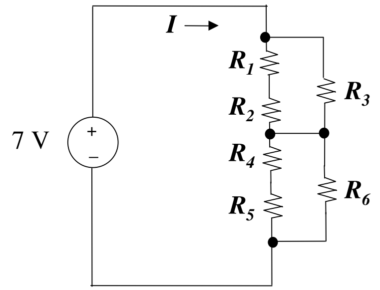
$$I = \frac{1V}{500\Omega + 500\Omega} = 1mA, \quad I_{meas} = ?$$

Compare to $R_2 + R_2$

Using Equivalent Resistances

Simplify a circuit before applying KCL and/or KVL:

Example: Find ***I***



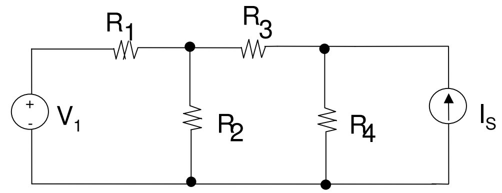
$R_1 = R_2 = 3\ k\Omega$
$R_3 = 6\ k\Omega$

$R_4 = R_5 = 5\ k\Omega$
$R_6 = 10\ k\Omega$

Node-Voltage Circuit Analysis Method

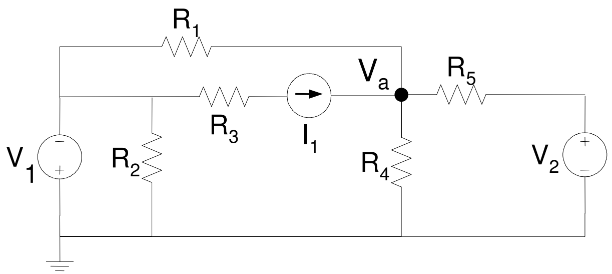
1. **Choose a reference node** ("ground")
   Look for the one with the most connections!
2. **Define unknown node voltages**
   those which are not fixed by voltage sources
3. **Write KCL at each unknown node**, expressing current in terms of the node voltages (using the *I-V* relationships of branch elements)
   Special cases: floating voltage sources
4. **Solve the set of independent equations**
   N equations for *N* unknown node voltages

Nodal Analysis: Example #1



1. Choose a reference node.
2. Define the node voltages (except reference node and the one set by the voltage source).
3. Apply KCL at the nodes with unknown voltage.
4. Solve for unknown node voltages.

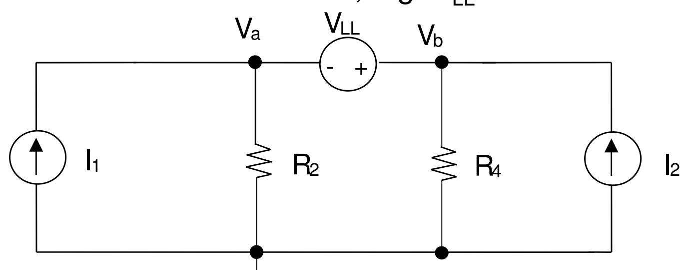
Nodal Analysis: Example #2



Challenges:
Determine number of nodes needed
Deal with different types of sources

Nodal Analysis w/ "Floating Voltage Source"

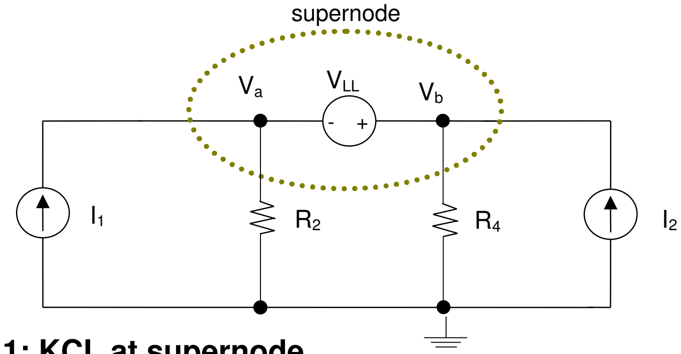
A "floating" voltage source is one for which neither side is connected to the reference node, e.g. V_{LL} in the circuit below:



Problem: We cannot write KCL at nodes a or b because there is no way to express the current through the voltage source in terms of V_a-V_b.

Solution: Define a "supernode" – that chunk of the circuit containing nodes a and b. Express KCL for this supernode. Incorporate voltage source constraint into KCL equation.

Nodal Analysis: Example #3



<u>Eq'n 1</u>: KCL at supernode

Substitute property of voltage source:

Formal Circuit Analysis Methods

<u>NODAL ANALYSIS</u>

("Node-Voltage Method")

0) Choose a reference node

1) Define unknown node voltages

2) Apply KCL to each unknown node, expressing current in terms of the node voltages

=> N equations for N unknown node voltages

3) Solve for node voltages

=> determine branch currents

<u>MESH ANALYSIS</u>

("Mesh-Current Method")

1) Select M independent mesh currents such that at least one mesh current passes through each branch*

M = #branches - #nodes + 1

2) Apply KVL to each mesh, expressing voltages in terms of mesh currents

=> M equations for M unknown mesh currents

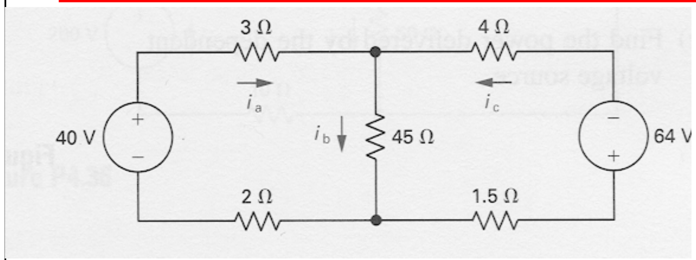
3) Solve for mesh currents

=> determine node voltages

*Simple method for *planar* circuits

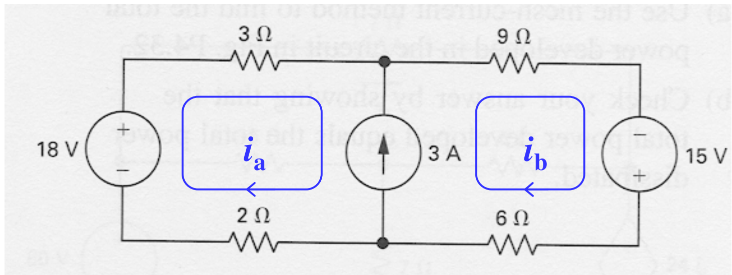
A mesh current is not necessarily identified with a branch current.

Mesh Analysis: Example #1



1. Select M mesh currents.
2. Apply KVL to each mesh.
3. Solve for mesh currents.

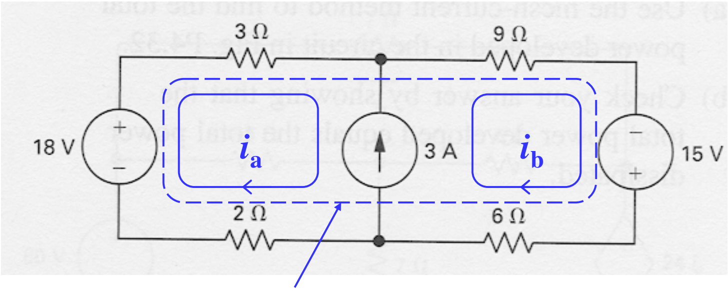
Mesh Analysis with a Current Source



<u>Problem</u>: We cannot write KVL for meshes a and b because there is no way to express the voltage drop across the current source in terms of the mesh currents.

<u>Solution</u>: Define a "supermesh" – a mesh which avoids the branch containing the current source. Apply KVL for this supermesh.

Mesh Analysis: Example #2



Eq'n 1: KVL for supermesh

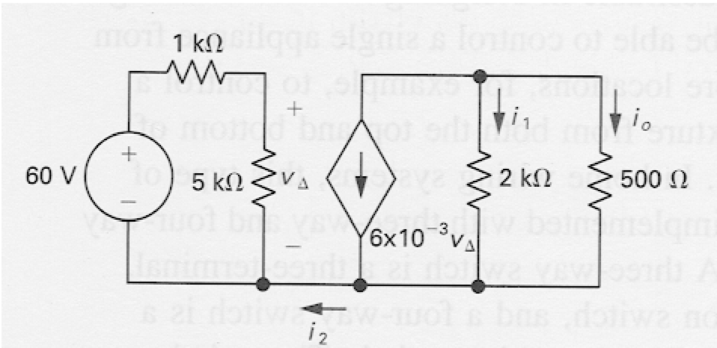
Eq'n 2: Constraint due to current source:

Mesh Analysis with Dependent Sources

- Exactly analogous to Node Analysis
- Dependent Voltage Source: (1) Formulate and write KVL mesh eqns. (2) Include and express dependency constraint in terms of mesh currents
- Dependent Current Source: (1) Use supermesh. (2) Include and express dependency constraint in terms of mesh currents

Circuit w/ Dependent Source Example

Find i_2, i_1 and i_o



Superposition

A ***linear circuit*** is one constructed only of linear elements (linear resistors, **and linear capacitors and inductors,** linear dependent sources) and independent sources. Linear

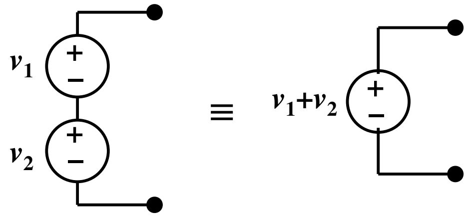
means I-V charcteristic of elements/sources are straight lines when plotted

Principle of Superposition:

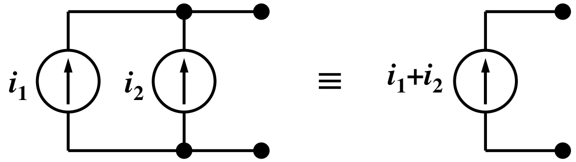
- In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

Source Combinations

- Voltage sources in series can be replaced by an equivalent voltage source:



- Current sources in parallel can be replaced by an equivalent current source:



Superposition

Procedure:

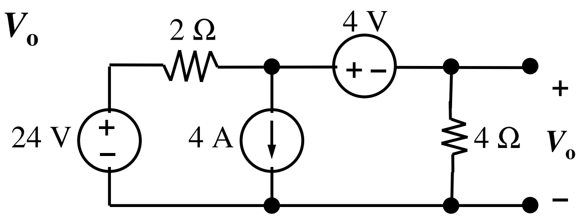
1. Determine contribution due to one independent source
   - Set all other sources to 0: Replace independent voltage source by short circuit, independent current source by open circuit
2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

Open Circuit and Short Circuit

- Open circuit → i=0 ; Cut off the branch
- Short circuit → v=0 ; replace the element by wire
- Turn off an independent voltage source means
  - V=0
  - Replace by wire
  - Short circuit
- Turn off an independent current source means
  - i=0
  - Cut off the branch
  - open circuit

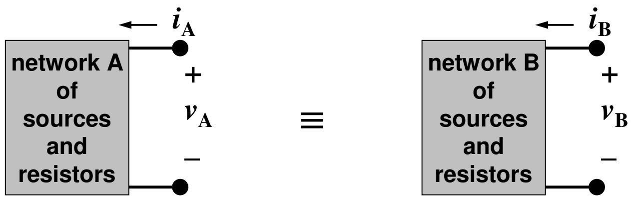
Superposition Example

- Find V_o



Equivalent Circuit Concept

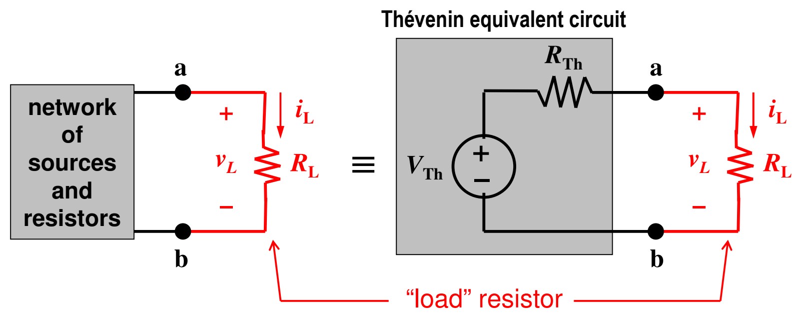
- A network of voltage sources, current sources, and resistors can be replaced by an ***equivalent circuit*** which has identical terminal properties (*I-V* characteristics) without affecting the operation of the rest of the circuit.



$$i_A(v_A) = i_B(v_B)$$

Thévenin Equivalent Circuit

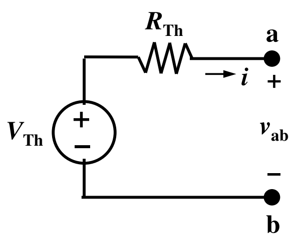
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent voltage source in series with a resistor without affecting the operation of the rest of the circuit.

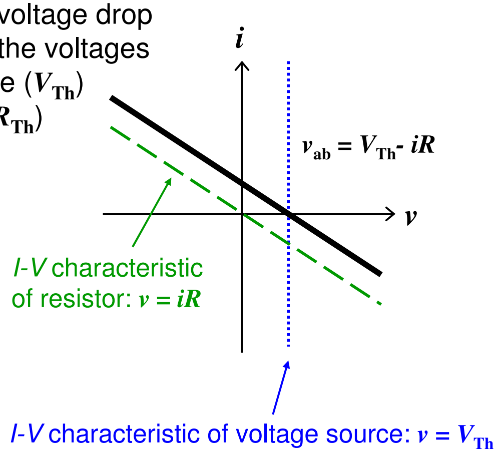


I-V Characteristic of Thévenin Equivalent

- **The *I-V* characteristic for the series combination of elements is obtained by adding their voltage drops:**

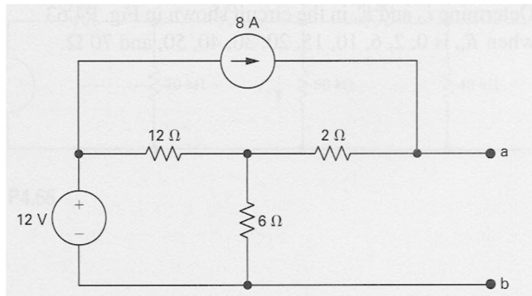
For a given current i, the voltage drop v_{ab} is equal to the sum of the voltages dropped across the source (V_{Th}) and across the resistor (iR_{Th})



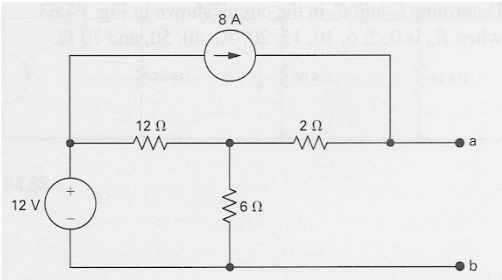


Thévenin Equivalent Example

Find the Thevenin equivalent with respect to the terminals a,b:



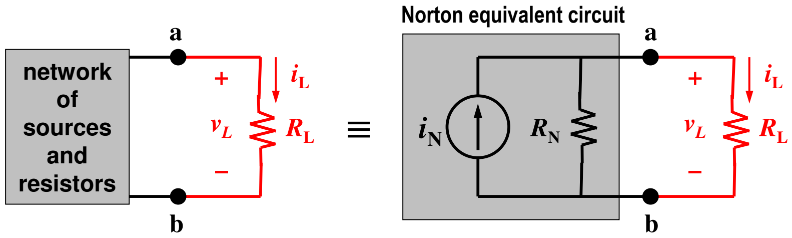
R_{Th} Calculation Example #1



Set all independent sources to 0:

Norton Equivalent Circuit

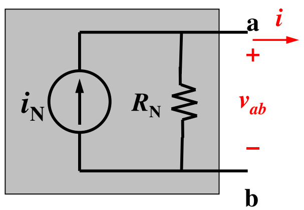
- Any* *linear* 2-terminal (1-port) network of indep. voltage sources, indep. current sources, and linear resistors can be replaced by an equivalent circuit consisting of an independent current source in parallel with a resistor without affecting the operation of the rest of the circuit.

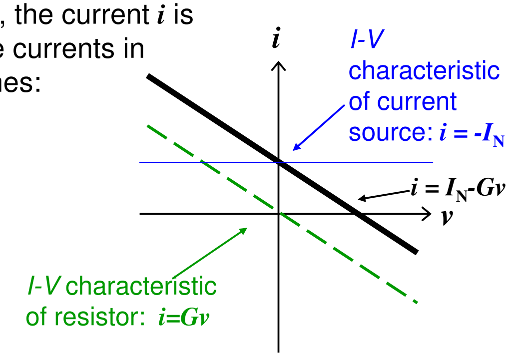


I-V Characteristic of Norton Equivalent

- **The *I-V* characteristic for the parallel combination of elements is obtained by adding their currents:**

For a given voltage v_{ab}, the current i is equal to the sum of the currents in each of the two branches:





Finding I_N and $R_N = R_{Th}$

Analogous to calculation of Thevenin Eq. Ckt:

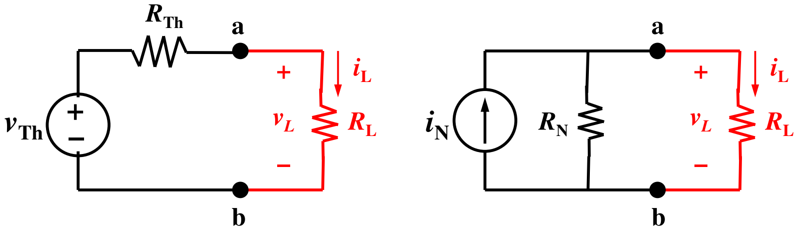
1) Find o.c voltage and s.c. current

$$I_N \equiv i_{sc} = V_{Th}/R_{Th}$$

2) Or, find s.c. current and Norton (Thev) resistance

Finding I_N and R_N

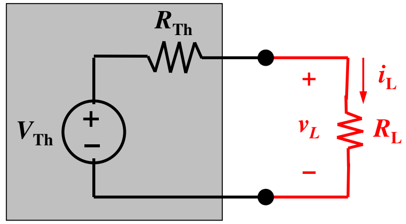
- We can derive the Norton equivalent circuit from a Thévenin equivalent circuit simply by making a source transformation:



$$\boxed{R_N = R_{Th} = \frac{v_{oc}}{i_{sc}}; \quad i_N = \frac{v_{Th}}{R_{Th}} = i_{sc}}$$

Maximum Power Transfer Theorem

Thévenin equivalent circuit



Power absorbed by load resistor:

$$p = i_L^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

To find the value of R_L for which p is maximum, set $\frac{dp}{dR_L}$ to 0:

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] = 0$$

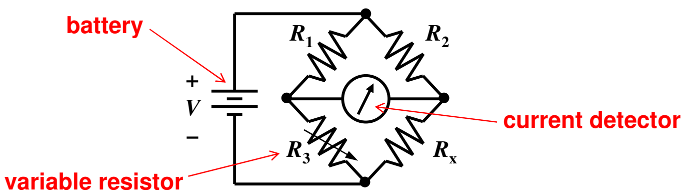
$$\Rightarrow (R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L) = 0$$

$$\Rightarrow \boxed{R_{Th} = R_L}$$

A resistive load receives maximum power from a circuit if the load resistance equals the Thévenin resistance of the circuit.

The Wheatstone Bridge

- Circuit used to precisely measure resistances in the range from 1 Ω to 1 MΩ, with ±0.1% accuracy
  - R_1 and R_2 are resistors with known values
  - R_3 is a variable resistor (typically 1 to 11,000Ω)
  - R_x is the resistor whose value is to be measured

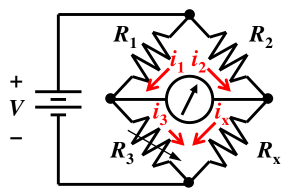


Finding the value of R_x

- Adjust R_3 until there is no current in the detector

Then, $\boxed{R_x = \frac{R_2}{R_1} R_3}$

Derivation:

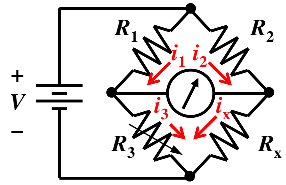


Typically, R_2 / R_1 can be varied from 0.001 to 1000 in decimal steps

Finding the value of R_x

- Adjust R_3 until there is no current in the detector

Then, $\boxed{R_x = \dfrac{R_2}{R_1} R_3}$



Typically, R_2 / R_1 can be varied from 0.001 to 1000 in decimal steps

Derivation:

KCL => $i_1 = i_3$ and $i_2 = i_x$

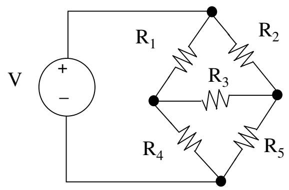
KVL => $i_3 R_3 = i_x R_x$ and $i_1 R_1 = i_2 R_2$

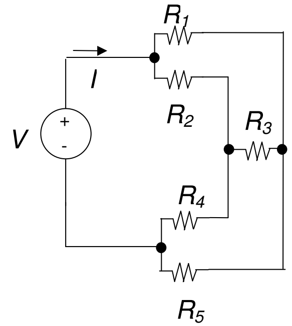
$i_1 R_3 = i_2 R_x$

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

Identifying Series and Parallel Combinations

Some circuits *must* be analyzed (not amenable to simple inspection)

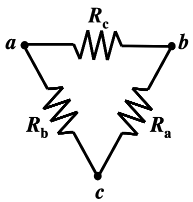


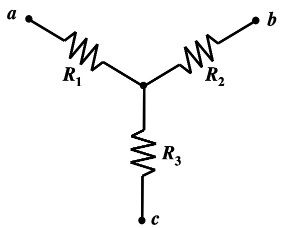


Special cases:
$R_3 = 0$ **OR** $R_3 = \infty$

Y-Delta Conversion

- These two resistive circuits are equivalent for voltages and currents <u>external</u> to the Y and Δ circuits. Internally, the voltages and currents are different.



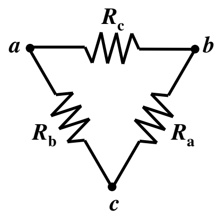


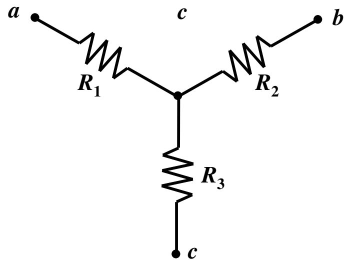
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Brain Teaser Category: Important for motors and electrical utilities.

Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

- In order for the Delta interconnection to be equivalent to the Wye interconnection, the resistance between corresponding terminal pairs must be the same





$$R_{ab} = \frac{R_c (R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

$$R_{bc} = \frac{R_a (R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3$$

$$R_{ca} = \frac{R_b (R_a + R_c)}{R_a + R_b + R_c} = R_1 + R_3$$

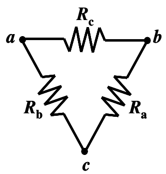
Δ-Y and Y-Δ Conversion Formulas

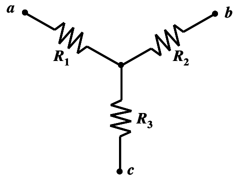
Delta-to-Wye conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$





Wye-to-Delta conversion

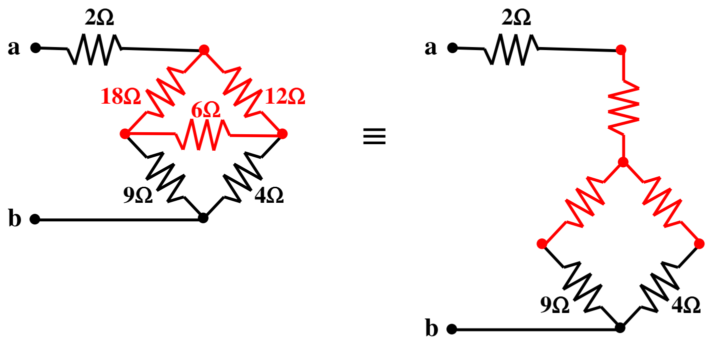
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Circuit Simplification Example

Find the equivalent resistance R_{ab}:



Dependent Sources

- Node-Voltage Method
  - Dependent current source:
    - treat as independent current source in organizing node eqns
    - substitute constraining dependency in terms of defined node voltages.
  - Dependent voltage source:
    - treat as independent voltage source in organizing node eqns
    - Substitute constraining dependency in terms of defined node voltages.
- Mesh Analysis
  - Dependent Voltage Source:
    - Formulate and write KVL mesh eqns.
    - Include and express dependency constraint in terms of mesh currents
  - Dependent Current Source:
    - Use supermesh.
    - Include and express dependency constraint in terms of mesh currents

Comments on Dependent Sources

A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.

(device model, used to model behavior of transistors & amplifiers)

To specify a dependent source, we must identify:

1. the controlling voltage or current (must be calculated, in general)
2. the relationship between the controlling voltage or current and the supplied voltage or current
3. the reference direction for the supplied voltage or current

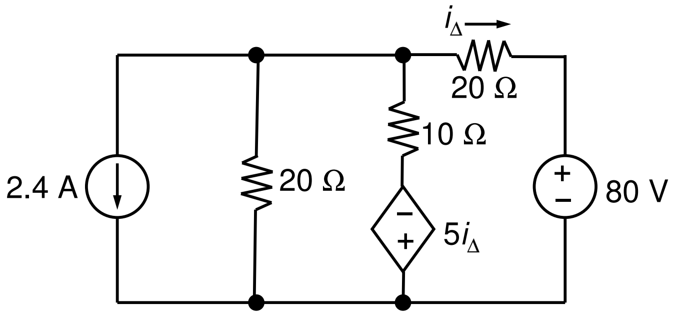
The relationship between the dependent source and its reference cannot be broken!

– Dependent sources cannot be turned off for various purposes (*e.g.* to find the Thévenin resistance, or in analysis using Superposition).

Node-Voltage Method and Dependent Sources

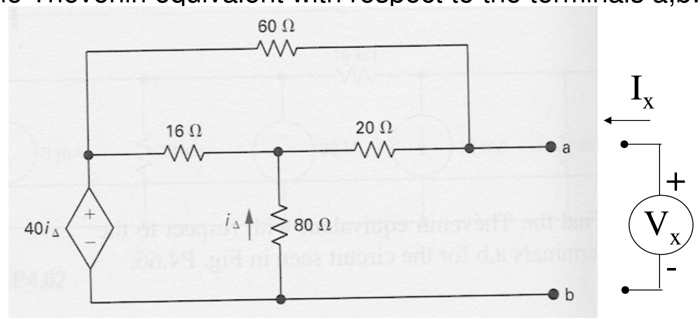
- If a circuit contains dependent sources, what to do?

<u>Example</u>:



R_{Th} Calculation Example #2

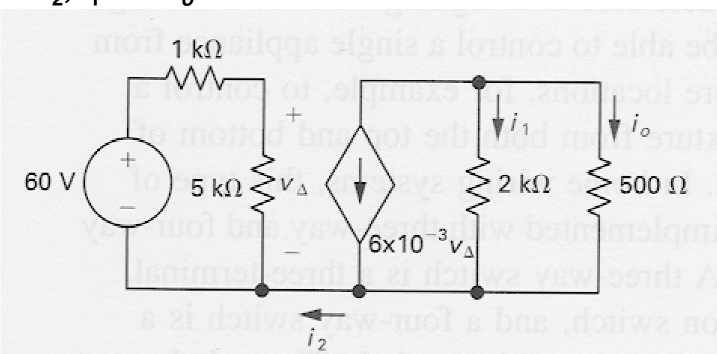
Find the Thevenin equivalent with respect to the terminals a,b:



Since there is no independent source and we cannot arbitrarily turn off the dependence source, we can add a voltage source V_x across terminals a-b and measure the current through this terminal I_x . $R_{th}= V_x/ I_x$

Circuit w/ Dependent Source Example

Find i_2, i_1 and i_o



Summary of Techniques for Circuit Analysis -1 (Chap 2)

- Resistor network
  - Parallel resistors
  - Series resistors
  - Y-delta conversion
  - "Add" current source and find voltage (or vice versa)
- Superposition
  - Leave one independent source on at a time
  - Sum over all responses
  - Voltage off → SC
  - Current off → OC

Summary of Techniques for Circuit Analysis -2 (Chap 2)

- Node Analysis
  - Node voltage is the unknown
  - Solve for KCL
  - Floating voltage source using super node
- Mesh Analysis
  - Loop current is the unknown
  - Solve for KVL
  - Current source using super mesh
- Thevenin and Norton Equivalent Circuits
  - Solve for OC voltage
  - Solve for SC current

Chapter 3

- Outline
  - The capacitor
  - The inductor

The Capacitor

Two conductors (a,b) separated by an insulator:

difference in potential = V_{ab}

=> equal & opposite charge Q on conductors

$$\boxed{Q = CV_{ab}}$$

(stored charge in terms of voltage)

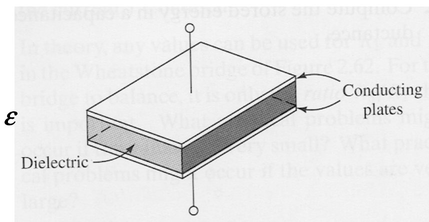
where C is the **capacitance** of the structure,

➢ positive (+) charge is on the conductor at higher potential

Parallel-plate capacitor:

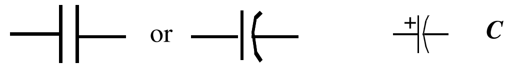
- area of the plates = A *(m²)*
- separation between plates = d *(m)*
- ***dielectric permittivity*** of insulator = ε *(F/m)*

=> capacitance $\boxed{C = \frac{A\varepsilon}{d}}$ (F)



Capacitor

Symbol:



C C Electrolytic (polarized) capacitor

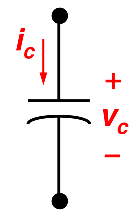
Units: Farads (Coulombs/Volt)

(typical range of values: 1 pF to 1 μF; for "supercapa-citors" up to a few F!)

Current-Voltage relationship:

$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt} + v_c\frac{dC}{dt}$$

If C (geometry) is unchanging, $i_C = C\, dv_C/dt$



Note: Q (v_c) must be a continuous function of time

Voltage in Terms of Current

$$Q(t) = \int_0^t i_c(t)dt + Q(0)$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t)dt + \frac{Q(0)}{C} = \frac{1}{C}\int_0^t i_c(t)dt + v_c(0)$$

Uses: Capacitors are used to store energy for camera flashbulbs, in filters that separate various frequency signals, and they appear as undesired "parasitic" elements in circuits where they usually degrade circuit performance

Stored Energy

CAPACITORS STORE ELECTRIC ENERGY

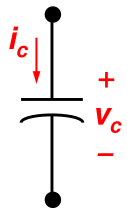
You might think the energy stored on a capacitor is ***QV = CV²***, which has the dimension of Joules. But during charging, the average voltage across the capacitor was only half the final value of ***V*** for a linear capacitor.

Thus, energy is $\frac{1}{2}QV = \boxed{\frac{1}{2}CV^2}$.

Example: A 1 pF capacitance charged to 5 Volts has ½(5V)² (1pF) = 12.5 pJ (A 5F supercapacitor charged to 5 volts stores 63 J; if it discharged at a constant rate in 1 ms energy is discharged at a 63 kW rate!)

A more rigorous derivation

This derivation holds independent of the circuit!

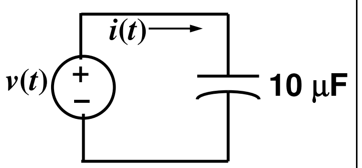


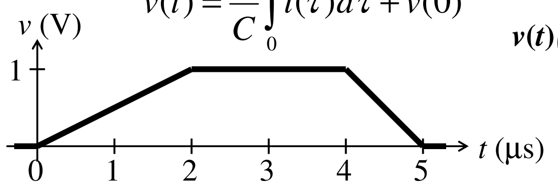
$$w = \int_{t=t_{Initial}}^{t=t_{Final}} v_c \cdot i_c \, dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \frac{dQ}{dt} dt = \int_{v=V_{Initial}}^{v=V_{Final}} v_c \, dQ$$

$$w = \int_{v=V_{Initial}}^{v=V_{Final}} C v_c \, dv_c = \frac{1}{2} C V_{Final}^2 - \frac{1}{2} C V_{Initial}^2$$

Example: Current, Power & Energy for a Capacitor

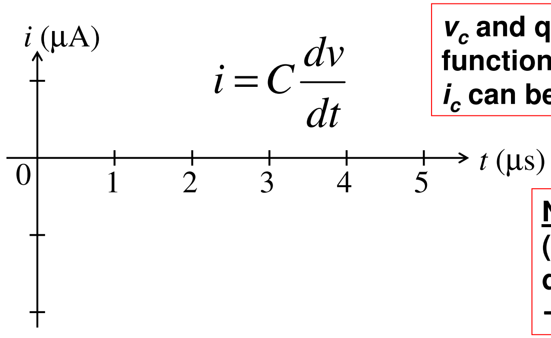
$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v(0)$$



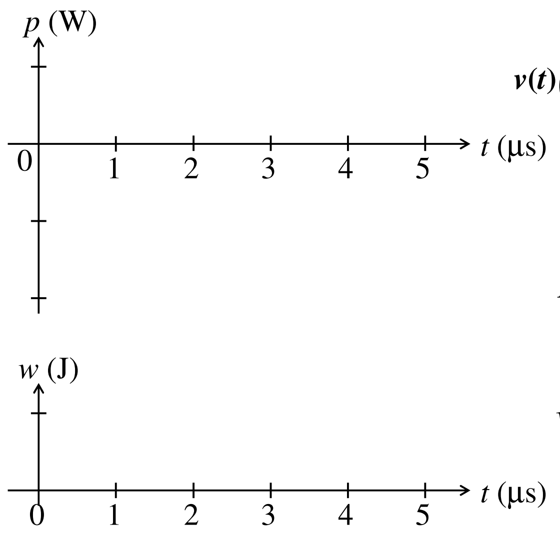


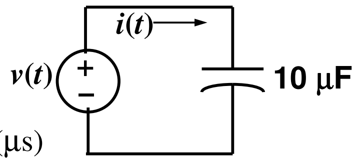
v_c and q must be continuous functions of time; however, i_c can be discontinuous.

$$i = C\frac{dv}{dt}$$



Note: In "steady state" (dc operation), time derivatives are zero → *C* is an open circuit

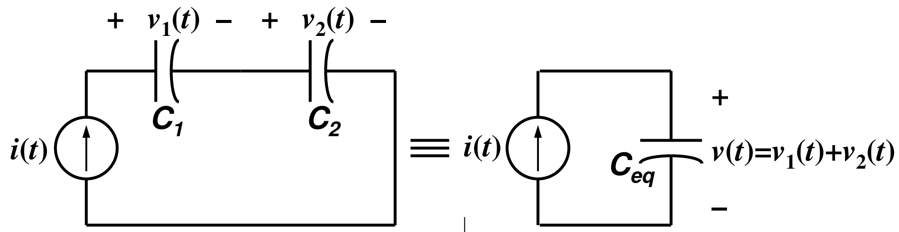




$$p = vi$$

$$w = \int_0^t p\,d\tau = \frac{1}{2}Cv^2$$

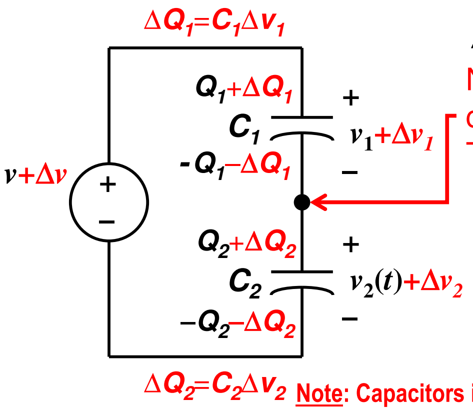
Capacitors in Series



$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Capacitive Voltage Divider

Q: Suppose the voltage applied across a series combination of capacitors is changed by Δv. How will this affect the voltage across each individual capacitor?



$$\Delta v = \Delta v_1 + \Delta v_2$$

Note that no net charge can can be introduced to this node. Therefore, $-\Delta Q_1 + \Delta Q_2 = 0$

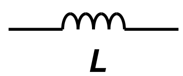
$$\Rightarrow C_1 \Delta v_1 = C_2 \Delta v_2$$

$$\boxed{\Delta v_2 = \frac{C_1}{C_1 + C_2}\Delta v}$$

Note: Capacitors in series have the same incremental charge.

Inductor

Symbol:



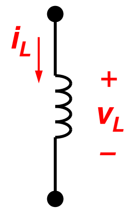
Units: Henrys (Volts • second / Ampere)

(typical range of values: μH to 10 H)

Current in terms of voltage:

$$di_L = \frac{1}{L} v_L(t)\,dt$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v_L(\tau)\,d\tau + i(t_0)$$



Note: i_L must be a continuous function of time

Stored Energy

INDUCTORS STORE MAGNETIC ENERGY

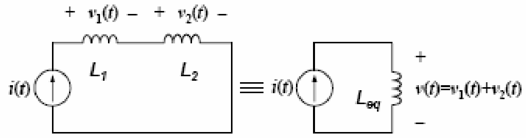
Consider an inductor having an initial current $i(t_0) = i_0$

$$p(t) = v(t)i(t) =$$

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau =$$

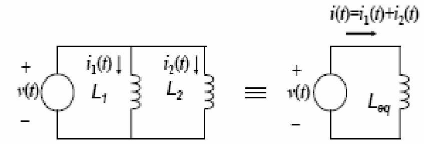
$$w(t) = \frac{1}{2}Li^2 - \frac{1}{2}Li_0^{\ 2}$$

Inductors in Series and Parallel



Common Current

$$\boxed{L_{eq} = L_1 + L_2}$$



Common Voltage

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

Summary

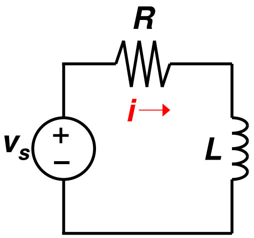
Capacitor	**Inductor**
$i = C\frac{dv}{dt}; w = \frac{1}{2}Cv^2$	$v = L\frac{di}{dt}; w = \frac{1}{2}Li^2$
v cannot change instantaneously ***i*** can change instantaneously Do not short-circuit a charged capacitor (-> infinite current!)	***i*** cannot change instantaneously ***v*** can change instantaneously Do not open-circuit an inductor with current (-> infinite voltage!)
n cap.'s in series: $\frac{1}{C_{eq}} = \sum_{i=1}^{n}\frac{1}{C_i}$	*n* ind.'s in series: $L_{eq} = \sum_{i=1}^{n} L_i$
n cap.'s in parallel: $C_{eq} = \sum_{i=1}^{n} C_i$	*n* ind.'s in parallel: $\frac{1}{L_{eq}} = \sum_{i=1}^{n}\frac{1}{L_i}$
In steady state (not time-varying), a capacitor behaves like an open circuit.	In steady state, an inductor behaves like a short circuit.

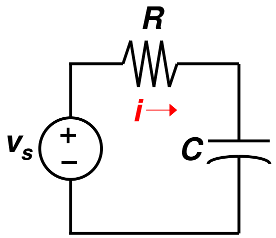
Chapter 4

- OUTLINE
  - First Order Circuits
    - RC and RL Examples
    - General Procedure
  - RC and RL Circuits with General Sources
    - Particular and complementary solutions
    - Time constant
  - Second Order Circuits
    - The differential equation
    - Particular and complementary solutions
    - The natural frequency and the damping ratio
- Reading
  - Chapter 4

First-Order Circuits

- A circuit that contains only sources, resistors and an inductor is called an ***RL circuit***.
- A circuit that contains only sources, resistors and a capacitor is called an ***RC circuit***.
- RL and RC circuits are called first-order circuits because their voltages and currents are described by first-order differential equations.



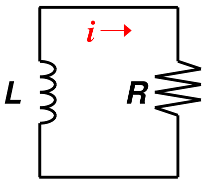


Response of a Circuit

- ***Transient response*** of an RL or RC circuit is
  – Behavior when voltage or current source are **suddenly** applied to or removed from the circuit due to switching.
  – Temporary behavior
- ***Steady-state response (aka. forced response)***
  – Response that persists long after transient has decayed
- ***Natural response*** of an RL or RC circuit is
  – Behavior (*i.e.*, current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources).

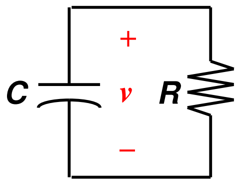
Natural Response Summary

RL Circuit



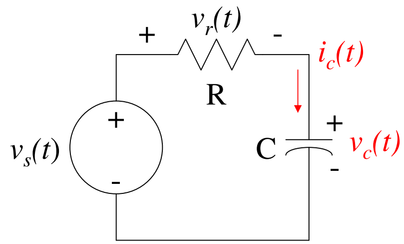
- Inductor current cannot change instantaneously
- In steady state, an inductor behaves like a short circuit.

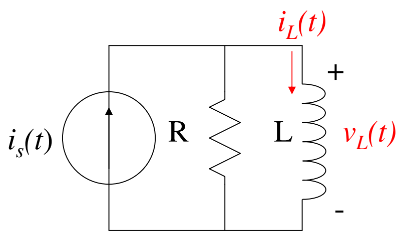
RC Circuit



- Capacitor voltage cannot change instantaneously
- In steady state, a capacitor behaves like an open circuit

First Order Circuits





KVL around the loop:

$v_r(t) + v_c(t) = v_s(t)$

$$RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t)$$

KCL at the node:

$$\frac{v(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v(x)dx = i_s(t)$$

$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = i_s(t)$$

Procedure for Finding Transient Response

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$
2. **Determine the initial value (at $t = t_0^-$ and t_0^+) of the variable**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

     $i_L(t_0^+) = i_L(t_0^-)$ and $v_c(t_0^+) = v_c(t_0^-)$
   - Assuming that the circuit reached steady state before t_0, use the fact that an inductor behaves like a short circuit in steady state or that a capacitor behaves like an open circuit in steady state

Procedure (cont'd)

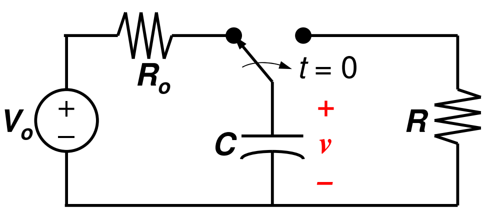
3. **Calculate the final value of the variable** (its value as $t \rightarrow \infty$)
   - Again, make use of the fact that an inductor behaves like a short circuit in steady state ($t \rightarrow \infty$) or that a capacitor behaves like an open circuit in steady state ($t \rightarrow \infty$)
4. **Calculate the time constant for the circuit**

   $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor

   $\tau = RC$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor

Natural Response of an RC Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:



Notation:

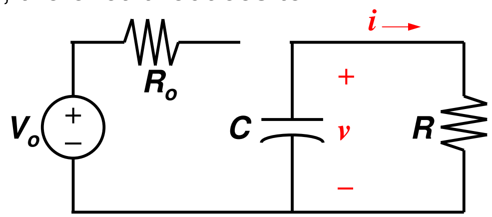
0⁻ is used to denote the time just prior to switching

0⁺ is used to denote the time immediately after switching

- The voltage on the capacitor at $t = 0^-$ is V_o

Solving for the Voltage ($t \geq 0$)

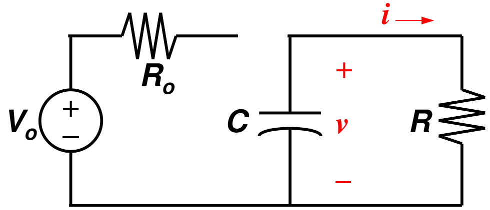
- For t > 0, the circuit reduces to



- Applying KCL to the RC circuit:

- Solution: $$v(t) = v(0)e^{-t/RC}$$

Solving for the Current ($t > 0$)



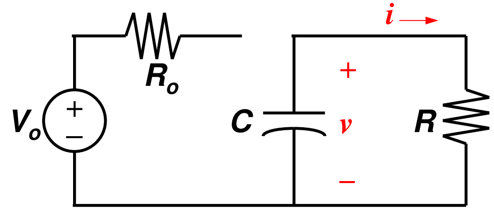
$$v(t) = V_o e^{-t/RC}$$

- Note that the current changes abruptly:
  $i(0^-) = 0$

$$\text{for } t > 0,\ i(t) = \frac{v}{R} = \frac{V_o}{R} e^{-t/RC}$$

$$\Rightarrow i(0^+) = \frac{V_o}{R}$$

Solving for Power and Energy Delivered ($t > 0$)



$$v(t) = V_o e^{-t/RC}$$

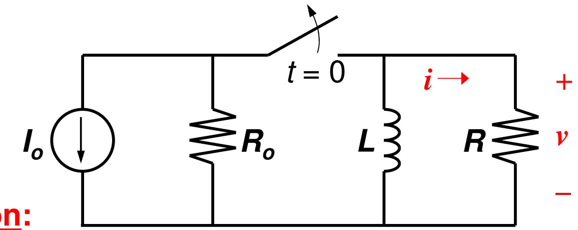
$$p = \frac{v^2}{R} = \frac{V_o^2}{R} e^{-2t/RC}$$

$$w = \int_0^t p(x)dx = \int_0^t \frac{V_o^2}{R} e^{-2x/RC} dx$$

$$= \frac{1}{2} C V_o^2 \left(1 - e^{-2t/RC}\right)$$

Natural Response of an RL Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:



Notation:

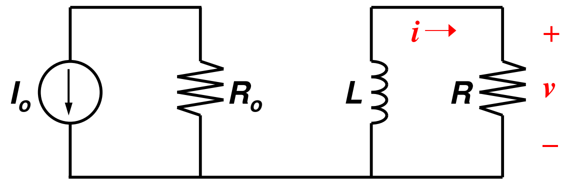
0⁻ is used to denote the time just prior to switching

0⁺ is used to denote the time immediately after switching

- t<0 the entire system is at steady-state; and the inductor is → like short circuit
- The current flowing in the inductor at $t = 0^-$ is I_o and V across is 0.

Solving for the Current ($t \geq 0$)

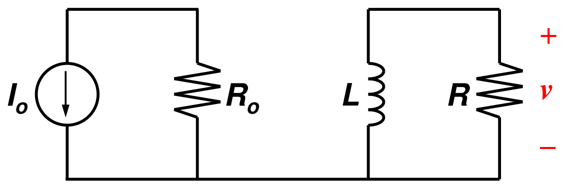
- For t > 0, the circuit reduces to



- Applying KVL to the LR circuit:
- $v(t)=i(t)R$
- At $t=0^+$, $i=I_0$,
- At arbitrary $t>0$, $i=i(t)$ and $v(t) = -L\dfrac{di(t)}{dt}$
- Solution: $i(t) = i(0)e^{-(R/L)t} = I_0 e^{-(R/L)t}$

Solving for the Voltage ($t > 0$)

$$i(t) = I_o e^{-(R/L)t}$$



- Note that the voltage changes abruptly:

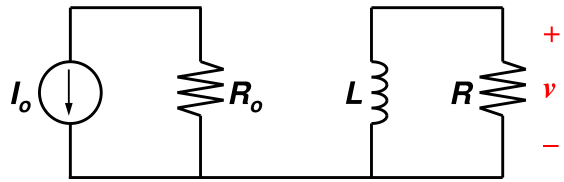
  $v(0^-) = 0$

  for $t > 0$, $v(t) = iR = I_o R e^{-(R/L)t}$

  $\Rightarrow v(0^+) = I_0 R$

Solving for Power and Energy Delivered ($t > 0$)

$$i(t) = I_o e^{-(R/L)t}$$



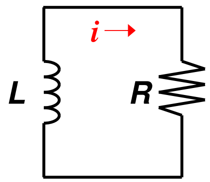
$$p = i^2 R = I_o^2 R e^{-2(R/L)t}$$

$$w = \int_0^t p(x)dx = \int_0^t I_o^2 R e^{-2(R/L)x} dx$$

$$= \frac{1}{2} L I_o^2 \left(1 - e^{-2(R/L)t}\right)$$

Natural Response Summary

RL Circuit



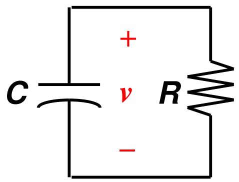
- Inductor current cannot change instantaneously

  $i(0^-) = i(0^+)$

  $i(t) = i(0)e^{-t/\tau}$

- time constant $\tau = \frac{L}{R}$

RC Circuit



- Capacitor voltage cannot change instantaneously

  $v(0^-) = v(0^+)$

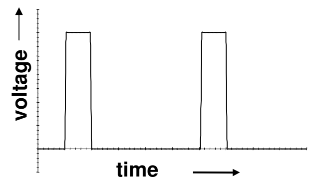
  $v(t) = v(0)e^{-t/\tau}$

- time constant $\tau = RC$

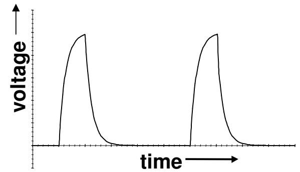
Digital Signals

We compute with pulses.

We send beautiful pulses in:



But we receive lousy-looking pulses at the output:

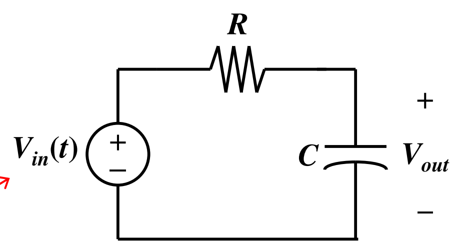


Capacitor charging effects are responsible!

- Every node in a real circuit has capacitance; it's the charging of these capacitances that limits circuit performance (speed)

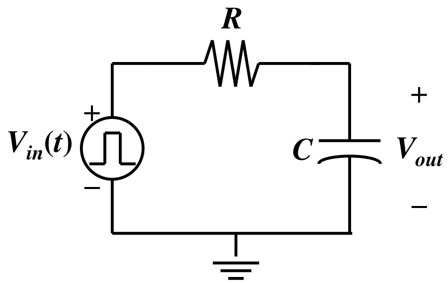
Circuit Model for a Logic Gate

- Recall (from Lecture 1) that electronic building blocks referred to as “logic gates” are used to implement logical functions (NAND, NOR, NOT) in digital ICs
  - Any logical function can be implemented using these gates.
- A logic gate can be modeled as a simple RC circuit:



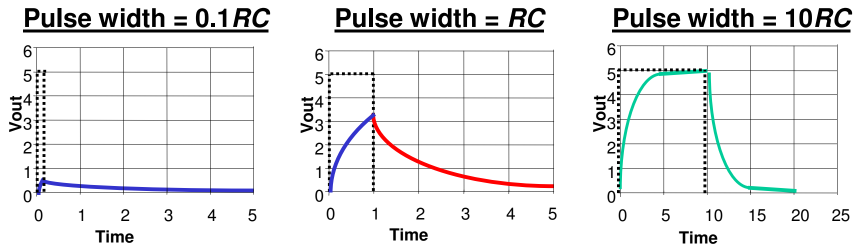
switches between “low” (logic 0) and “high” (logic 1) voltage states

Pulse Distortion



The input voltage pulse width must be large enough; otherwise the output pulse is distorted.

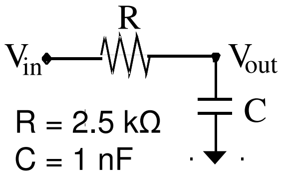
(We need to wait for the output to reach a recognizable logic level, before changing the input again.)



Example

Suppose a voltage pulse of width 5 µs and height 4 V is applied to the input of this circuit beginning at $t = 0$:

$\tau = RC = 2.5\ \mu s$



R = 2.5 kΩ
C = 1 nF

- First, V_{out} will increase exponentially toward 4 V.
- When V_{in} goes back down, V_{out} will decrease exponentially back down to 0 V.

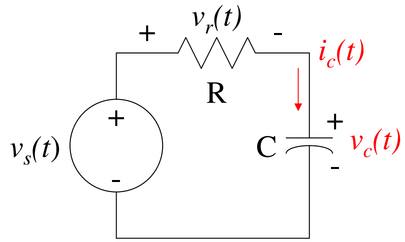
What is the peak value of V_{out}?

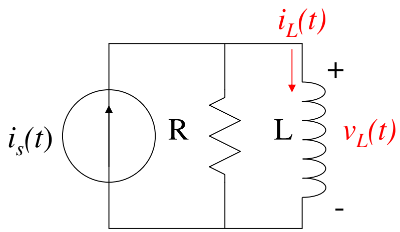
The output increases for 5 µs, or 2 time constants.

→ It reaches $1-e^{-2}$ or 86% of the final value.

0.86 x 4 V = 3.44 V is the peak value

First Order Circuits: Forced Response





KVL around the loop:

$v_r(t) + v_c(t) = v_s(t)$

$$RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t)$$

KCL at the node:

$$\frac{v(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v(x)dx = i_s(t)$$

$$\frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = i_s(t)$$

Complete Solution

- Voltages and currents in a 1st order circuit satisfy a differential equation of the form

$$x(t)+\tau\frac{dx(t)}{dt}=f(t)$$

  - *f(t)* is called the forcing function.
- The complete solution is the sum of particular solution (forced response) and complementary solution (natural response).

$$x(t)=x_p(t)+x_c(t)$$

  - Particular solution satisfies the forcing function
  - Complementary solution is used to satisfy the initial conditions.
  - The initial conditions determine the value of *K.*

$$x_p(t)+\tau\frac{dx_p(t)}{dt}=f(t)$$

$$x_c(t)+\tau\frac{dx_c(t)}{dt}=0$$ Homogeneous equation

$$x_c(t)=Ke^{-t/\tau}$$

The Time Constant

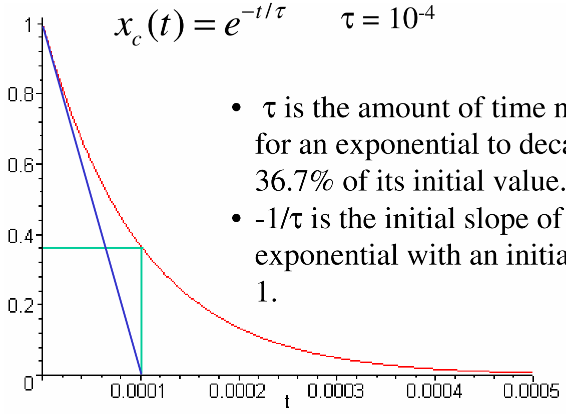
- The complementary solution for any 1st order circuit is

$$x_c(t)=Ke^{-t/\tau}$$

- For an RC circuit, $\tau = RC$
- For an RL circuit, $\tau = L/R$

What Does $X_c(t)$ Look Like?

$$x_c(t)=e^{-t/\tau} \qquad \tau = 10^{-4}$$



- τ is the amount of time necessary for an exponential to decay to 36.7% of its initial value.
- $-1/\tau$ is the initial slope of an exponential with an initial value of 1.

The Particular Solution

- The particular solution $x_p(t)$ is usually a weighted sum of *f(t)* and its first derivative.
- If *f(t)* is constant, then $x_p(t)$ is constant.
- If *f(t)* is sinusoidal, then $x_p(t)$ is sinusoidal.

The Particular Solution: F(t) Constant

$$x_P(t)+\tau\frac{dx_P(t)}{dt}=F$$

Guess a solution

$$x_P(t)=A+Bt$$

$$(A+Bt)+\tau\frac{d(A+Bt)}{dt}=F$$

Equation holds for all time and time variations are independent and thus each time variation coefficient is individually zero

$$(A+Bt)+\tau B=F$$

$$(A+\tau B-F)+(B)t=0$$

$$(B)=0 \qquad (A+\tau B-F)=0$$

$$B=0 \qquad A=F$$

The Particular Solution: F(t) Sinusoid

$$x_P(t)+\tau\frac{dx_P(t)}{dt}=F_A\sin(wt)+F_B\cos(wt)$$

Guess a solution $x_P(t)=A\sin(wt)+B\cos(wt)$

$$(A\sin(wt)+B\cos(wt))+\tau\frac{d(A\sin(wt)+B\cos(wt))}{dt}=F_A\sin(wt)+F_B\cos(wt)$$

$$(A-\tau\omega B-F_A)\sin(\omega t)+(B+\tau\omega A-F_B)\cos(\omega t)=0$$

$$(A-\tau\omega B-F_A)=0 \qquad (B+\tau\omega A-F_B)=0$$

Equation holds for all time and time variations are independent and thus each time variation coefficient is individually zero

$$A=\frac{F_A+\tau\omega F_B}{(\tau\omega)^2+1} \qquad B=-\frac{\tau\omega F_A-F_B}{(\tau\omega)^2+1}$$

$$x_P(t)=\frac{1}{\sqrt{(\tau\omega)^2+1}}\left[\frac{\tau\omega}{\sqrt{(\tau\omega)^2+1}}\sin(\omega t)+\frac{1}{\sqrt{(\tau\omega)^2+1}}\cos(\omega t)\right]$$

$$=\frac{1}{\sqrt{(\tau\omega)^2+1}}\cos(\omega t-\theta);\quad where\ \theta=\tan^{-1}(\tau\omega)$$

The Particular Solution: F(t) Exp.

$$x_P(t)+\tau\frac{dx_P(t)}{dt}=F_1e^{-\alpha t}+F_2$$

Guess a solution

$$x_P(t)=A+Be^{-\alpha t}$$

$$(A+Be^{-\alpha t})+\tau\frac{d(A+Be^{-\alpha t})}{dt}=F_1e^{-\alpha t}+F_2$$

Equation holds for all time and time variations are independent and thus each time variation coefficient is individually zero

$$(A+Be^{-\alpha t})-\alpha\tau Be^{-\alpha t}=F_1e^{-\alpha t}+F_2$$

$$(A-F_2)+(B-\alpha\tau-F_1)e^{-\alpha t}=0$$

$$(B-\alpha\tau-F_1)=0 \qquad (A-F_2)=0$$

$$B=\alpha\tau+F1 \qquad A=F_2$$

The Total Solution: F(t) Sinusoid

$$x_P(t)+\tau\frac{dx_P(t)}{dt}=F_A\sin(wt)+F_B\cos(wt)$$

$$x_P(t)=A\sin(wt)+B\cos(wt) \qquad A=\frac{F_A+\tau\omega F_B}{(\tau\omega)^2+1} \qquad B=-\frac{\tau\omega F_A-F_B}{(\tau\omega)^2+1}$$

$$x_C(t)=Ke^{-t/\tau}$$

$$x_T(t)=A\sin(wt)+B\cos(wt)+Ke^{-t/\tau}$$

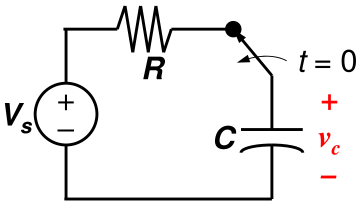
Only K is unknown and is determined by the initial condition at t =0

Example: $x_T(t=0) = V_C(t=0)$

$$x_T(0)=A\sin(0)+B\cos(0)+Ke^{-0/\tau}=V_C(t=0)$$

$$x_T(0)=B+K=V_C(t=0) \qquad K=V_C(t=0)-B$$

Example

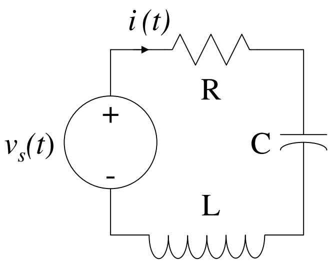


- Given $v_c(0^-)=1$, $V_s=2\cos(\omega t)$, $\omega=200$.
- *Find i(t)*, $v_c(t)=?$

2nd Order Circuits

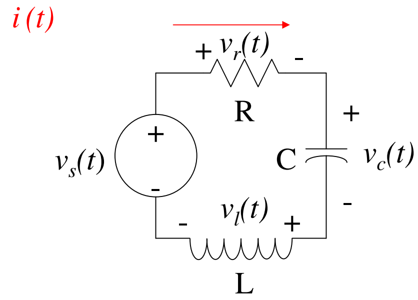
- Any circuit with a single capacitor, a single inductor, an arbitrary number of sources, and an arbitrary number of resistors is a circuit of order 2.
- Any voltage or current in such a circuit is the solution to a 2nd order differential equation.

A 2nd Order RLC Circuit



- Application: Filters
  - A bandpass filter such as the IF amp for the AM radio.
  - A lowpass filter with a sharper cutoff than can be obtained with an RC circuit.

The Differential Equation



KVL around the loop:

$$v_r(t) + v_c(t) + v_l(t) = v_s(t)$$

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(x)dx + L\frac{di(t)}{dt} = v_s(t)$$

$$\frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) + \frac{d^2 i(t)}{dt^2} = \frac{1}{L}\frac{dv_s(t)}{dt}$$

The Differential Equation

The voltage and current in a second order circuit is the solution to a differential equation of the following form:

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

$$x(t) = x_p(t) + x_c(t)$$

$X_p(t)$ is the particular solution (forced response) and $X_c(t)$ is the complementary solution (natural response).

The Particular Solution

- The particular solution $x_p(t)$ is usually a weighted sum of *f(t)* and its first and second derivatives.
- If *f(t)* is constant, then $x_p(t)$ is constant.
- If *f(t)* is sinusoidal, then $x_p(t)$ is sinusoidal.

The Complementary Solution

The complementary solution has the following form:

$$x_c(t) = Ke^{st}$$

K is a constant determined by initial conditions.
s is a constant determined by the coefficients of the differential equation.

$$\frac{d^2Ke^{st}}{dt^2} + 2\alpha\frac{dKe^{st}}{dt} + \omega_0^2 Ke^{st} = 0$$

$$s^2Ke^{st} + 2\alpha sKe^{st} + \omega_0^2 Ke^{st} = 0$$

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Characteristic Equation

- To find the complementary solution, we need to solve the characteristic equation:

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$\alpha = \zeta\omega_0$$

- The characteristic equation has two roots-call them s_1 and s_2.

$$x_c(t) = K_1e^{s_1t} + K_2e^{s_2t}$$

$$s_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$$

Damping Ratio and Natural Frequency

$$\zeta = \frac{\alpha}{\omega_0} \qquad s_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$$

damping ratio

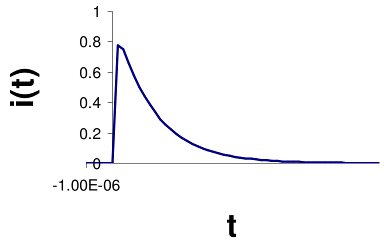
$$s_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$$

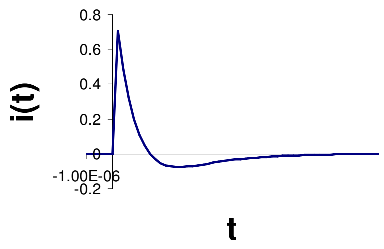
- The damping ratio determines what type of solution we will get:
  - Exponentially decreasing ($\zeta > 1$)
  - Exponentially decreasing sinusoid ($\zeta < 1$)
- The natural frequency is ω_0
  - It determines how fast sinusoids wiggle.

Overdamped : Real Unequal Roots

- If $\zeta > 1$, s_1 and s_2 are real and not equal.

$$i_c(t) = K_1 e^{\left(-\varsigma\omega_0 + \omega_0\sqrt{\varsigma^2 - 1}\right)t} + K_2 e^{\left(-\varsigma\omega_0 - \omega_0\sqrt{\varsigma^2 - 1}\right)t}$$



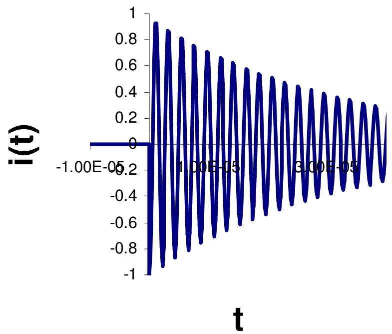


Underdamped: Complex Roots

- If $\zeta < 1$, s_1 and s_2 are complex.
- Define the following constants:

$$\alpha = \zeta\omega_0 \qquad \omega_d = \omega_0\sqrt{1 - \zeta^2}$$

$$x_c(t) = e^{-\alpha t}\left(A_1 \cos\omega_d t + A_2 \sin\omega_d t\right)$$



Critically damped: Real Equal Roots

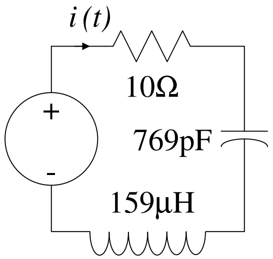
- If $\zeta = 1$, s_1 and s_2 are real and equal.

$$x_c(t) = K_1 e^{-\varsigma\omega_0 t} + K_2 t e^{-\varsigma\omega_0 t}$$

Note: The degeneracy of the roots results in the extra factor of 't'

Example

For the example, what are ζ and ω_0?



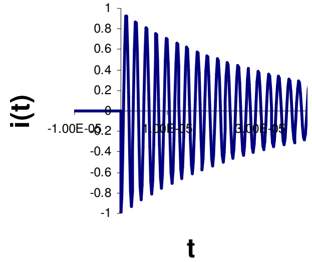
$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC} i(t) = \frac{1}{L}\frac{dv_s(t)}{dt}$$

$$\frac{d^2 x_c(t)}{dt^2} + 2\zeta\omega_0 \frac{dx_c(t)}{dt} + \omega_0^2 x_c(t) = 0$$

$$\omega_0^2 = \frac{1}{LC} \;,\; 2\zeta\omega_0 = \frac{R}{L} \;,\; \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

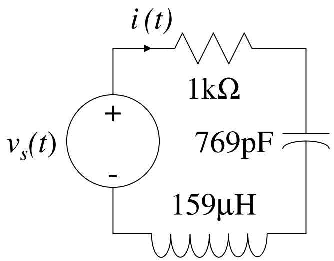
Example

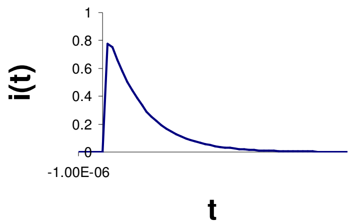
- $\zeta = 0.011$
- $\omega_0 = 2\pi 455000$
- Is this system over damped, under damped, or critically damped?
- What will the current look like?



Slightly Different Example

- Increase the resistor to 1kΩ
- What are ζ and ω_0?

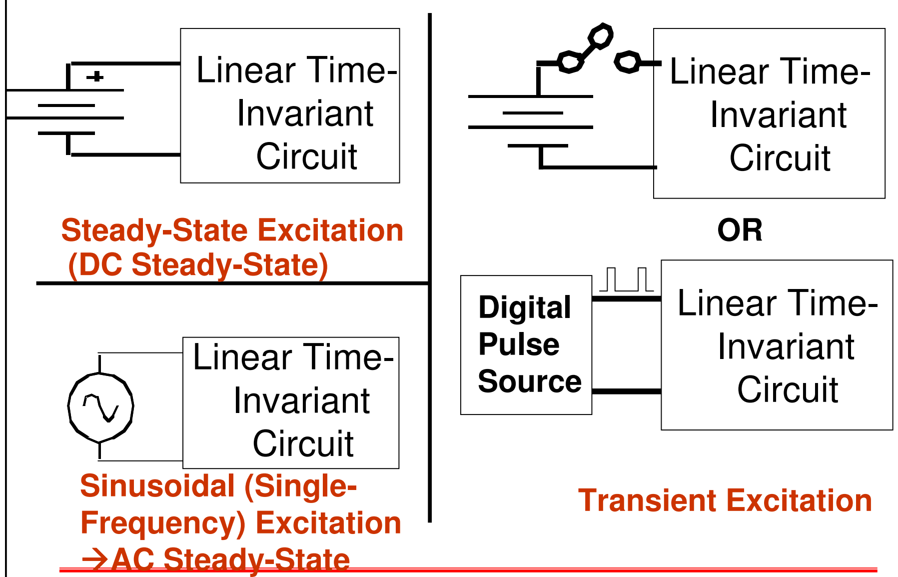




$\zeta = 2.2$

$\omega_0 = 2\pi 455000$

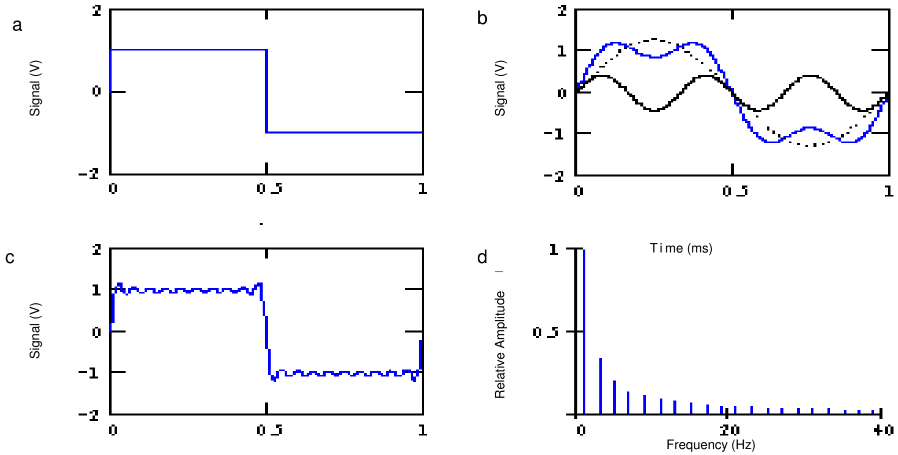
Types of Circuit Excitation



Why is Single-Frequency Excitation Important?

- Some circuits are driven by a single-frequency sinusoidal source.
- Some circuits are driven by sinusoidal sources whose frequency changes slowly over time.
- You can express any periodic electrical signal as a sum of single-frequency sinusoids – so you can analyze the response of the (linear, time-invariant) circuit to each individual frequency component and then sum the responses to get the total response.
- This is known as Fourier Transform and is tremendously important to all kinds of engineering disciplines!

Representing a Square Wave as a Sum of Sinusoids



(a)Square wave with 1-second period. (b) Fundamental component (dotted) with 1-second period, third-harmonic (solid black) with1/3-second period, and their sum (blue). (c) Sum of first ten components. (d) Spectrum with 20 terms.

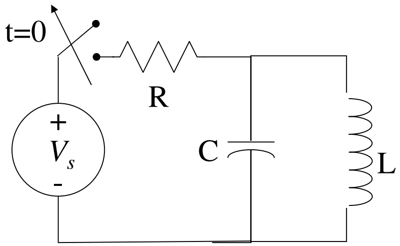
Steady-State Sinusoidal Analysis

- Also known as AC steady-state
- Any steady state voltage or current in a linear circuit with a sinusoidal source is a sinusoid.
  - This is a consequence of the nature of particular solutions for sinusoidal forcing functions.
- All AC steady state voltages and currents have the same frequency as the source.
- In order to find a steady state voltage or current, all we need to know is its magnitude and its phase relative to the source
  - We already know its frequency.
- Usually, an AC steady state voltage or current is given by the particular solution to a differential equation.

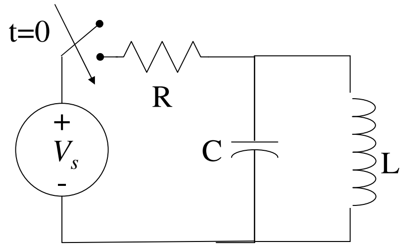
Chapter 5

- OUTLINE
  - Phasors as notation for Sinusoids
  - Arithmetic with Complex Numbers
  - Complex impedances
  - Circuit analysis using complex impdenaces
  - Dervative/Integration as multiplication/division
  - Phasor Relationship for Circuit Elements
- Reading
  - Chap 5
  - Appendix A

Example 1: 2nd Order RLC Circuit



Example 2: 2nd Order RLC Circuit



Sinusoidal Sources Create Too Much Algebra

$$x_P(t) + \tau \frac{dx_P(t)}{dt} = F_A \sin(wt) + F_B \cos(wt)$$

Guess a solution

Two terms to be general

$$x_P(t) = A\sin(wt) + B\cos(wt)$$

Dervatives

Addition

$$(A\sin(wt) + B\cos(wt)) + \tau \frac{d(A\sin(wt) + B\cos(wt))}{dt} = F_A \sin(wt) + F_B \cos(wt)$$

$$(A - \tau B - F_A)\sin(wt) + (B + \tau A - F_B)\cos(wt) = 0$$

Equation holds for all time and time variations are independent and thus each time variation coefficient is individually zero

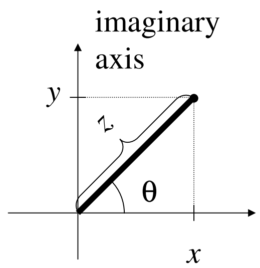
$$(A - \tau B - F_A) = 0$$

$$(B + \tau A - F_B) = 0$$

$$A = \frac{F_A + \tau F_B}{\tau^2 + 1} \qquad B = -\frac{\tau F_A - F_B}{\tau^2 + 1}$$

Phasors (vectors that rotate in the complex plane) are a clever alternative.

Complex Numbers (1)



$j = \sqrt{(-1)}$

- x is the real part
- y is the imaginary part
- z is the magnitude
- θ is the phase

$x = z\cos\theta \qquad y = z\sin\theta$

$z = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\frac{y}{x}$

- Rectangular Coordinates

  Z = $x + jy$

- Polar Coordinates:

  Z = $z \angle \theta$

- Exponential Form:

$$\mathbf{Z} = |\mathbf{Z}| e^{j\theta} = z e^{j\theta}$$

$\mathbf{Z} = z(\cos\theta + j\sin\theta)$

$1 = 1e^{j0} = 1\angle 0°$

$j = 1e^{j\frac{\pi}{2}} = 1\angle 90°$

Complex Numbers (2)

Euler's Identities

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\left|e^{j\theta}\right| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

Exponential Form of a complex number

$$\mathbf{Z} = |\mathbf{Z}|e^{j\theta} = ze^{j\theta} = z\angle\theta$$

Arithmetic With Complex Numbers

- To compute phasor voltages and currents, we need to be able to perform computation with complex numbers.
  - Addition
  - Subtraction
  - Multiplication
  - Division
- (And later use multiplication by jω to replace
  - Diffrentiation
  - Integration

Addition

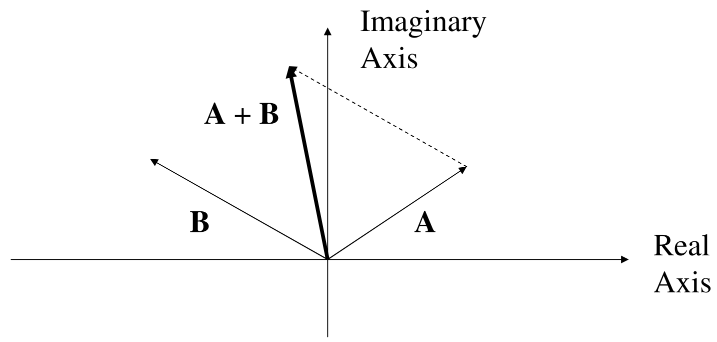
- Addition is most easily performed in rectangular coordinates:

$$\mathbf{A} = x + jy$$

$$\mathbf{B} = z + jw$$

$$\mathbf{A} + \mathbf{B} = (x + z) + j(y + w)$$

Addition



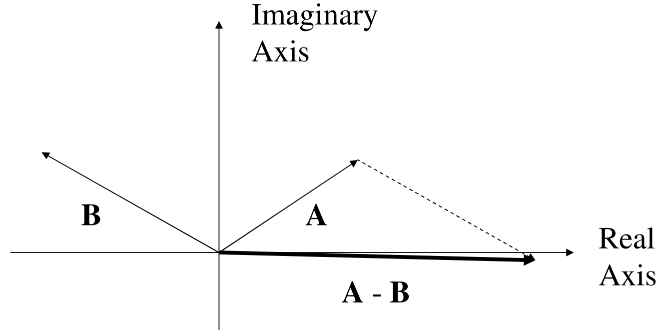
Subtraction

- Subtraction is most easily performed in rectangular coordinates:

$$\mathbf{A} = x + jy$$
$$\mathbf{B} = z + jw$$

$$\mathbf{A} - \mathbf{B} = (x - z) + j(y - w)$$

Subtraction



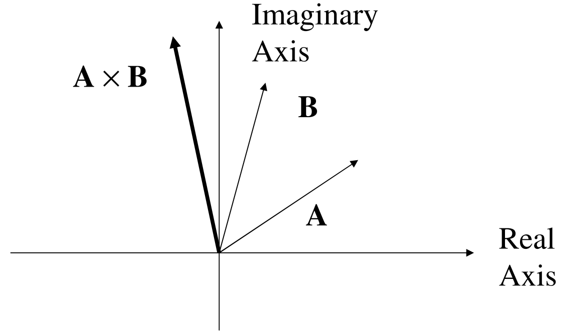
Multiplication

- Multiplication is most easily performed in polar coordinates:

$$\mathbf{A} = A_M \angle \theta$$
$$\mathbf{B} = B_M \angle \phi$$

$$\mathbf{A} \times \mathbf{B} = (A_M \times B_M) \angle (\theta + \phi)$$

Multiplication



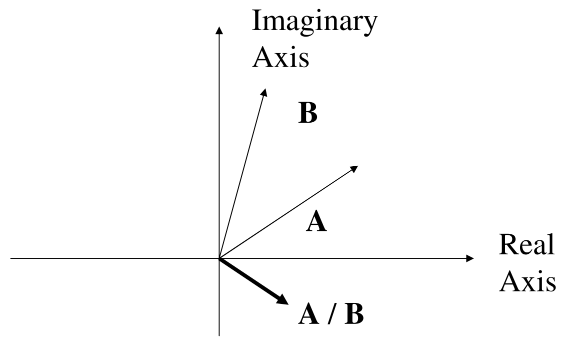
Division

- Division is most easily performed in polar coordinates:

$$\mathbf{A} = A_M \angle \theta$$
$$\mathbf{B} = B_M \angle \phi$$

$$\mathbf{A} / \mathbf{B} = (A_M / B_M) \angle (\theta - \phi)$$

Division



Arithmetic Operations of Complex Numbers

- Add and Subtract: it is easiest to do this in rectangular format
  - Add/subtract the real and imaginary parts separately
- Multiply and Divide: it is easiest to do this in exponential/polar format
  - Multiply (divide) the magnitudes
  - Add (subtract) the phases

$$\mathbf{Z}_1 = z_1 e^{j\theta_1} = z_1 \angle \theta_1 = z_1 \cos\theta_1 + jz_1 \sin\theta_1$$
$$\mathbf{Z}_2 = z_2 e^{j\theta_2} = z_2 \angle \theta_2 = z_2 \cos\theta_2 + jz_2 \sin\theta_2$$
$$\mathbf{Z}_1 + \mathbf{Z}_2 = (z_1 \cos\theta_1 + z_2 \cos\theta_2) + j(z_1 \sin\theta_1 + z_2 \sin\theta_2)$$
$$\mathbf{Z}_1 - \mathbf{Z}_2 = (z_1 \cos\theta_1 - z_2 \cos\theta_2) + j(z_1 \sin\theta_1 - z_2 \sin\theta_2)$$
$$\mathbf{Z}_1 \times \mathbf{Z}_2 = (z_1 \times z_2) e^{j(\theta_1 + \theta_2)} = (z_1 \times z_2) \angle (\theta_1 + \theta_2)$$
$$\mathbf{Z}_1 / \mathbf{Z}_2 = (z_1 / z_2) e^{j(\theta_1 - \theta_2)} = (z_1 / z_2) \angle (\theta_1 - \theta_2)$$

Phasors

- Assuming a source voltage is a sinusoid time-varying function

$$v(t) = V \cos(\omega t + \theta)$$

- We can write:

$$v(t) = V\cos(\omega t + \theta) = V\,\mathrm{Re}\left[e^{j(\omega t + \theta)}\right] = \mathrm{Re}\left[Ve^{j(\omega t + \theta)}\right]$$

$$Define\ \ Phasor\ \ as\ \ Ve^{j\theta} = V\angle\theta$$

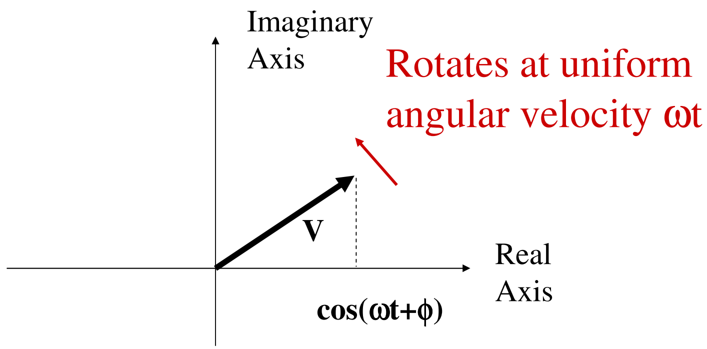
- Similarly, if the function is $v(t) = V \sin(\omega t + \theta)$

$$v(t) = V\sin(\omega t + \theta) = V\cos(\omega t + \theta - \frac{\pi}{2}) = \mathrm{Re}\left[Ve^{j(\omega t + \theta - \frac{\pi}{2})}\right]$$

$$Phasor = V\angle\left(\theta - \frac{\pi}{2}\right)$$

Phasor: Rotating Complex Vector

$$v(t) = V\cos(\omega t + \phi) = \mathrm{Re}\{Ve^{j\phi}e^{jwt}\} = \mathrm{Re}(\mathbf{V}e^{j\omega t})$$

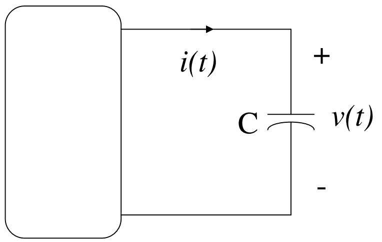


The head start angle is ϕ.

Complex Exponentials

- We represent a real-valued sinusoid as the real part of a complex exponential after multiplying by $e^{j\omega t}$.
- Complex exponentials
  - provide the link between time functions and phasors.
  - Allow derivatives and integrals to be replaced by multiplying or dividing by $j\omega$
  - make solving for AC steady state simple algebra with complex numbers.
- Phasors allow us to express current-voltage relationships for inductors and capacitors much like we express the current-voltage relationship for a resistor.

I-V Relationship for a Capacitor



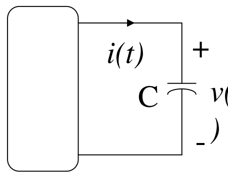
$$i(t) = C\frac{dv(t)}{dt}$$

Suppose that *v(t)* is a sinusoid:

$$v(t) = \mathrm{Re}\{V_M e^{j(\omega t+\theta)}\}$$

Find *i(t)*.

Capacitor Impedance (1)



$$i(t) = C\frac{dv(t)}{dt}$$

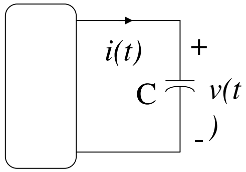
$$v(t) = V\boxed{\cos(\omega t+\theta)} = \frac{V}{2}\left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right]$$

$$i(t) = C\frac{dv(t)}{dt} = \frac{CV}{2}\frac{d}{dt}\left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right] = \frac{CV}{2}j\omega\left[e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}\right]$$

$$= \frac{-\omega CV}{2j}\left[e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}\right] = -\omega CV\sin(\omega t+\theta) = \omega CV\boxed{\cos(\omega t+\theta+\frac{\pi}{2})}$$

$$Z_c = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V\angle\theta}{I\angle\left(\theta+\frac{\pi}{2}\right)} = \frac{V}{\omega CV}\angle(\theta-\theta-\frac{\pi}{2}) = \frac{1}{\omega C}\angle(-\frac{\pi}{2}) = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$$

Capacitor Impedance (2)



$$i(t) = C\frac{dv(t)}{dt}$$

Phasor definition

$$v(t) = V\cos(\omega t + \theta) = \mathrm{Re}\left[Ve^{j(\omega t+\theta)}\right] \Rightarrow \mathbf{V} = V\angle\theta$$

$$i(t) = C\frac{dv(t)}{dt} = \mathrm{Re}\left[CV\frac{de^{j(\omega t+\theta)}}{dt}\right] = \mathrm{Re}\left[j\omega CVe^{j(\omega t+\theta)}\right] \Rightarrow \mathbf{I} = I\angle\theta$$

$$Z_c = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V\angle\theta}{I\angle\theta} = \frac{V}{j\omega CV}\angle(\theta-\theta) = \frac{1}{j\omega C}$$

Example

$v(t)$ = 120V cos(377t + 30°)

C = 2μF

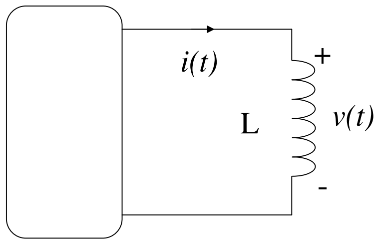
- What is **V**?
- What is **I**?
- What is $i(t)$?

Computing the Current

Note: The differentiation and integration operations become algebraic operations

$$\frac{d}{dt} \Rightarrow j\omega \qquad \int dt \Rightarrow \frac{1}{j\omega}$$

Inductor Impedance



$$v(t) = L\frac{di(t)}{dt}$$

V = $j\omega L$ **I**

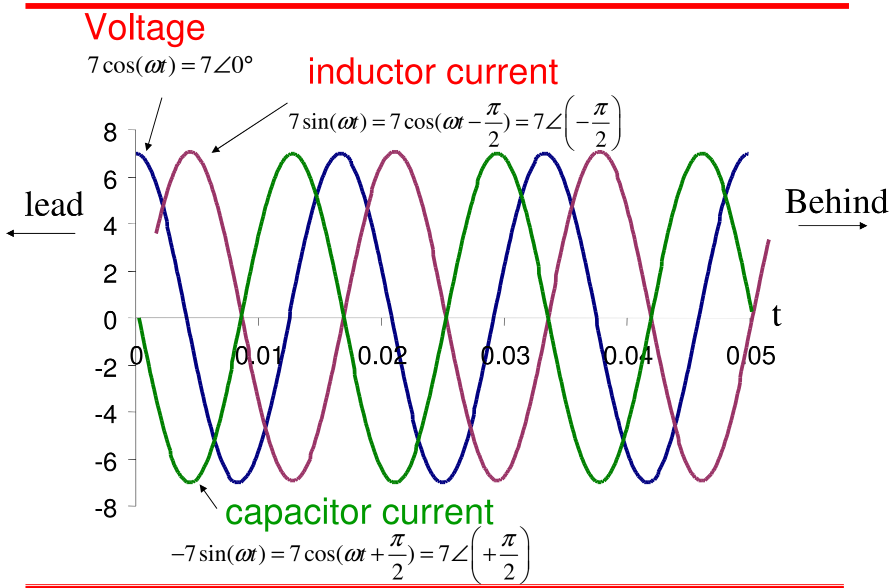
Example

$i(t) = 1\mu A \cos(2\pi\, 9.15\, 10^7 t + 30°)$

$L = 1\mu H$

- What is **I**?
- What is **V**?
- What is *v(t)*?

Phase



Phasor Diagrams

- A phasor diagram is just a graph of several phasors on the complex plane (using real and imaginary axes).
- A phasor diagram helps to visualize the relationships between currents and voltages.
- Capacitor: I leads V by 90°
- Inductor: V leads I by 90°

Impedance

- AC steady-state analysis using phasors allows us to express the relationship between current and voltage using a formula that looks likes Ohm's law:

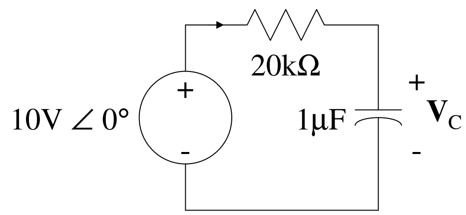
$$\mathbf{V} = \mathbf{I}\,\mathbf{Z}$$

- **Z** is called impedance.

Some Thoughts on Impedance

- Impedance depends on the frequency ω.
- Impedance is (often) a complex number.
- Impedance allows us to use the same solution techniques for AC steady state as we use for DC steady state.

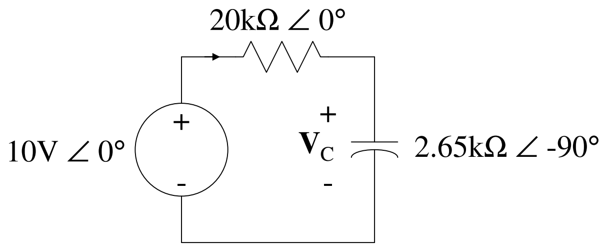
Example: Single Loop Circuit



f=60 Hz, $\mathbf{V}_C$=?

How do we find $\mathbf{V}_C$?
First compute impedances for resistor and capacitor:
$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°
$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

Impedance Example

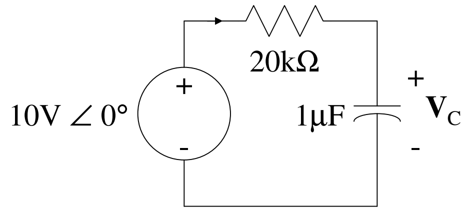


Now use the voltage divider to find $\mathbf{V}_C$:

$$\mathbf{V}_C = 10\text{V} \angle 0° \left(\frac{2.65\text{k}\Omega \angle -90°}{2.65\text{k}\Omega \angle -90° + 20\text{k}\Omega \angle 0°} \right)$$

$$\mathbf{V}_C = 1.31\text{V} \angle -82.4°$$

What happens when ω changes?



ω = 10

Find $\mathbf{V}_C$

Circuit Analysis Using Complex Impedances

- Suitable for AC steady state.
- KVL

$$v_1(t) + v_2(t) + v_3(t) = 0$$

$$V_1 \cos(\omega t + \theta_1) + V_2 \cos(\omega t + \theta_2) + V_3 \cos(\omega t + \theta_3) = 0$$

$$\mathrm{Re}\left[V_1 e^{j(\omega t + \theta_1)} + V_2 e^{j(\omega t + \theta_2)} + V_3 e^{j(\omega t + \theta_3)}\right] = 0$$

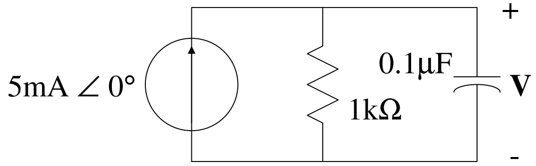
Phasor Form KVL

$$V_1 e^{j(\theta_1)} + V_2 e^{j(\theta_2)} + V_3 e^{j(\theta_3)} = 0$$

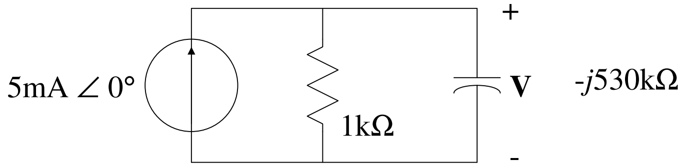
$$\mathbf{V_1} + \mathbf{V_2} + \mathbf{V_3} = 0$$

- Phasor Form KCL $\quad \mathbf{I_1} + \mathbf{I_2} + \mathbf{I_3} = 0$
- Use complex impedances for inductors and capacitors and follow same analysis as in chap 2.

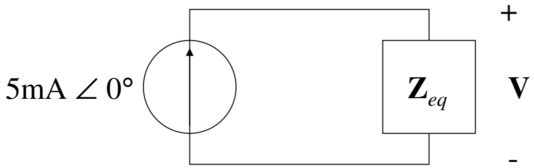
Steady-State AC Analysis



Find $v(t)$ for $\omega = 2\pi$ 3000



Find the Equivalent Impedance



$$\mathbf{Z}_{eq} = \frac{1000(-j530)}{1000 - j530} = \frac{10^3\angle 0° \times 530\angle -90°}{1132\angle -27.9°}$$

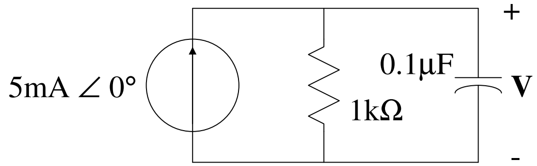
$$\mathbf{Z}_{eq} = 468.2\Omega\angle -62.1°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = 5\text{mA}\angle 0° \times 468.2\Omega\angle -62.1°$$

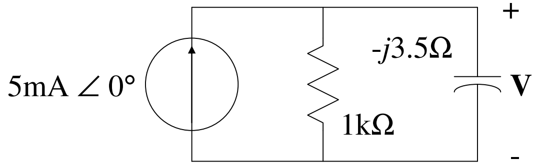
$$\mathbf{V} = 2.34\text{V}\angle -62.1°$$

$$v(t) = 2.34\text{V}\cos(2\pi 3000\text{t} - 62.1°)$$

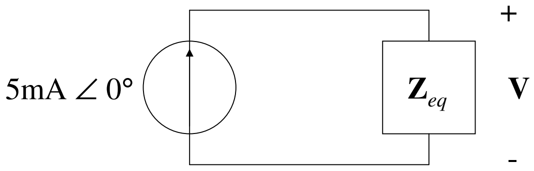
Change the Frequency



Find $v(t)$ for $\omega = 2\pi$ 455000



Find an Equivalent Impedance



$$\mathbf{Z}_{eq} = \frac{1000(-j3.5)}{1000 - j3.5} = \frac{10^3\angle 0° \times 3.5\angle -90°}{1000\angle -0.2°}$$

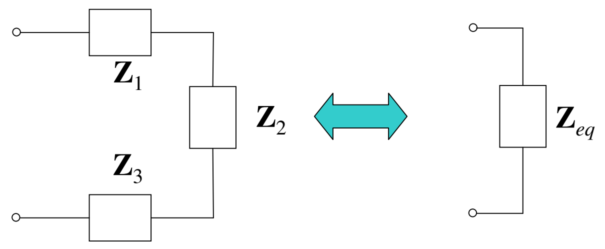
$$\mathbf{Z}_{eq} = 3.5\Omega\angle -89.8°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = 5\text{mA}\angle 0° \times 3.5\Omega\angle -89.8°$$

$$\mathbf{V} = 17.5\text{mV}\angle -89.8°$$

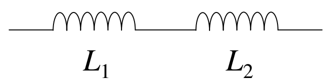
$$v(t) = 17.5\text{mV}\cos(2\pi 455000\text{t} - 89.8°)$$

Series Impedance

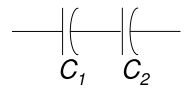


$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$

For example:

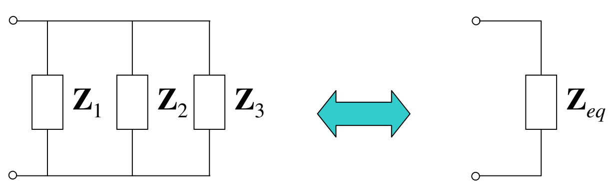


$$\mathbf{Z}_{eq} = j\omega(L_1 + L_2)$$



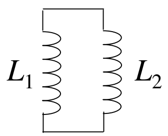
$$\mathbf{Z}_{eq} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}$$

Parallel Impedance

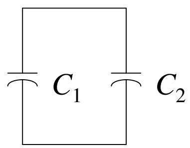


$$1/\mathbf{Z}_{eq} = 1/\mathbf{Z}_1 + 1/\mathbf{Z}_2 + 1/\mathbf{Z}_3$$

For example:

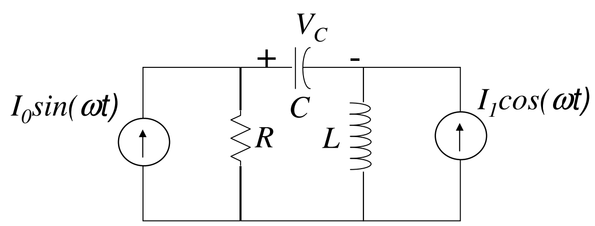


$$\mathbf{Z}_{eq} = j\omega\frac{L_1L_2}{(L_1 + L_2)}$$



$$\mathbf{Z}_{eq} = \frac{1}{j\omega(C_1 + C_2)}$$

Steady-State AC Node-Voltage Analysis



- Try using Thevinin equivalent circuit.
- What happens if the sources are at different frequencies?

Resistor I-V relationship

$v_R = i_R R$ ....$\mathbf{V}_R = \mathbf{I}_R R$ where R is the resistance in ohms,
$\mathbf{V}_R$ = phasor voltage, $\mathbf{I}_R$ = phasor current
(boldface indicates complex quantity)

Capacitor I-V relationship

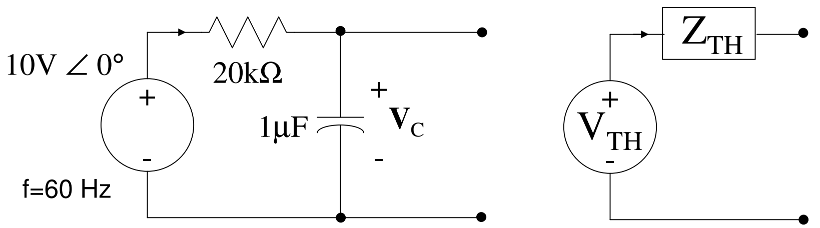
$i_C = C dv_C/dt$ ....Phasor current $\mathbf{I}_C$ = phasor voltage $\mathbf{V}_C$ / capacitive impedance $\mathbf{Z}_C$: → $\mathbf{I}_C = \mathbf{V}_C/\mathbf{Z}_C$
where $\mathbf{Z}_C = 1/j\omega C$, $j = (-1)^{1/2}$ and boldface indicates complex quantity

Inductor I-V relationship

$v_L = L di_L/dt$ ....Phasor voltage $\mathbf{V}_L$ = phasor current $\mathbf{I}_L$/ inductive impedance $\mathbf{Z}_L$ → $\mathbf{V}_L = \mathbf{I}_L \mathbf{Z}_L$
where $\mathbf{Z}_L = j\omega L$, $j = (-1)^{1/2}$ and boldface indicates complex quantity

R	C	L
$v_0(t) = V_0 \cos(\omega t)$	$v_0(t) = V_0 \cos(\omega t)$	$v_0(t) = V_0 \cos(\omega t)$
$\vec{V}_0 = V_0 \angle 0^\circ$	$\vec{V}_0 = V_0 \angle 0^\circ$	$\vec{V}_0 = V_0 \angle 0^\circ$
$i_0(t) = \frac{V_0}{R}\cos(\omega t)$	$i_0(t) = -\omega C V_0 \sin(\omega t)$	$i_0(t) = \frac{V_0}{\omega L}\sin(\omega t)$
$\vec{I}_0 = \frac{V_0}{R}\angle 0^\circ$	$\vec{I}_0 = \omega C V_0 \angle 90^\circ$	$\vec{I}_0 = \frac{V_0}{\omega L}\angle -90^\circ$

Thevenin Equivalent



$\mathbf{Z}_R$ = R= 20kΩ = 20kΩ ∠ 0°
$\mathbf{Z}_C$ = 1/*j* (2πf x 1μF) = 2.65kΩ ∠ -90°

$$\mathbf{V}_{TH} = \mathbf{V}_{OC} = 10\text{V} \angle 0^\circ \left(\frac{2.65\text{k}\Omega \angle -90^\circ}{2.65\text{k}\Omega \angle -90^\circ + 20\text{k}\Omega \angle 0^\circ} \right) = 1.31 \angle -82.4$$

$$\mathbf{Z}_{TH} = \mathbf{Z}_R \parallel \mathbf{Z}_C = {}^\circ \left(\frac{20\text{k}\Omega \angle 0^\circ \cdot 2.65\text{k}\Omega \angle -90^\circ}{2.65\text{k}\Omega \angle -90^\circ + 20\text{k}\Omega \angle 0^\circ} \right) = 2.62 \angle -82.4$$

Root Mean Square (rms) Values

- rms valued defined as

$$v_{RMS} = \sqrt{\frac{1}{T}\int_o^T v^2(t)dt} \qquad T = \text{period}$$

- Assuming a sinusoid gives

$$v_{RMS} = \sqrt{\frac{1}{T}\int_o^T v_m{}^2 \cos^2(\omega t + \theta)dt}$$

- Using an identity gives

$$v_{RMS} = \sqrt{\frac{v_m{}^2}{2T}\int_o^T [1 + \cos(2\omega t + 2\theta)]dt}$$

- Evaluating at limits gives

$$v_{RMS} = \sqrt{\frac{v_m{}^2}{2T}[T + \frac{1}{2\omega}\sin(2\omega T + 2\theta) - \frac{1}{2w}\sin(2\theta)]} \qquad v_{RMS} = \frac{v_m}{\sqrt{2}}$$

Power: Instantaneous and Time-Average

For a Resistor

- The instantaneous power is $$p(t) = v(t)i(t) = \frac{v(t)^2}{R}$$
- The time-average power is $$P_{AVE} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T \frac{v(t)^2}{R}dt = \frac{1}{R}[\frac{1}{T}\int_0^T v(t)^2 dt] = \frac{v_{rms}^2}{R}$$

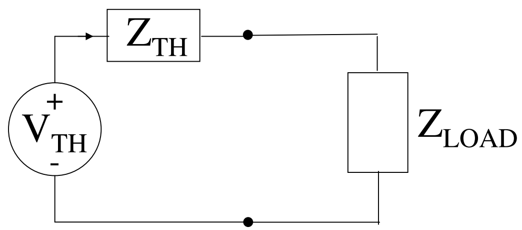
$v(t)i(t$

For an Impedance

- The instantaneous power is $$p(t) = v(t)i(t)$$
- The time-average power is $$P_{AVE} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v(t)i(t)dt = \mathrm{Re}\{\mathbf{V}_{rms}\cdot\mathbf{I}_{rms}{}^*\}$$
- The reactive power at 2ω is

$$Q = \mathrm{Im}\{\mathbf{V}_{rms}\cdot\mathbf{I}_{rms}^*\} \qquad P_{AVE}^2 + Q^2 = (V_{rms}\cdot I_{rms})^2$$

Maximum Average Power Transfer



- Maximum time average power occurs when $$\mathbf{Z}_{LOAD} = \mathbf{Z}_{TH}^*$$
- This presents a resistive impedance to the source $$\mathbf{Z}_{total} = \mathbf{Z}_{TH} + \mathbf{Z}_{TH}^*$$
- Power transferred is

$$P_{AVE} = \mathrm{Re}\{\mathbf{VI}^*\} = \mathrm{Re}\{\mathbf{V}\frac{\mathbf{V}^*}{2R}\} = \tfrac{1}{2}\frac{V_{rms}^2}{R}$$

Chapter 6

- OUTLINE
  - Frequency Response for Characterization
  - Asymptotic Frequency Behavior
  - Log magnitude vs log frequency plot
  - Phase vs log frequency plot
  - dB scale
  - Transfer function example

Bel and Decibel (dB)

- A **bel** (symbol **B**) is a unit of measure of ratios of power levels, i.e. relative power levels.
  - The name was coined in the early 20th century in honor of Alexander Graham Bell, a telecommunications pioneer.
  - The bel is a logarithmic measure. The number of bels for a given ratio of power levels is calculated by taking the logarithm, to the base 10, of the ratio.
  - one bel corresponds to a ratio of 10:1.
  - B = $\log_{10}(P_1/P_2)$ where P_1 and P_2 are power levels.
- The bel is too large for everyday use, so the **decibel (dB)**, equal to 0.1B, is more commonly used.
  - 1dB = $10 \log_{10}(P_1/P_2)$
- dB are used to measure
  - Electric power, Gain or loss of amplifiers, Insertion loss of filters.

Logarithmic Measure for Power

- To express a power in terms of decibels, one starts by choosing a reference power, $P_{reference}$, and writing

  Power P in decibels = $10 \log_{10}(P/P_{reference})$
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 watt.
  - P (dB) = $10 \log_{10} (50 \times 10^{-3}) = -13$ dB
- Exercise:
  - Express a power of 50 mW in decibels relative to 1 mW.
  - P (dB) = $10 \log_{10} (50) = 17$ dB.
- dBm to express absolute values of power relative to a milliwatt.
  - dBm = $10 \log_{10}$ (power in milliwatts / 1 milliwatt)
  - 100 mW = 20 dBm
  - 10 mW = 10 dBm

Logarithmic Measures for Voltage or Current

From the expression for power ratios in decibels, we can readily derive the corresponding expressions for voltage or current ratios.

Suppose that the voltage V (or current I) appears across (or flows in) a resistor whose resistance is R. The corresponding power dissipated, P, is V^2/R (or I^2R). We can similarly relate the reference voltage or current to the reference power, as

$$P_{reference} = (V_{reference})^2/R \text{ or } P_{reference} = (I_{reference})^2 R.$$

Hence,

Voltage, V in decibels = $20\log_{10}(V/V_{reference})$
Current, I, in decibels = $20\log_{10}(I/I_{reference})$

Logarithmic Measures for Voltage or Current

Note that the voltage and current expressions are just like the power expression except that they have 20 as the multiplier instead of 10 because power is proportional to the square of the voltage or current.

Exercise: How many decibels larger is the voltage of a 9-volt transistor battery than that of a 1.5-volt AA battery? Let $V_{reference} = 1.5$. The ratio in decibels is

$$20 \log_{10}(9/1.5) = 20 \log_{10}(6) = 16 \text{ dB.}$$

Logarithmic Measures for Voltage or Current

The gain produced by an amplifier or the loss of a filter is often specified in decibels.

The input voltage (current, or power) is taken as the reference value of voltage (current, or power) in the decibel defining expression:

Voltage gain in dB = $20 \log_{10}(V_{output}/V_{input})$
Current gain in dB = $20\log_{10}(I_{output}/I_{input})$
Power gain in dB = $10\log_{10}(P_{output}/P_{input})$

Example: The voltage gain of an amplifier whose input is 0.2 mV and whose output is 0.5 V is

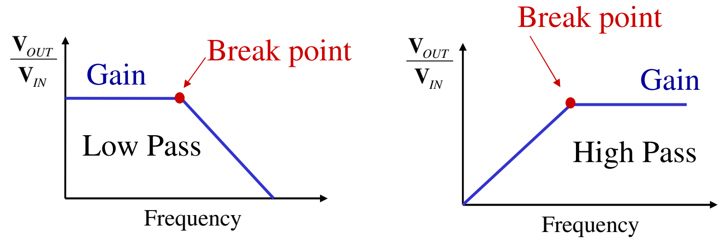
$$20\log_{10}(0.5/0.2\times10^{-3}) = 68 \text{ dB.}$$

Bode Plot

- Plot of magnitude of transfer function vs. frequency
  - Both x and y scale are in log scale
  - Y scale in dB
- Log Frequency Scale
  - Decade → Ratio of higher to lower frequency = 10
  - Octave → Ratio of higher to lower frequency = 2

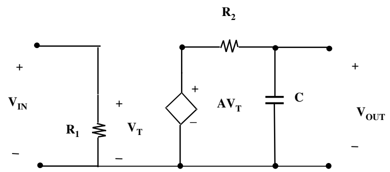
Frequency Response

- The shape of the frequency response of the complex ratio of phasors $\mathbf{V}_{OUT}/\mathbf{V}_{IN}$ is a convenient means of classifying a circuit behavior and identifying key parameters.



FYI: These are log ratio vs log frequency plots

Example Circuit



$$TransferFunction = \frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}}$$

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{AZ_c}{Z_R + Z_c}$$

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A(1/jwC)}{R_2 + 1/j\omega C)} = \frac{A}{(1 + j\omega R_2 C)}$$

A = 100

R_1 = 100,000 Ohms

R_2 = 1000 Ohms

C = 10 uF

Break Point Values

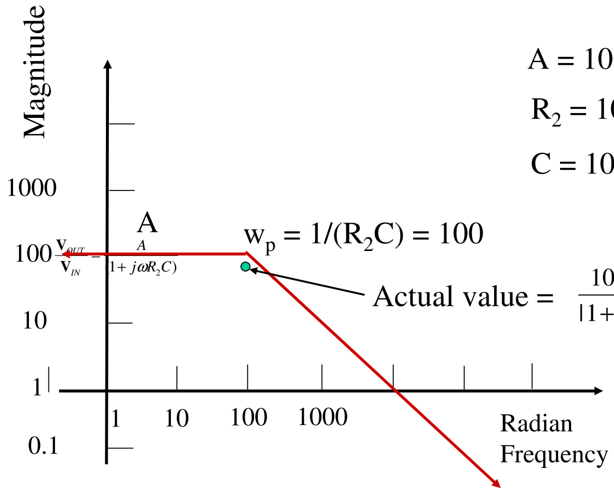
- **When dealing with resonant circuits it is convenient to refer to the frequency difference between points at which the power from the circuit is half that at the peak of resonance.**
- **Such frequencies are known as "half-power frequencies", and the power output there referred to the peak power (at the resonant frequency) is**
- **$10\log_{10}(P_{half\text{-}power}/P_{resonance}) = 10\log_{10}(1/2) = -3$ dB.**

Example: Circuit in Slide #3 Magnitude

A = 100

R_2 = 1000 Ohms

C = 10 uF



Actual value = $\frac{100}{|1+j|} = \frac{100}{\sqrt{2}}$

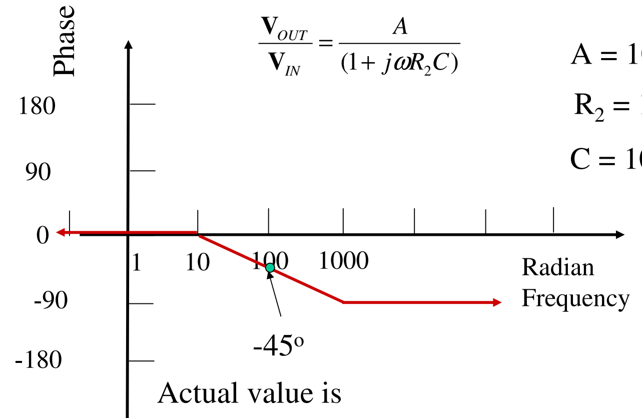
Example: Circuit in Slide #3 Phase

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A}{(1+j\omega R_2 C)}$$

A = 100

R_2 = 1000 Ohms

C = 10 uF



Actual value is

$$Phase\{\frac{100\angle 0}{|1+j|}\} = Phase\{\frac{100\angle 0}{\sqrt{2}\angle 45}\} = 0-45 = -45$$

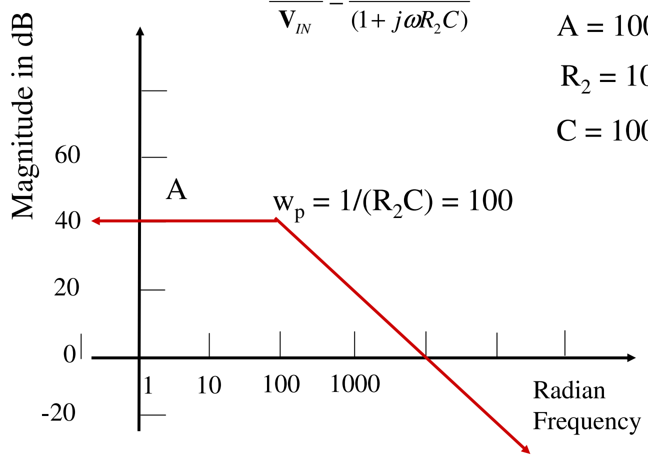
Bode Plot: Label as dB

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{A}{(1+j\omega R_2 C)}$$

A = 100

R_2 = 1000 Ohms

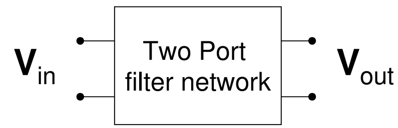
C = 100 uF



Note: Magnitude in dB = 20 $\log_{10}(V_{OUT}/V_{IN})$

Transfer Function

- Transfer function is a function of frequency
  - Complex quantity
  - Both magnitude and phase are function of frequency



$$\mathbf{H}(f) = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{V_{out}}{V_{in}} \angle(\theta_{out} - \theta_{in})$$

$$\mathbf{H(f)} = H(f)\angle\theta$$

Filters

- Circuit designed to retain a certain frequency range and discard others

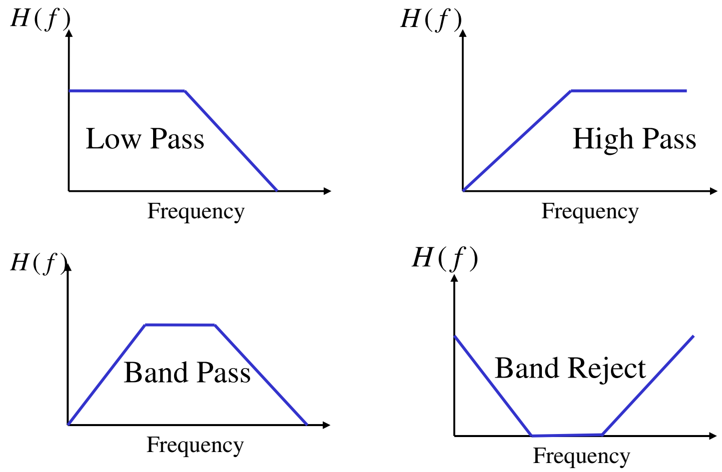
  Low-pass: pass low frequencies and reject high frequencies

  High-pass: pass high frequencies and reject low frequencies

  Band-pass: pass some particular range of frequencies, reject other frequencies outside that band

  Notch: reject a range of frequencies and pass all other frequencies

Common Filter Transfer Function vs. Freq



First-Order Lowpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V_C}}{\mathbf{V}} = \frac{1/(j\omega C)}{1/(j\omega C)+R} = \frac{1}{1+j\omega RC} = \frac{1}{\sqrt{1+(\omega RC)^2}}\angle -\tan^{-1}(\omega RC)$$

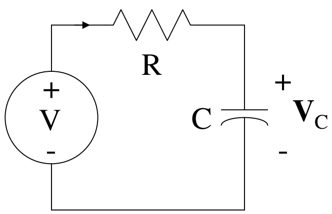
$$Let\ \omega_B = \frac{1}{RC} \quad and\ f_B = \frac{1}{2\pi RC}$$

$$\mathbf{H(f)} = H(f)\angle\theta$$

$$H(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}},\ \theta = -\tan^{-1}\left(\frac{f}{f_B}\right)$$

$$H(f_B) = \frac{1}{\sqrt{2}} = 2^{-1/2}$$

$$20\log_{10}\frac{H(f_B)}{H(0)} = 20(-\frac{1}{2})\log_{10}2 = -3\ dB$$



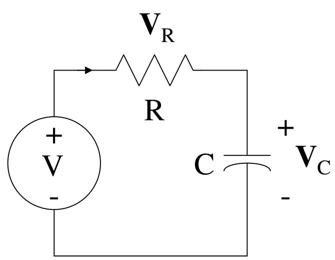
First-Order Highpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V_R}}{\mathbf{V}} = \frac{R}{1/(j\omega C)+R} = \frac{j\omega RC}{1+j\omega RC} = \frac{(\omega RC)}{\sqrt{1+(\omega RC)^2}}\angle\left[\frac{\pi}{2}-\tan^{-1}(\omega RC)\right]$$

$$H(f) = \frac{\left(\frac{f}{f_B}\right)}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}},\ \theta = \frac{\pi}{2}-\tan^{-1}\left(\frac{f}{f_B}\right)$$

$$H(f_B) = \frac{1}{\sqrt{2}} = 2^{-1/2}$$

$$20\log_{10}\frac{H(f_B)}{H(0)} = 20(-\frac{1}{2})\log_{10}2 = -3\ dB$$



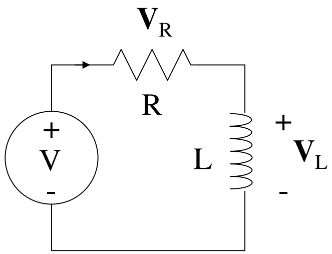
First-Order Lowpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V}_R}{\mathbf{V}} = \frac{1}{\dfrac{j\omega L}{R}+1} = \frac{1}{\sqrt{1+\left(\dfrac{\omega L}{R}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$Let\ \omega_B = \frac{R}{L} \quad and\ f_B = \frac{R}{2\pi L}$$

$$\mathbf{H(f)} = H(f)\angle\theta$$

$$H(f) = \frac{1}{\sqrt{1+\left(\dfrac{f}{f_B}\right)^2}}, \theta = -\tan^{-1}\left(\frac{f}{f_B}\right)$$



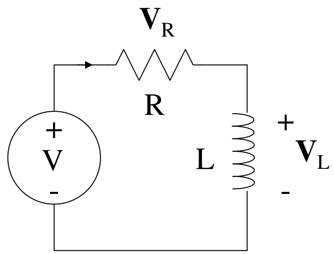
First-Order Highpass Filter

$$\mathbf{H(f)} = \frac{\mathbf{V}_L}{\mathbf{V}} = \frac{\dfrac{j\omega L}{R}}{\dfrac{j\omega L}{R}+1} = \frac{\dfrac{\omega L}{R}}{\sqrt{1+\left(\dfrac{\omega L}{R}\right)^2}} \angle\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]$$

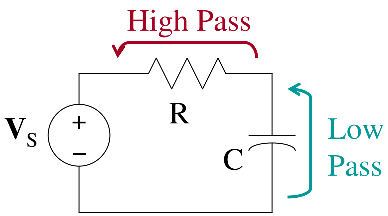
$$Let\ \omega_B = \frac{R}{L} \quad and\ f_B = \frac{R}{2\pi L}$$

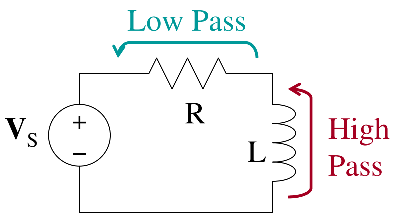
$$\mathbf{H(f)} = H(f)\angle\theta$$

$$H(f) = \frac{\left(\dfrac{f}{f_B}\right)}{\sqrt{1+\left(\dfrac{f}{f_B}\right)^2}}, \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{f}{f_B}\right)$$



First-Order Filter Circuits





$\mathbf{H}_R = \mathrm{R} / (\mathrm{R} + 1/j\omega C)$

$\mathbf{H}_C = (1/j\omega \mathrm{C}) / (\mathrm{R} + 1/j\omega C)$

$\mathbf{H}_R = \mathrm{R} / (\mathrm{R} + j\omega \mathrm{L})$

$\mathbf{H}_L = j\omega \mathrm{L} / (\mathrm{R} + j\omega \mathrm{L})$

Change of Voltage or Current with A Change of Frequency

One may wish to specify the change of a quantity such as the output voltage of a filter when the frequency changes by a factor of 2 (an octave) or 10 (a decade).

For example, a single-stage RC low-pass filter has at frequencies above ω = 1/RC an output that changes at the rate -20dB per decade.

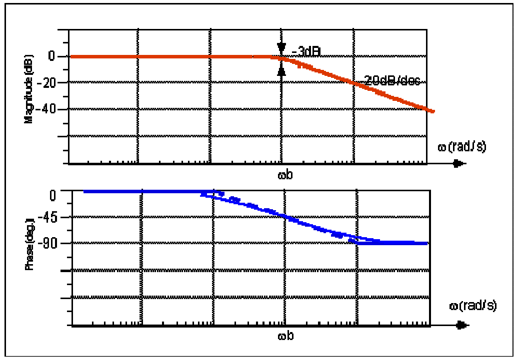
High-frequency asymptote of Lowpass filter

The high frequency asymptote of magnitude Bode plot assumes -20dB/decade slope

As $f \to \infty$

$$H(f) = \left(\frac{f}{f_B}\right)^{-1}$$

$$20\log_{10}\frac{H(10f_B)}{H(f_B)} = -20dB$$



Low-frequency asymptote of Highpass filter

As $f \to 0$

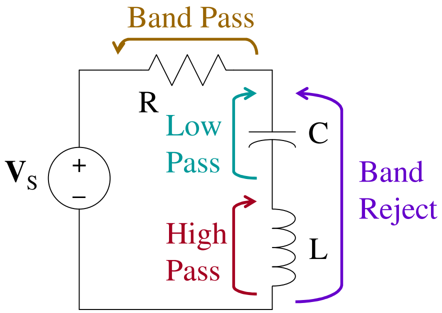
$$H(f) = \frac{\left(\frac{f}{f_B}\right)}{\sqrt{1+\left(\frac{f}{f_B}\right)^2}} \to \left(\frac{f}{f_B}\right)$$

$f \to \infty$

$$20\log_{10}\frac{H(f_B)}{H(0.1f_B)} = 20dB$$

The low frequency asymptote of magnitude Bode plot assumes 20dB/decade slope

Second-Order Filter Circuits



$\mathbf{Z} = \mathrm{R} + 1/j\omega\mathrm{C} + j\omega\mathrm{L}$

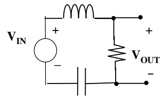
$\mathbf{H}_{BP} = \mathrm{R} / \mathbf{Z}$

$\mathbf{H}_{LP} = (1/j\omega\mathrm{C}) / \mathbf{Z}$

$\mathbf{H}_{HP} = j\omega\mathrm{L} / \mathbf{Z}$

$\mathbf{H}_{BR} = \mathbf{H}_{LP} + \mathbf{H}_{HP}$

Series Resonance



Voltage divider

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{\mathbf{Z}_R}{\mathbf{Z}_L + \mathbf{Z}_R + \mathbf{Z}_C}$$

Substitute branch elements

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{R}{j\omega L + R + 1/j\omega C}$$

Arrange in resonance form

$$\frac{\mathbf{V}_{OUT}}{\mathbf{V}_{IN}} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

Maximum when $w^2 = 1/(LC)$

Resonance quality factor

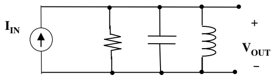
$$Q = \frac{\omega L}{R}$$

Ratio of reactance to resistance

Closely related to number of round trip cycles before 1/e decay.

Bandwidth is f_0/Q

Parallel Resonance



Admittance

$$\mathbf{V}_{OUT} = \frac{\mathbf{I}_S}{\mathbf{Y}_L + \mathbf{Y}_R + \mathbf{Y}_C}$$

Substitute branch elements

$$\mathbf{V}_{OUT} = \frac{\mathbf{I}_S}{1/j\omega L + 1/R + jwC}$$

Arrange in resonance form

$$\mathbf{V}_{OUT} = \frac{\mathbf{I}_S}{1/R + j(\omega C - 1/\omega L)}$$

Maximum = I_S/R when $w^2 = 1/(LC)$

Resonance quality factor

$$Q = \frac{\omega L}{R}$$

Ratio of reactance to resistance

Closely related to number of round trip cycles before 1/e decay.

Bandwidth is f_0/Q

Chapter 14

- OUTLINE
  - Op-Amp from 2-Port Blocks
  - Op-Amp Model and its Idealization
  - Negative Feedback for Stability
  - Components around Op-Amp define the Circuit Function
- Reading
  - Chap 14

The Operational Amplifier

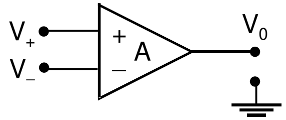
- The ***operational amplifier*** (“***op amp***”) is a basic building block used in analog circuits.
  - Its behavior is modeled using a dependent source.
  - When combined with resistors, capacitors, and inductors, it can perform various useful functions:
    - **amplification/scaling** of an input signal
    - **sign changing** (inversion) of an input signal
    - **addition** of multiple input signals
    - **subtraction** of one input signal from another
    - **integration** (over time) of an input signal
    - **differentiation** (with respect to time) of an input signal
    - **analog filtering**
    - **nonlinear functions** like exponential, log, sqrt, etc
  - Isolate input from output; allow cascading

High Quality Dependent Source In an Amplifier

AMPLIFIER SYMBOL

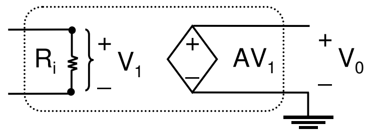
Differential Amplifier

$V_0 = A(V_+ - V_-)$



AMPLIFIER MODEL

Circuit Model *in linear region*

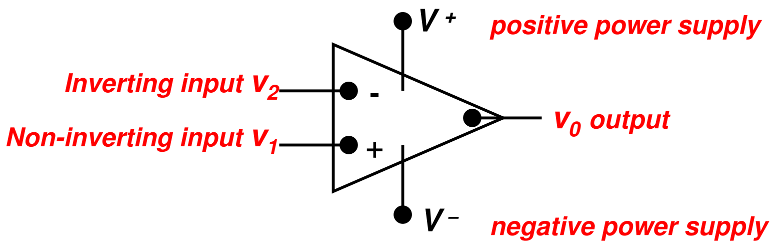


V_0 depends only on input ($V_+ - V_-$)

See the utility of this: this Model when used correctly mimics the behavior of an amplifier but omits the complication of the many many transistors and other components.

Op Amp Terminals

- 3 signal terminals: 2 inputs and 1 output
- IC op amps have 2 additional terminals for DC power supplies
- Common-mode signal= $(v_1+v_2)/2$
- Differential signal = v_1-v_2



Model for Internal Operation

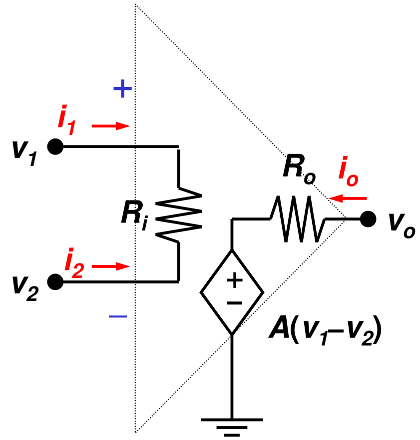
- A is differential gain or open loop gain
- Ideal op amp

$$A \to \infty$$
$$R_i \to \infty$$
$$R_o = 0$$

  – Common mode gain = 0

$$v_{cm} = \frac{(v_1+v_2)}{2} \quad , v_d = v_1 - v_2$$
$$v_o = A_{cm}v_{cm} + A_d v_d$$
$$Since \ \ v_o = A(v_1 - v_2) \quad , A_{cm} = 0$$

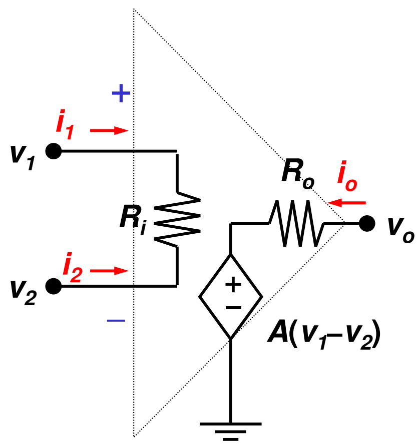
- Circuit Model



Model and Feedback

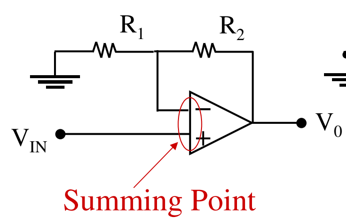
- Negative feedback
  – connecting the output port to the negative input (port 2)
- Positive feedback
  – connecting the output port to the positive input (port 1)
- Input impedance: R looking into the input terminals
- Output impedance: Impedance in series with the output terminals

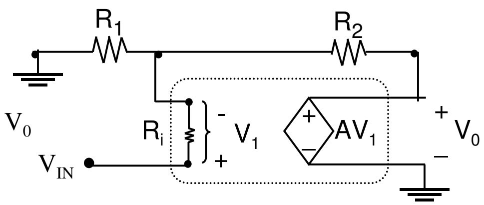
- Circuit Model



Op-Amp and Use of Feedback

A very high-gain differential amplifier can function in an extremely linear fashion as an operational amplifier by using negative feedback.





Circuit Model

Negative feedback $\Rightarrow$ **Stabilizes** the output

Hambley Example pp. 644 for Power Steering

We can show that that for $A \to \infty$ and $R_i \to \infty$,

$$V_0 \cong V_{IN} \cdot \frac{R_1 + R_2}{R_1}$$

Stable, finite, and independent of the properties of the OP AMP !

Summing-Point Constraint

- Check if under negative feedback
  - Small v_i result in large v_o
  - Output v_o is connected to the inverting input to reduce v_i
  - Resulting in v_i=0
- Summing-point constraint
  - $v_1 = v_2$
  - $i_1 = i_2 = 0$
- Virtual short circuit
  - Not only voltage drop is 0 (which is short circuit), input current is 0
  - This is different from short circuit, hence called “virtual” short circuit.

Ideal Op-Amp Analysis Technique

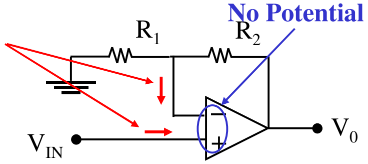
Applies only when Negative Feedback is present in circuit!

Assumption 1: The potential between the op-amp input terminals, $v_{(+)} - v_{(-)}$, equals zero.

Assumption 2: The currents flowing into the op-amp’s two input terminals both equal zero.

No Currents

No Potential Difference



EXAMPLE

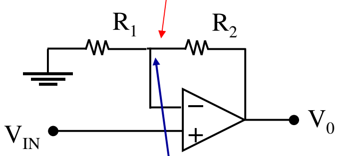
Ideal Op-Analysis: Non-Inverting Amplifier

Yes Negative Feedback is present in this circuit!

Assumption 1: The potential between the op-amp input terminals, $v_{(+)} - v_{(-)}$, equals zero.

Assumption 2: The currents flowing into the op-amp’s two input terminals both equal zero.

KCL with currents in only two branches



EXAMPLE

V_{IN} appears here

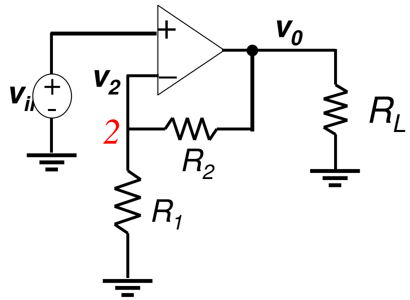
$$\frac{v_{in}}{R_1} + \frac{v_{in} - v_{out}}{R_2} = 0$$

$$v_{out} = \frac{R_1 + R_2}{R_1} v_{in}$$

Non-inverting Amplifier

Non-Inverting Amplifier

- Ideal voltage amplifier



$$Closed \ \ loop \ \ gain = A_v = \frac{v_o}{v_{in}}$$

$$v_1 = v_2 = v_{in} \ , i_1 = i_2 = 0$$

$$Use \ \ KCL \ \ At \ \ Node \ \ 2.$$

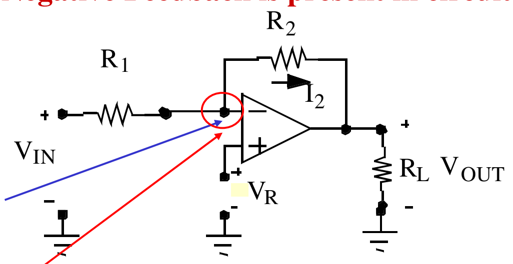
$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1}$$

$$Input \ \ impedance = \frac{v_{in}}{i} \rightarrow \infty$$

Ideal Op-Amp Analysis: Inverting Amplifier

Yes Negative Feedback is present in circuit!



Voltage is V_R

Only two currents for KCL

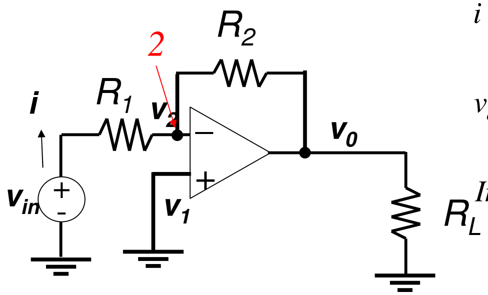
$$\frac{V_R - V_{IN}}{R_1} + \frac{V_R - V_{OUT}}{R_2} = 0$$

$$V_{OUT} = V_R - \frac{R_2}{R_1}(V_{in} - V_R)$$

Inverting Amplifier with reference voltage

Inverting Amplifier

- Negative feedback → checked
- Use summing-point constraint



$$Closed\ \ loop\ \ gain = A_v = \frac{v_o}{v_{in}}$$

$$v_1 = v_2 = 0\ ,i_1 = i_2 = 0$$

$Use\ \ KCL\ \ At\ \ Node\ \ 2.$

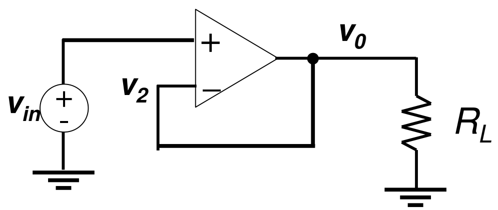
$$i = \frac{(v_{in} - v_2)}{R_1} = \frac{(v_{out} - v_2)}{R_2}$$

$$v_o = -\frac{R_2 v_o}{R_1}$$

$$Input\ \ impedance = \frac{v_{in}}{i} = R_1$$

Ideal voltage source – independent of load resistor

Voltage Follower



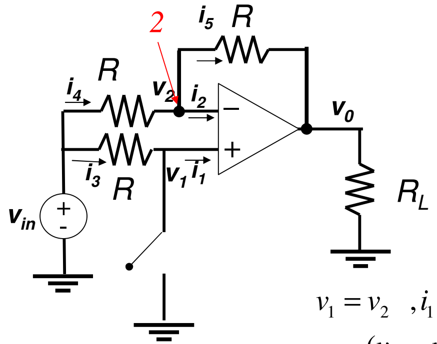
$$R_2 = 0$$

$$R_1 \to \infty$$

$$i = \frac{(v_0 - v_2)}{R_2} = \frac{(v_2 - 0)}{R_1}$$

$$A = \frac{v_o}{v_{in}} = \frac{(R_1 + R_2)}{R_1} = 1 + \frac{R_2}{R_1} = 1$$

Example 1



- Switch is open

$$v_1 = v_2\ \ ,i_1 = 0 \to i_3 = 0$$

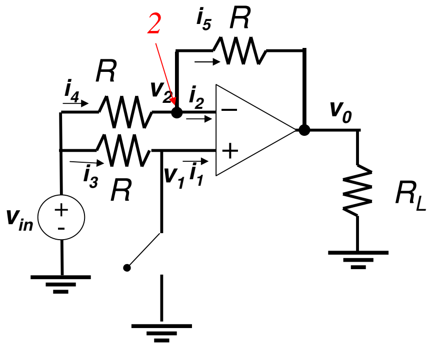
$$i_3 = \frac{(v_{in} - v_1)}{R} \to v_1 = v_2 = v_{in} \to i_4 = 0 \to i_5 = 0$$

$$i_5 = \frac{(v_0 - v_2)}{R} \to v_0 = v_2 = v_{in}$$

$$A = \frac{v_o}{v_{in}} = 1\ \ ,R_{in} \to \infty$$

Example 1

- Switch is closed



$$v_1 = v_2 = 0 \quad , i_1 = 0 \rightarrow i_3 = 0$$

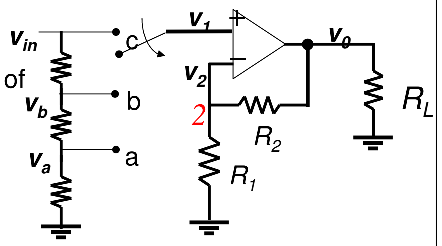
$$i_4 = \frac{(v_{in} - v_2)}{R} = i_5 = -\frac{(v_0 - v_2)}{R}$$

$$v_0 = -v_{in}$$

$$A = \frac{v_o}{v_{in}} = -1 \quad , R_{in} = R/2$$

Example 2

- Design an analog front end circuit to an instrument system
  - Requires to work with 3 full-scale of input signals (by manual switch): $0 \sim \pm 1, 0 \sim \pm 10, 0 \sim \pm 100$ V
  - For each input range, the output needs to be $0 \sim \pm 10$ V
  - The input resistance is 1MΩ



$$v_o = (1 + \frac{R_2}{R_1}) v_1$$

$$v_1 = v_{in} \quad Switch \ \ at \ \ c$$

$$v_1 = \frac{R_a + R_b}{R_a + R_b + R_c} v_{in} \quad Switch \ \ at \ \ b$$

$$v_1 = \frac{R_a}{R_a + R_b + R_c} v_{in} \quad Switch \ \ at \ \ a$$

Example 2 (cont'd)

$$R_{in} = R_a + R_b + R_c = 1M\Omega$$

$$Max \ A_v = 10 = (1 + \frac{R_2}{R_1}) \quad Switch \ \ at \ \ c$$

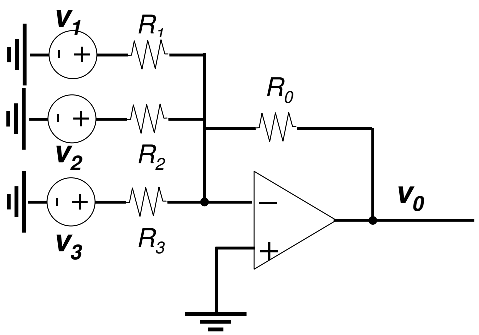
$$A_v = 1 = \frac{R_a + R_b}{R_a + R_b + R_c}(1 + \frac{R_2}{R_1}) \quad Switch \ \ at \ \ b \therefore \frac{R_a + R_b}{R_a + R_b + R_c} = 0.1$$

$$A_v = 0.1 = \frac{R_a}{R_a + R_b + R_c}(1 + \frac{R_2}{R_1}) \quad Switch \ \ at \ \ a \therefore \frac{R_a}{R_a + R_b + R_c} = 0.01$$

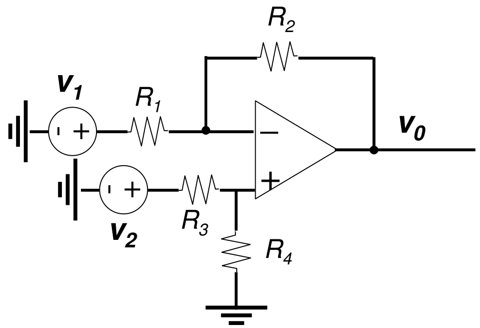
$$\therefore R_a = 10k\Omega, R_b = 90k\Omega, R_c = 900k\Omega$$

$$R_2 = 9R_1$$

Summing Amplifier

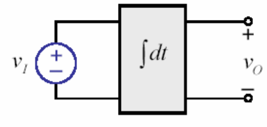


Difference Amplifier

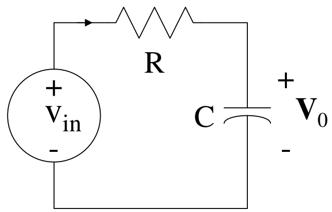


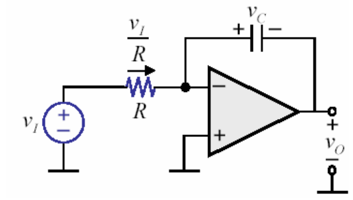
Integrator

- Want $v_o = K\int v_{in}dt$



- What is the difference between:



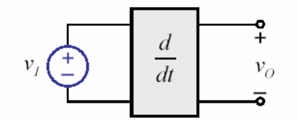


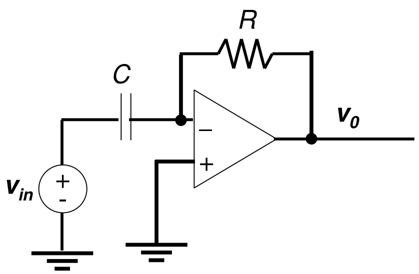
$$v_O \approx \frac{1}{RC}\int_{-\infty}^{t} v_I\, dt$$

$$v_O = -\frac{1}{C}\int_{-\infty}^{t}\frac{v_I}{R}dt$$

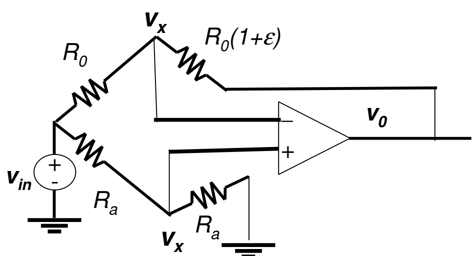
Differentiator

- Want





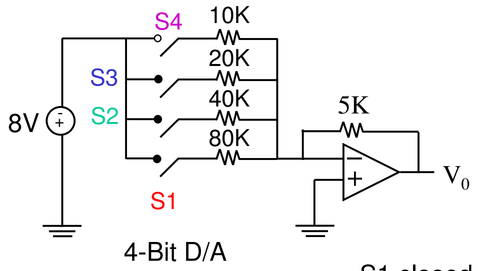
Bridge Amplifier



Application: Digital-to-Analog Conversion

A DAC can be used to convert the digital representation of an audio signal into an analog voltage that is then used to drive speakers -- so that you can hear it!

“Weighted-adder D/A converter”



4-Bit D/A

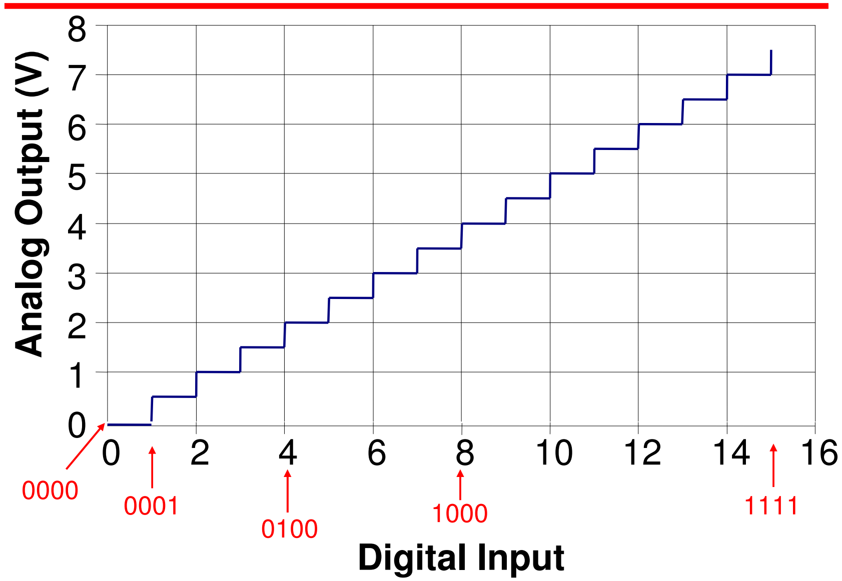
(Transistors are used as electronic switches)

S1 closed if LSB =1
S2 " if next bit = 1
S3 " if " " = 1
S4 " if MSB = 1

Binary number	Analog output (volts)
0 0 0 0	0
0 0 0 1	.5
0 0 1 0	1
0 0 1 1	1.5
0 1 0 0	2
0 1 0 1	2.5
0 1 1 0	3
0 1 1 1	3.5
1 0 0 0	4
1 0 0 1	4.5
1 0 1 0	5
1 0 1 1	5.5
1 1 0 0	6
1 1 0 1	6.5
1 1 1 0	7
1 1 1 1	7.5

MSB LSB

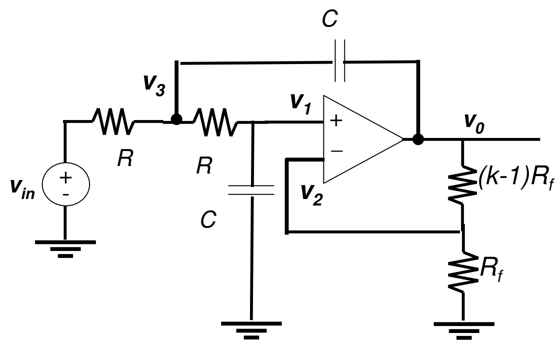
Characteristic of 4-Bit DAC



Active Filter

- Contain few components
- Transfer function that is insensitive to component tolerance
- Easily adjusted
- Require a small spread of components values
- Allow a wide range of useful transfer functions

Active Filter Example



Active Filter Solution

$$v_1 = v_2 = \frac{v_o}{k}$$

$$Use\ \ KCL\ \ At\ \ Node\ \ A \Rightarrow \frac{(v_3 - v_1)}{R} = j\omega C v_1$$

$$Use\ \ KCL\ \ At\ \ Node\ \ B \Rightarrow \frac{(v_{in} - v_3)}{R} = j\omega C(v_3 - v_o) + \frac{(v_3 - v_1)}{R}$$

$$\frac{v_o}{v_{in}} = \frac{k}{1 - \omega^2 R^2 C^2 + j\omega RC(3-k)}$$

$$Let\ \omega_B = 1/RC$$

$$|H(\omega)| = \left|\frac{v_o}{v_{in}}\right| = \frac{k}{\sqrt{\left(1 - \frac{\omega^2}{\omega_B^{\ 2}}\right)^2 + \frac{\omega^2}{\omega_B^{\ 2}}(3-k)^2}}$$

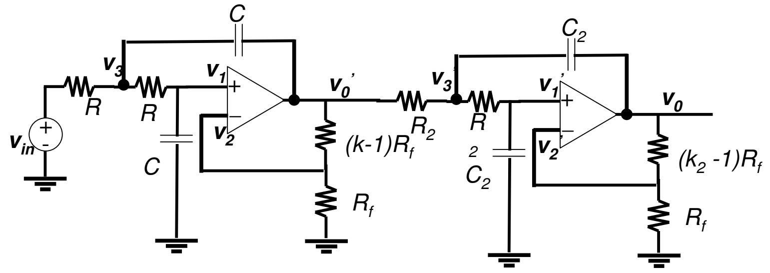
$$\omega = 0, |H(\omega)| = k \quad DC\ \ gain$$

$$\omega = \omega_B, |H(\omega)| = \frac{k}{3-k}$$

$$\omega >> \omega_B, |H(\omega)| = \frac{k}{\left(\frac{\omega^2}{\omega_B^{\ 2}}\right)} \sim \omega^{-2}$$

$$20\log|H(\omega)|\ \ decays\ at\ a\ rate\ of\ 40dB/decade$$

Cascaded Active Filter Example



Cascaded Active Filter Solution

$$\frac{v_o}{v_{in}} = \frac{k_2}{1-\omega^2 R_2^{\ 2} C_2^{\ 2} + j\omega R_2 C_2 (3-k_2)} \frac{k}{1-\omega^2 R^2 C^2 + j\omega RC(3-k)}$$

$$Let\ \omega_B = 1/RC,\ \ \omega_{B2} = 1/R_2 C_2$$

$$|H(\omega)| = \left|\frac{v_o}{v_{in}}\right| = \frac{k_2}{\sqrt{\left(1-\frac{\omega^2}{\omega_{B2}^{\ 2}}\right)^2 + \frac{\omega^2}{\omega_{B2}^{\ 2}}(3-k_2)^2}} \frac{k}{\sqrt{\left(1-\frac{\omega^2}{\omega_B^{\ 2}}\right)^2 + \frac{\omega^2}{\omega_B^{\ 2}}(3-k)^2}}$$

$$\omega = 0, |H(\omega)| = k_2 k \quad DC\ \ gain$$

$$\omega = \omega_B, |H(\omega)| = \frac{k_2}{3-k_2} \frac{k}{3-k}$$

$$\omega >> \omega_B, |H(\omega)| = \frac{k_2 k}{\left(\frac{\omega^4}{\omega_{B2}^{\ 2}\omega_B^{\ 2}}\right)} \sim \omega^{-4}$$

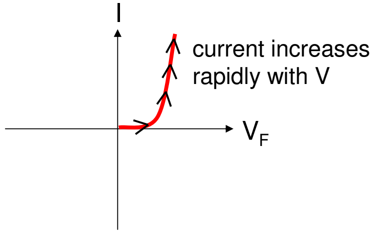
$$20\log|H(\omega)|\ \ decays\ at\ a\ rate\ of\ 80dB/decade$$

Chapter 10

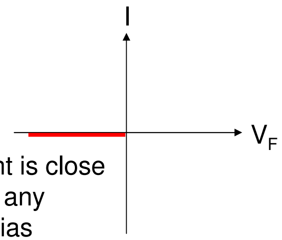
- OUTLINE
  - Diode Current and Equation
  - Some Interesting Circuit Applications
  - Load Line Analysis
  - Solar Cells, Detectors, Zener Diodes
  - Circuit Analysis with Diodes
  - Half-wave Rectifier
  - Clamps and Voltage Doublers using Capacitors
- Reading
  - Hambley 10.1-10.8
  - Supplementary Notes Chapter 2

I-V Characteristics

In forward bias (+ on p-side) we have almost unlimited flow (very low resistance). Qualitatively, the I-V characteristics must look like:



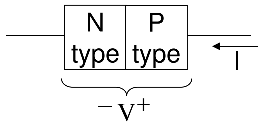
In reverse bias (+ on n-side) almost no current can flow. Qualitatively, the I-V characteristics must look like:



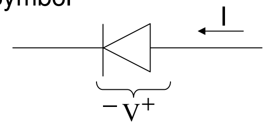
The current is close to zero for any negative bias

Diode Physical Behavior and Equation

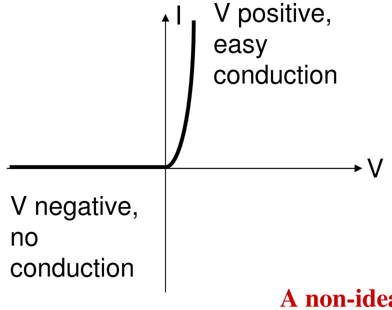
Schematic Device



Symbol



Qualitative I-V characteristics:



Quantitative I-V characteristics:

$$I = I_0(e^{qV/kT} - 1)$$

In which kT/q is 0.026V and I_O is a constant depending on diode area. Typical values: 10^{-12} to 10^{-16} A. Interestingly, the graph of this equation looks just like the figure to the left.

A non-ideality factor n times kT/q is often included.

The pn Junction I vs. V Equation

I-V characteristic of PN junctions

In EECS 105, 130, and other courses you will learn why the I vs. V relationship for PN junctions is of the form

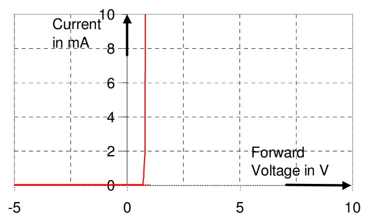
$$\boxed{I = I_0(e^{qV/kT} - 1)}$$

where I_0 is a constant proportional to junction area and depending on doping in P and N regions, $q = \text{electronic charge} = 1.6\times10^{-19}$, k is Boltzman constant, and T is absolute temperature.
$KT/q = 0.026\text{V at } 300°\text{K}$, a typical value for I_0 is $10^{-12} - 10^{-15}$ A

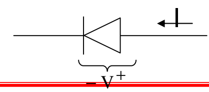
We note that in forward bias, I increases **exponentially** and is in the µA-mA range for voltages typically in the range of 0.6-0.8V. In reverse bias, the current is essentially zero.

Diode Ideal (Perfect Rectifier) Model

The equation $I = I_0 \exp(qV/kT - 1)$ is graphed below for $I_0 = 10^{-15}$ A

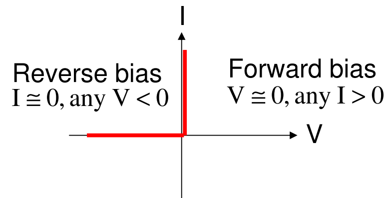


The characteristic is described as a "rectifier" – that is, a device that permits current to pass in only one direction. (The hydraulic analog is a "check value".) Hence the symbol:



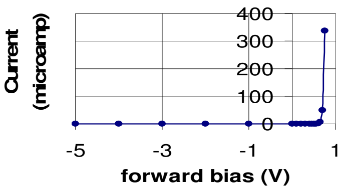
Simple "Perfect Rectifier" Model

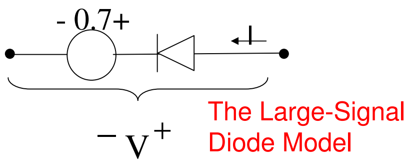
If we can ignore the small forward-bias voltage drop of a diode, a simple effective model is the "perfect rectifier," whose I-V characteristic is given below:



A perfect rectifier

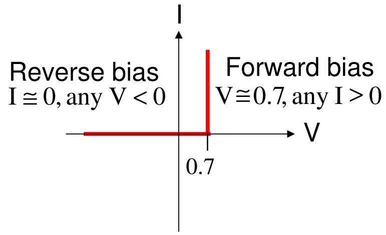
Diode Large-Signal Model (0.7 V Drop)





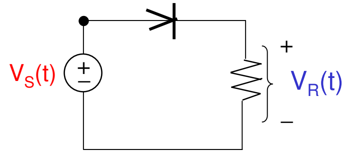
Improved "Large-Signal Diode" Model:

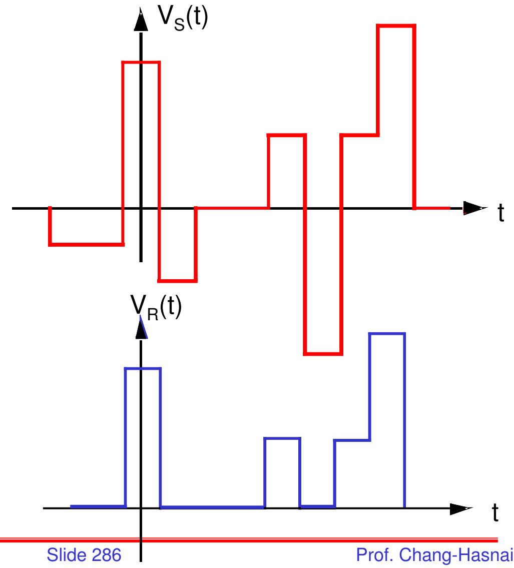
If we choose not to ignore the small forward-bias voltage drop of a diode, it is a very good approximation to regard the voltage drop in forward bias as a constant, about 0.7V. the "Large signal model" results.



Rectifier Circuit

Assume the ideal (perfect rectifier) model.

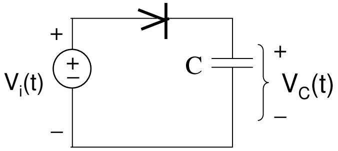


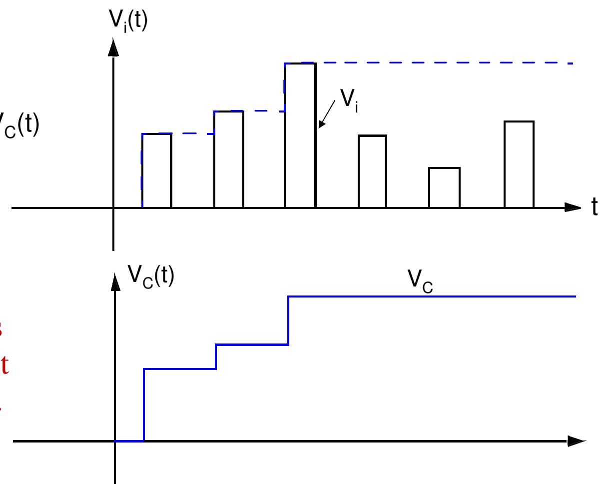


"rectified" version of input waveform

Peak Detector Circuit

Assume the ideal (perfect rectifier) model.

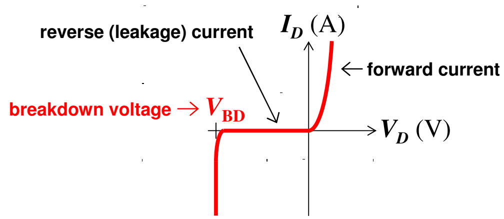




Key Point:
The capacitor charges due to one way current behavior of the diode.

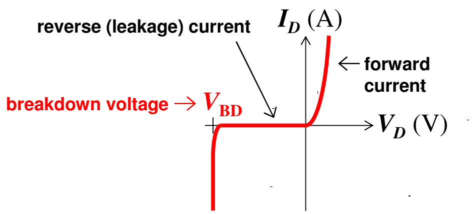
pn-Junction Reverse Breakdown

- As the reverse bias voltage increases, the peak electric field in the depletion region increases. When the electric field exceeds a critical value ($E_{crit} \cong 2\text{x}10^5$ V/cm), the reverse current shows a dramatic increase:

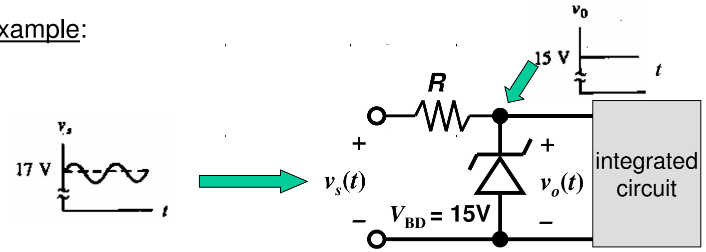


Zener Diode

A ***Zener diode*** is designed to operate in the breakdown mode.

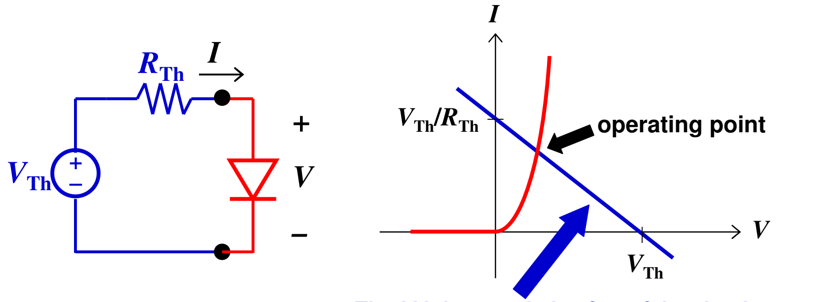


Example:



Load Line Analysis Method

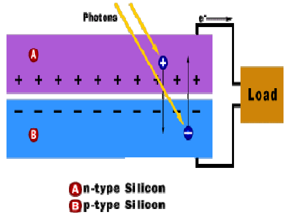
1. Graph the *I-V* relationships for the non-linear element and for the rest of the circuit
2. The operating point of the circuit is found from the intersection of these two curves.



The *I-V* characteristic of all of the circuit except the non-linear element is called the ***load line***

Solar cell: Example of simple PN junction

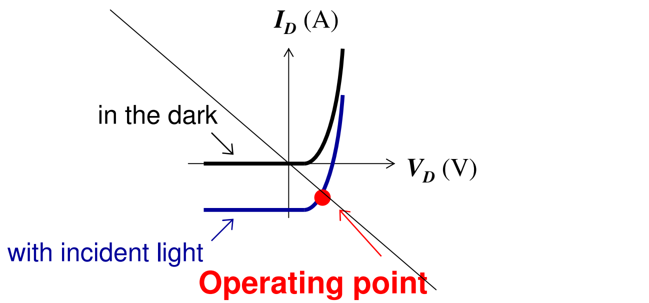
- What is a solar cell?
  - Device that converts sunlight into electricity
- How does it work?
  - In simple configuration, it is a diode made of PN junction
  - Incident light is absorbed by material
  - Creates electron-hole pairs that transport through the material through
    - Diffusion (concentration gradient)
    - Drift (due to electric field)



PN Junction Diode

Photovoltaic (Solar) Cell

$$I_D = I_S(e^{qV_D/kT} - 1) - I_{optical}$$



Operating point

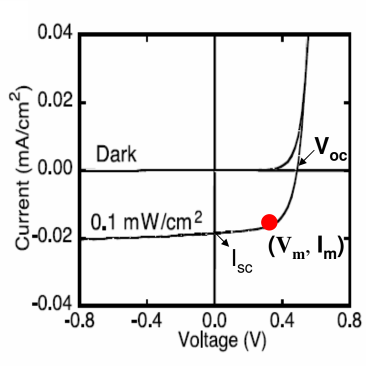
The load line a simple resistor.

I-V characteristics of the device

- I-V characteristics of a PN junction is given by

$$I = I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right] - I_L$$

where I_s is the saturation intensity depending on band gap and doping of the material and I_L is the photocurrent generated due to light



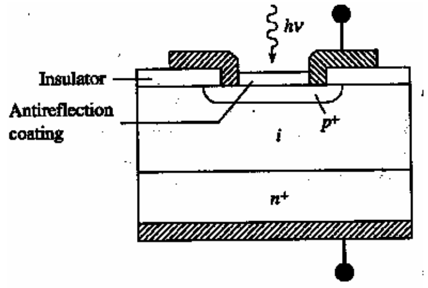
- Efficiency is defined as

$$\eta = \frac{I_m . V_m}{Light\ Intensity} = \frac{FF * V_{oc} * I_{sc}}{Light\ Intensity}$$

FF is the Fill Factor

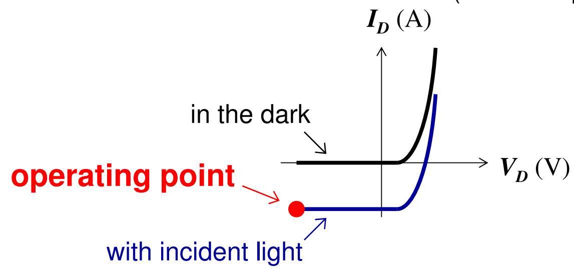
V_{oc} - Open circuit voltage
I_{sc} - Short circuit current
I_{mp}, V_{mp} - Current and voltage at maximum power

Example 2: Photodiode

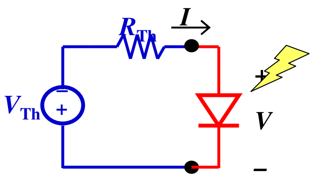


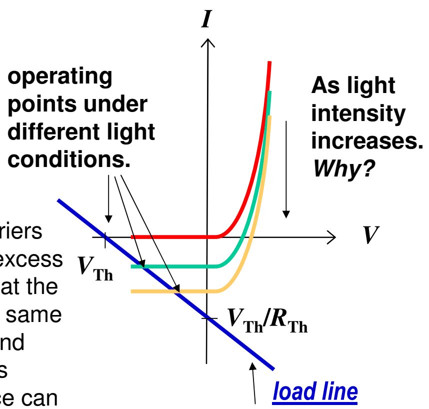
- An intrinsic region is placed between the p-type and n-type regions
  - $W_j \cong W_{i\text{-region}}$, so that most of the electron-hole pairs are generated in the depletion region

→ faster response time (~10 GHz operation)



Photodetector Circuit Using Load Line



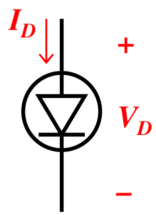


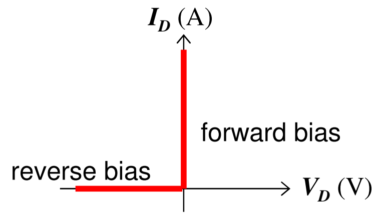
As light shines on the photodiode, carriers are generated by absorption. These excess carriers are swept by the electric field at the junction creating drift current, which is same direction as the reverse bias current and hence negative current. The current is proportional to light intensity and hence can provide a direct measurement of light intensity → photodetector.

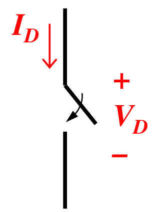
- What happens when R_{th} is too large?
- Why use Vth?

Ideal Diode Model of PN Diode

Circuit symbol **I-V characteristic** **Switch model**





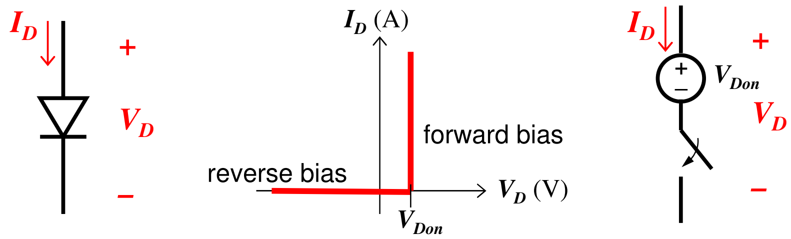


- An ideal diode passes current only in one direction.
- An ***ideal diode*** has the following properties:
  - when $I_D > 0$, $V_D = 0$
  - when $V_D < 0$, $I_D = 0$

Diode behaves like a switch:
• closed in forward bias mode
• open in reverse bias mode

Large-Signal Diode Model

Circuit symbol | ***I-V* characteristic** | **Switch model**



For a Si pn diode, $V_{Don} \cong 0.7$ V

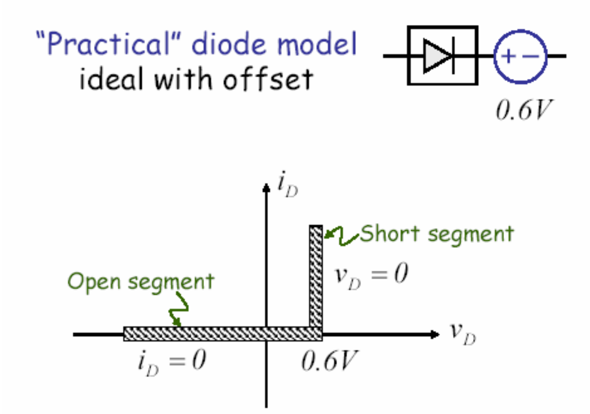
RULE 1: When $I_D > 0$, $V_D = V_{Don}$

RULE 2: When $V_D < V_{Don}$, $I_D = 0$

Diode behaves like a voltage source in series with a switch:
- closed in forward bias mode
- open in reverse bias mode

Diode: Large Signal Model

- Use piece-wise linear model



How to Analyze Circuits with Diodes

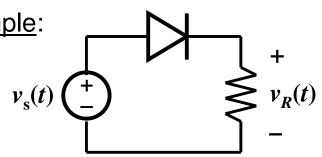
A diode has only two states:

- **forward biased:** $I_D > 0$, $V_D = 0$ V (or 0.7 V)
- **reverse biased:** $I_D = 0$, $V_D < 0$ V (or 0.7 V)

Procedure:

1. Guess the state(s) of the diode(s)
2. Check to see if KCL and KVL are obeyed.
3. If KCL and KVL are not obeyed, refine your guess
4. Repeat steps 1-3 until KCL and KVL are obeyed.

Example:



If $v_s(t) > 0$ V, diode is forward biased (else KVL is disobeyed – try it)

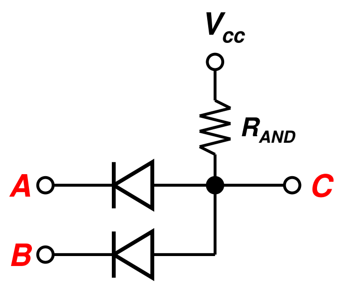
If $v_s(t) < 0$ V, diode is reverse biased (else KVL is disobeyed – try it)

Diode Logic: AND Gate

- Diodes can be used to perform logic functions:

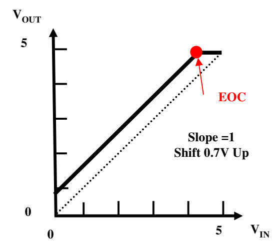
AND gate

output voltage is high only if both A and B are high



Inputs A and B vary between 0 Volts ("low") and V_{cc} ("high")

Between what voltage levels does C vary?

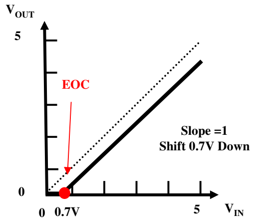


Diode Logic: OR Gate

- Diodes can be used to perform logic functions:

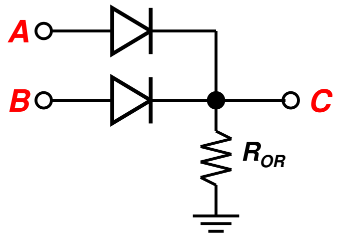
Inputs A and B vary between 0 Volts (“low”) and V_{cc} (“high”)

Between what voltage levels does C vary?



<u>OR gate</u>

output voltage is high if either (or both) A and B are high



Diode Logic: Incompatibility and Decay

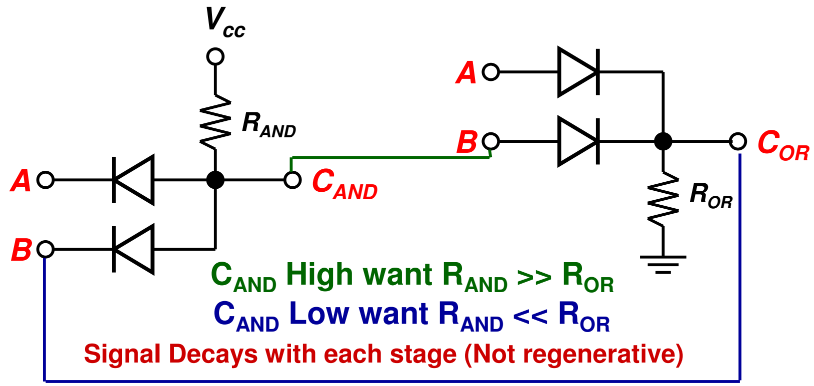
- Diode Only Gates are Basically Incompatible:

<u>AND gate</u>

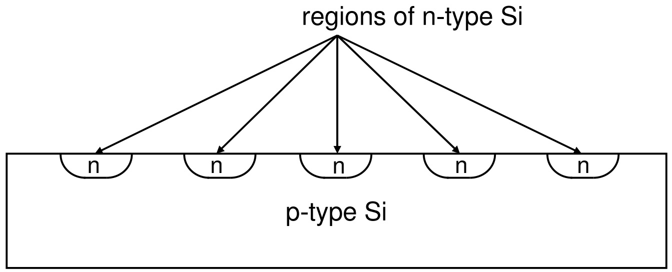
output voltage is high only if both A and B are high

<u>OR gate</u>

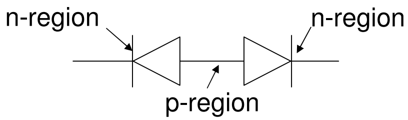
output voltage is high if either (or both) A and B are high



Device Isolation using pn Junctions



No current flows if voltages are applied between n-type regions, because two pn junctions are “back-to-back”



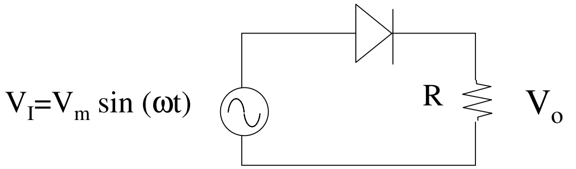
=> n-type regions **<u>isolated</u>** in p-type substrate and vice versa

Why are pn Junctions Important for ICs?

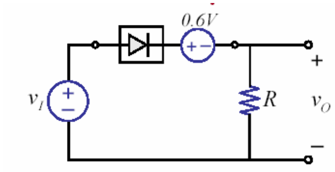
- The basic building block in digital ICs is the MOS transistor, whose structure contains reverse-biased diodes.
  - pn junctions are important for electrical isolation of transistors located next to each other at the surface of a Si wafer.
  - The junction capacitance of these diodes can limit the performance (operating speed) of digital circuits

Power Conversion Circuits

- Converting AC to DC
- Potential applications: Charging a battery



Rectifier Equivalent circuit

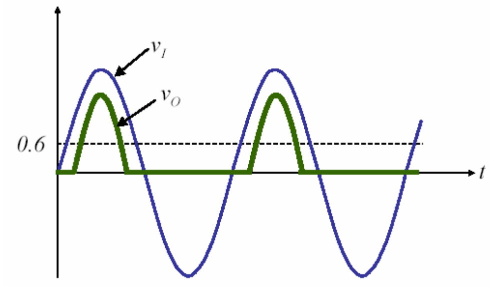


V>0.6V, diode = short circuit

→ $V_o = V_I - 0.6$

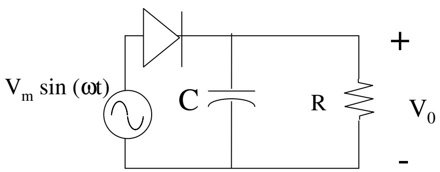
V<0.6V, diode = open circuit

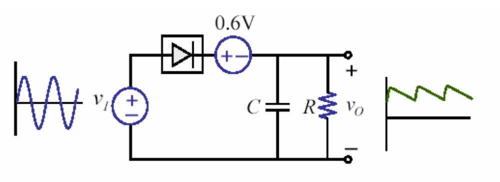
→ Vo=0



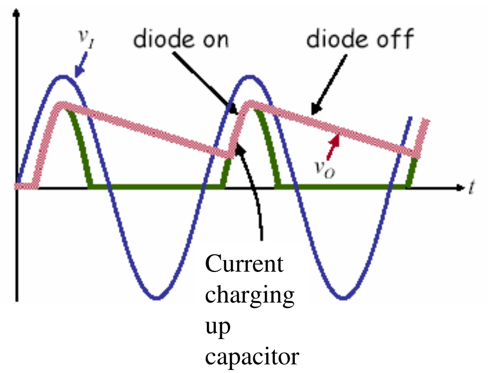
Half-wave Rectifier Circuits

- Adding a capacitor: what does it do?

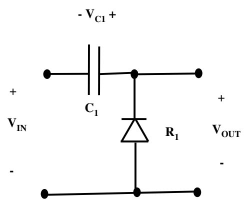


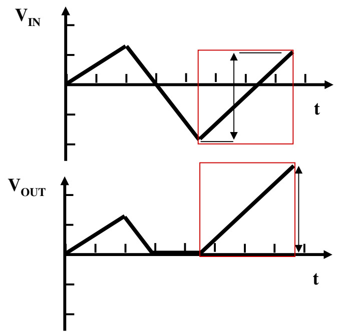


Half-Wave Rectifier



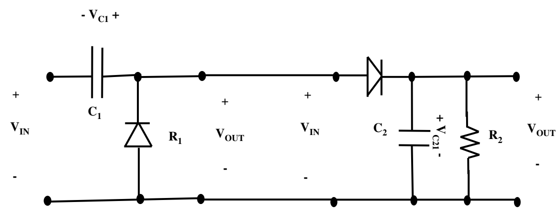
Level Shift Circuit





Once the capacitor is charged by the negative most voltage the rest of the signal is shifted up by that amount.

Voltage Doubler Circuit



Level Shift Peak Detect

The final output is the peak to peak voltage of the input.

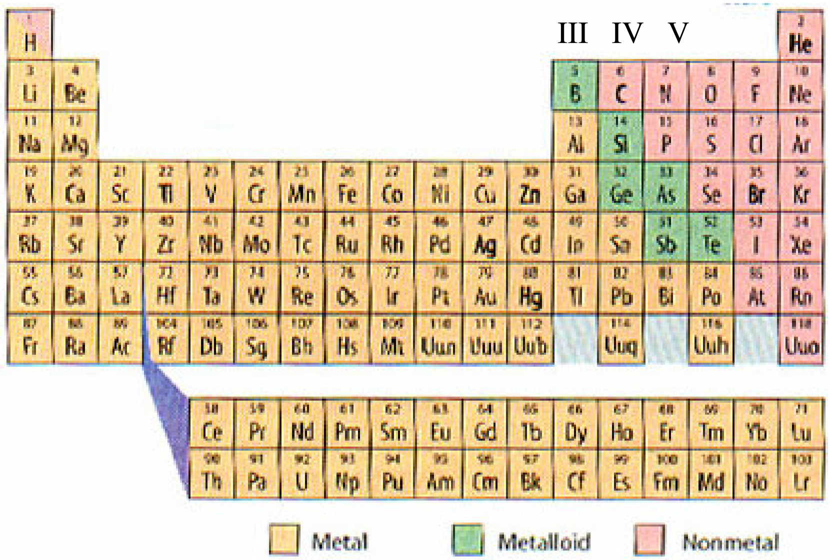
Week 12

- OUTLINE
  - Basic Semiconductor Materials
  - n and p doping
  - Bandgap
  - Gauss's Law
  - Poisson Equation
  - Depletion approximation
  - Diode I-V characteristics
  - Lasers and LEDs
  - Solar Cells
- Reading
  - Supplementary Notes Chap 3

Conductors, Insulators and Semiconductors

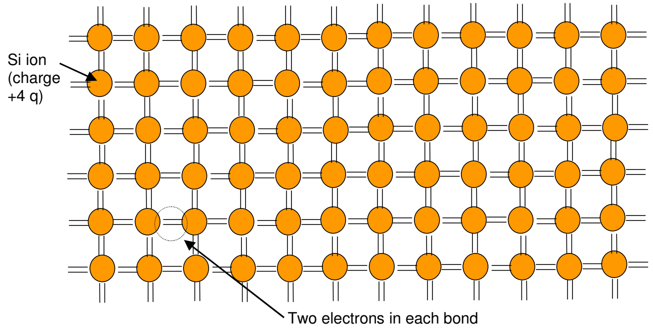
- Solids with "free electrons" – that is electrons not directly involved in the inter-atomic bonding- are the familiar metals (Cu, Al, Fe, Au, etc).
- Solids with no free electrons are the familiar insulators (glass, quartz crystals, ceramics, etc.)
- Silicon is an insulator, but at higher temperatures some of the bonding electrons can get free and make it a little conducting – hence the term "semiconductor"
- Pure silicon is a poor conductor (and a poor insulator). It has 4 valence electrons, all of which are needed to bond with nearest neighbors. No free electrons.

The Periodic Table



Electronic Bonds in Silicon

2-D picture of perfect crystal of pure silicon; double line is a Si-Si bond with each line representing an electron



Actual structure is 3-dimensional tetrahedral- just like carbon bonding in organic and inorganic materials.

Essentially no free electrons, and no conduction → insulator

How to get conduction in Si?

We must either:

1) Chemically modify the Si to produce free carriers (permanent) or

2) Electrically "induce" them by the field effect (switchable)

For the first approach controlled impurities, "dopants", are added to Si:

Add group V elements (5 bonding electrons vs four for Si), such as phosphorus or arsenic

(Extra electrons produce "free electrons" for conduction.)

or

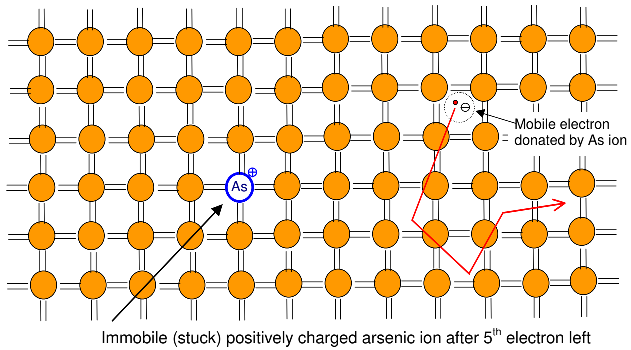
Add group III elements (3 bonding electrons), such as boron

Deficiency of electrons results in "free holes"

Doping Silicon with Donors (n-type)

Donors donate mobile electrons (and thus "n-type" silicon)

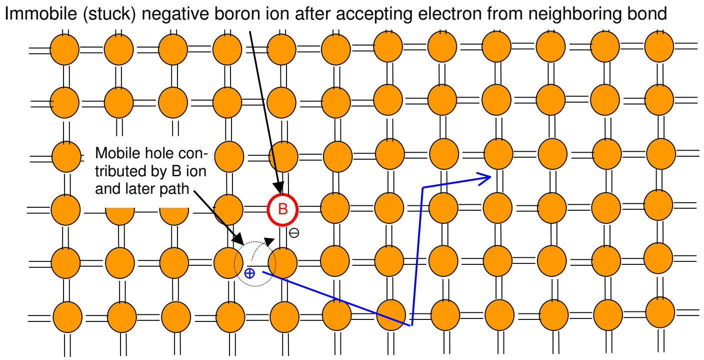
Example: add arsenic (As) to the silicon crystal:



The extra electron with As, "breaks free" and becomes a free electron for conduction

Doping with Acceptors (p-type)

Group III element (boron, typically) is added to the crystal



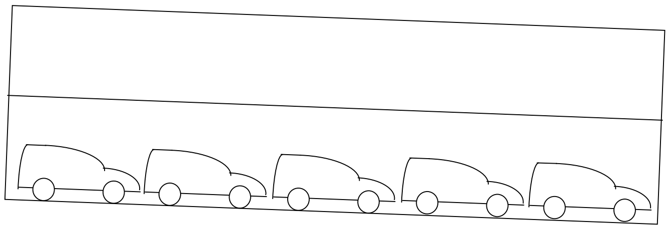
The “hole” which is a missing bonding electron, breaks free from the B acceptor and becomes a roaming positive charge, free to carry current in the semiconductor. It is positively charged.

Doping

- Typical doping densities: 10^{16}~10^{19} cm^{-3}
- Atomic density for Si: 5 x 10^{22} atoms/cm^3
- 10^{18} cm^{-3} is 1 in 50,000
  - two persons in entire Berkeley wearing a green hat
- P-n junction effect is like

Shockley’s Parking Garage Analogy for Conduction in Si

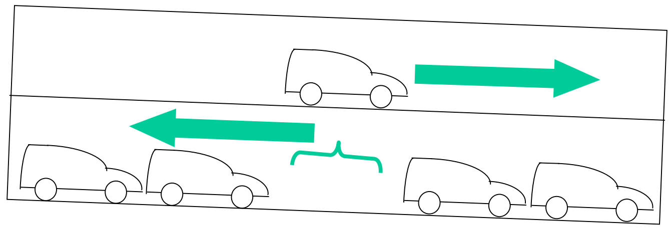
Two-story parking garage on a hill:



If the lower floor is full and top one is empty, no traffic is possible. Analog of an insulator. All electrons are locked up.

Shockley’s Parking Garage Analogy for Conduction in Si

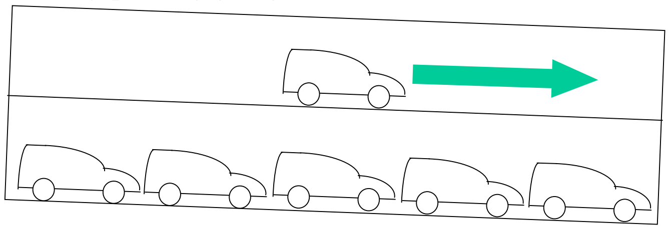
Two-story parking garage on a hill:



If one car is moved upstairs, it can move AND THE HOLE ON THE LOWER FLOOR CAN MOVE. Conduction is possible. Analog to warmed-up semiconductor. Some electrons get free (and leave “holes” behind).

Shockley's Parking Garage Analogy for Conduction in Si

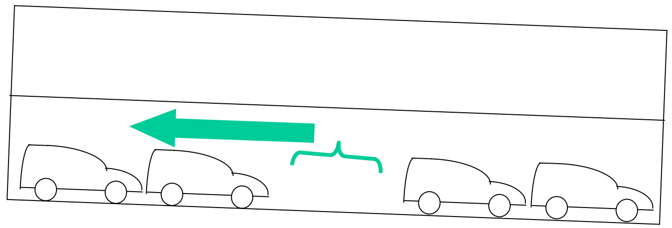
Two-story parking garage on a hill:



If an extra car is "donated" to the upper floor, it can move. Conduction is possible. *Analog to N-type semiconductor.* (An electron donor is added to the crystal, creating free electrons).

Shockley's Parking Garage Analogy for Conduction in Si

Two-story parking garage on a hill:



If a car is removed from the lower floor, it leaves a HOLE which can move. Conduction is possible. *Analog to P-type semiconductor.* (Acceptors are added to the crystal, "consuming" bonding electrons,creating free holes).

Summary of n- and p-type silicon

Pure silicon is an insulator. At high temperatures it conducts weakly.

If we add an impurity with extra electrons (e.g. arsenic, phosphorus) these extra electrons are set free and we have a pretty good conductor (n-type silicon).

If we add an impurity with a deficit of electrons (e.g. boron) then bonding electrons are missing (holes), and the resulting holes can move around … again a pretty good conductor (p-type silicon)

Now what is really interesting is when we join n-type and p-type silicon, that is make a pn junction. It has interesting electrical properties.

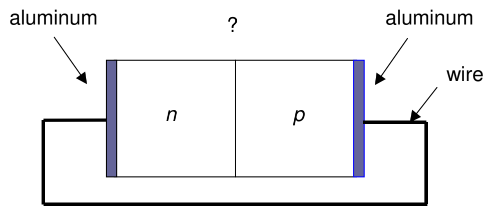
Junctions of n- and p-type Regions

p-n junctions form the essential basis of all semiconductor devices.

A silicon chip may have 10^8 to 10^9 p-n junctions today.

How do they behave*? What happens to the electrons and holes? What is the electrical circuit model for such junctions?

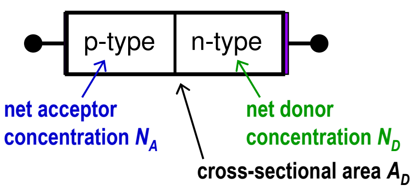
***n* and *p* regions are brought into contact :**



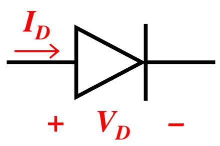
*Note that the textbook has a very good explanation.

The *pn* Junction Diode

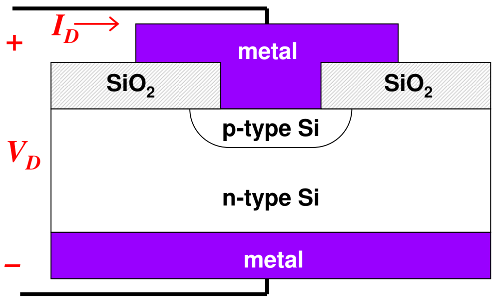
Schematic diagram



Circuit symbol



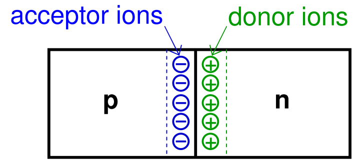
Physical structure:
(an example)



For simplicity, assume that the doping profile changes abruptly at the junction.

Depletion Region *Approximation*

- When the junction is first formed, mobile carriers ***diffuse*** across the junction (due to the concentration gradients)
  - Holes diffuse from the p side to the n side, leaving behind negatively charged immobile acceptor ions
  - Electrons diffuse from the n side to the p side, leaving behind positively charged immobile donor ions

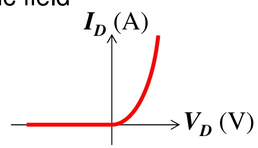


→A region depleted of mobile carriers is formed at the junction.

- The space charge due to immobile ions in the depletion region establishes an electric field that opposes carrier diffusion.

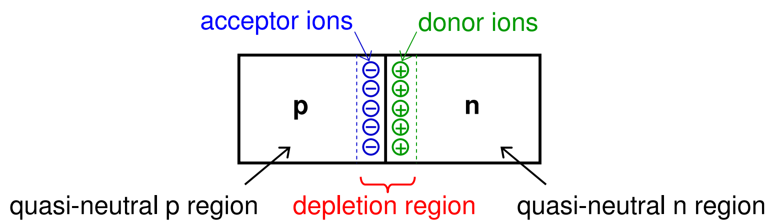
Summary: *pn*-Junction Diode *I-V*

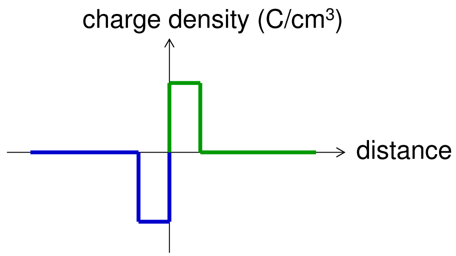
- Under forward bias, the potential barrier is reduced, so that carriers flow (by diffusion) across the junction
  - Current increases exponentially with increasing forward bias
  - The carriers become minority carriers once they cross the junction; as they diffuse in the quasi-neutral regions, they recombine with majority carriers (supplied by the metal contacts)
    "injection" of minority carriers
- Under reverse bias, the potential barrier is increased, so that negligible carriers flow across the junction
  - If a minority carrier enters the depletion region (by thermal generation or diffusion from the quasi-neutral regions), it will be swept across the junction by the built-in electric field
    "collection" of minority carriers → reverse current



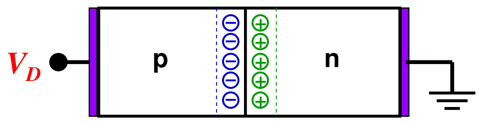
Charge Density Distribution

Charge is stored in the depletion region.





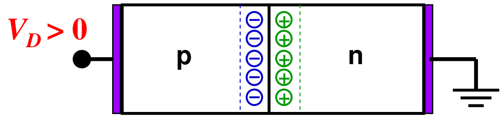
Effect of Applied Voltage



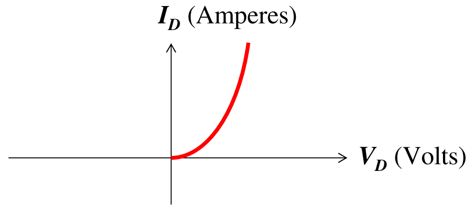
- The quasi-neutral p and n regions have low resistivity, whereas the depletion region has high resistivity. Thus, when an external voltage V_D is applied across the diode, almost all of this voltage is dropped across the depletion region. (Think of a voltage divider circuit.)
- If $V_D > 0$ (***forward bias***), the potential barrier to carrier diffusion is reduced by the applied voltage.
- If $V_D < 0$ (***reverse bias***), the potential barrier to carrier diffusion is increased by the applied voltage.

Forward Bias

- As V_D increases, the potential barrier to carrier diffusion across the junction decreases*, and current increases exponentially.



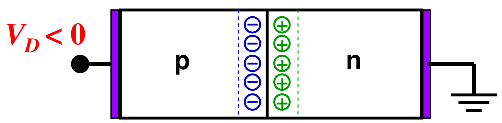
The carriers that diffuse across the junction become minority carriers in the quasi-neutral regions; they then recombine with majority carriers, "dying out" with distance.



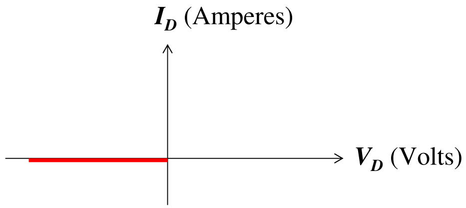
* Hence, the width of the depletion region decreases.

Reverse Bias

- As $|V_D|$ increases, the potential barrier to carrier diffusion across the junction increases*; thus, no carriers diffuse across the junction.



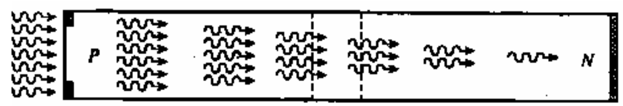
A very small amount of reverse current ($I_D < 0$) does flow, due to minority carriers diffusing from the quasi-neutral regions into the depletion region and drifting across the junction.



* Hence, the width of the depletion region increases.

Optoelectronic Diodes

- Light incident on a pn junction generates electron-hole pairs
- Carriers are generated in the depletion region as well as n-doped and p-doped quasi-neutral regions.
- The carriers that are generated in the quasi-neutral regions diffuse into the depletion region, together with the carriers generated in the depletion region, are swept across the junction by the electric field

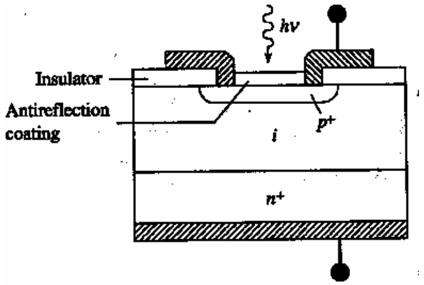


- This results in an additional component of current flowing in the diode:

$$I_D = I_S(e^{qV_D/kT} - 1) - I_{optical}$$

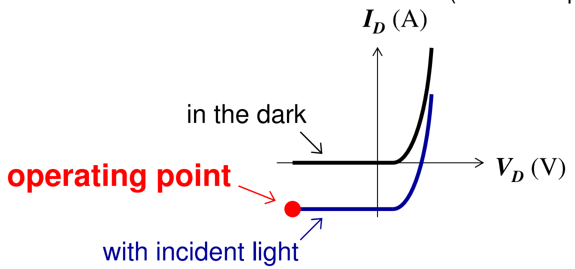
where $I_{optical}$ is proportional to the intensity of the light

Example: Photodiode



- An intrinsic region is placed between the p-type and n-type regions
  - $W_j \cong W_{i\text{-region}}$, so that most of the electron-hole pairs are generated in the depletion region

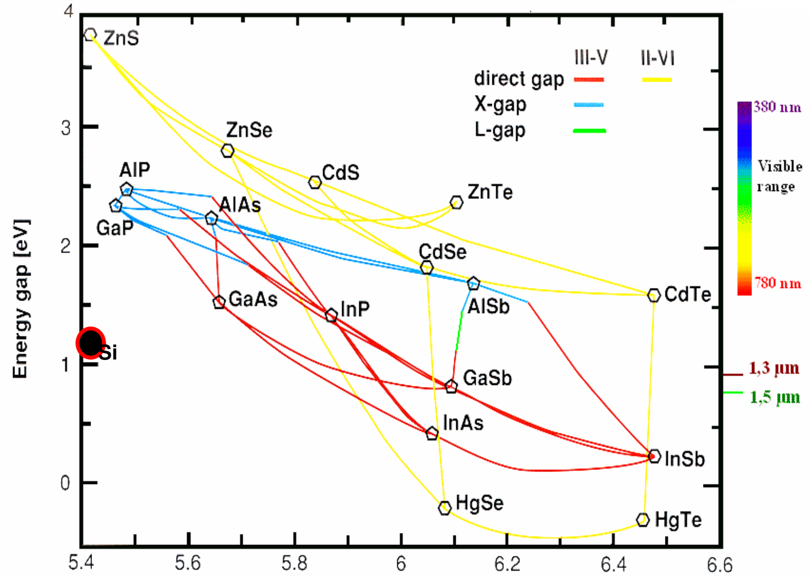
    → faster response time (~10 GHz operation)



Planck Constant

- Planck's constant h = $6.625 \cdot 10^{-34}$ J·s
- E=hν=hc/λ
- C is speed of light and hν is photon energy
- The first type of quantum effect is the quantization of certain physical quantities.
- Quantization first arose in the mathematical formulae of Max Planck in 1900. Max Planck was analyzing how the radiation emitted from a body was related to its temperature, in other words, he was analyzing the energy of a wave.
- The energy of a wave could not be infinite, so Planck used the property of the wave we designate as the frequency to define energy. Max Planck discovered a constant that when multiplied by the frequency of any wave gives the energy of the wave. This constant is referred to by the letter h in mathematical formulae. It is a cornerstone of physics.

Bandgap Versus Lattice Constant



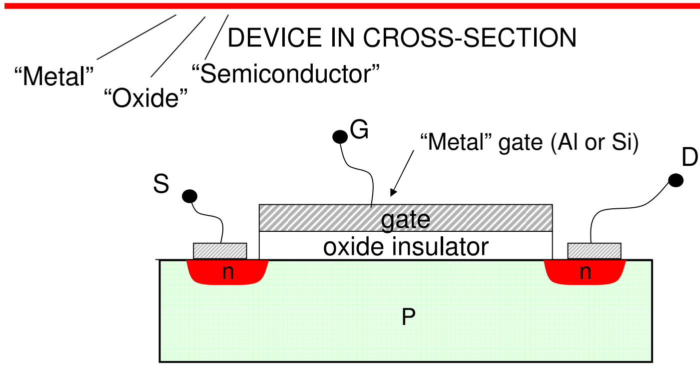
Chapter 12 MOSFET

- OUTLINE
  - The MOSFET as a controlled resistor
  - Pinch-off and current saturation
  - MOSFET ID vs. VGS characteristic
  - NMOS and PMOS I-V characteristics
  - Load-line analysis; Q operating point; Bias circuits
  - Small-signal equivalent circuits
  - Common source amplifier
  - Source follower
  - Common gate amplifier
  - Gain
- Reading
  - Supplementary Notes Chapter 4
  - Hambley: Chapter 12.1-12.5

MOSFET Terminals

- The voltage applied to the GATE terminal determines whether current can flow between the SOURCE & DRAIN terminals.
  - For an n-channel MOSFET, the SOURCE is biased at a *lower* potential (often 0 V) than the DRAIN
    (Electrons flow from SOURCE to DRAIN when $V_G > V_T$)
  - For a p-channel MOSFET, the SOURCE is biased at a *higher* potential (often the supply voltage V_{DD}) than the DRAIN
    (Holes flow from SOURCE to DRAIN when $V_G < V_T$)
- The BODY terminal is usually connected to a fixed potential.
  - For an n-channel MOSFET, the BODY is connected to 0 V
  - For a p-channel MOSFET, the BODY is connected to V_{DD}

MOSFET Structure



- In the absence of gate voltage, no current can flow between S and D.
- Above a certain gate to source voltage V_t (the "threshold"), electrons are induced at the surface beneath the oxide. (Think of it as a capacitor.)
- These electrons can carry current between S and D if a voltage is applied.

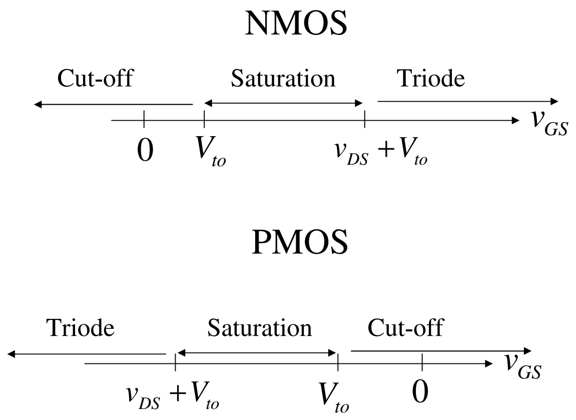
MOSFET

- Symbol and subscript convention
  - Upper case for both (e.g. V_D) = DC signal (often as bias)
  - Lower case for both (e.g. v_d) = AC signal (often small signal)
  - Lower symbol and upper sub (e.g. v_D) = total signal = V_D+v_d
- NMOS: Three regions of operation
  - V_{DS} and V_{GS} normally positive valus
  - $V_{GS}<V_t$:cut off mode, I_{DS}=0 for any V_{DS}
  - $V_{GS}>V_t$:transistor is turned on
    - $V_{DS}< V_{GS}-V_t$: Triode Region $i_D = K\left[2(v_{GS}-V_t)v_{DS}-v_{DS}^2\right]$
    - $V_{DS}> V_{GS}-V_t$: Saturation Region $i_D = K\left[2(v_{GS}-V_t)^2\right]$
    - Boundary $v_{GS}-V_t = v_{DS}$ $\quad K=\frac{W}{L}\frac{KP}{2}$

MOSFET

- PMOS: Three regions of operation (interchange > and < from NMOS)
  - V_{DS} and V_{GS} Normally negative values
  - $V_{GS}>V_t$:cut off mode, I_{DS}=0 for any V_{DS}
  - $V_{GS}<V_t$:transistor is turned on
    - $V_{DS}> V_{GS}-V_t$: Triode Region $i_D = K\left[2(v_{GS}-V_t)v_{DS}-v_{DS}^2\right]$
    - $V_{DS}< V_{GS}-V_t$: Saturation Region $i_D = K\left[2(v_{GS}-V_t)^2\right]$
    - Boundary $v_{GS}-V_t = v_{DS}$ $\quad K=\frac{W}{L}\frac{KP}{2}$

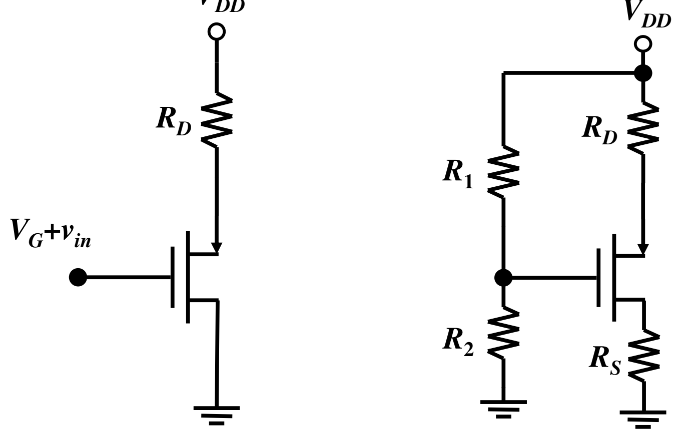
MOSFET Operating Regions



Bias Circuits

- Use load line to find Quiescent operating point.
- Remember no current flow through the gate.

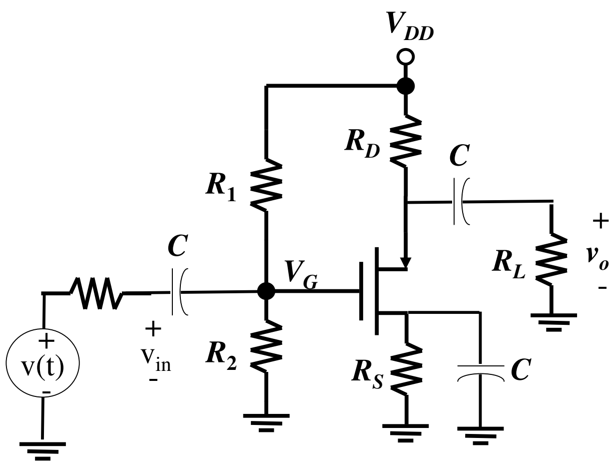
Fixed-plus Self-Bias CKT



MOSFET Circuit

- First look at DC case to find Q point
  - Use load line technique
  - All capacitors are open circuit
  - From Q-point, get g_m and r_d for small signal AC model
- AC Small signal analysis
  - DC source is AC ground (because there is no AC signal variation).
  - All capacitors are short circuit (unless otherwise specified).

Common Source Amplifier

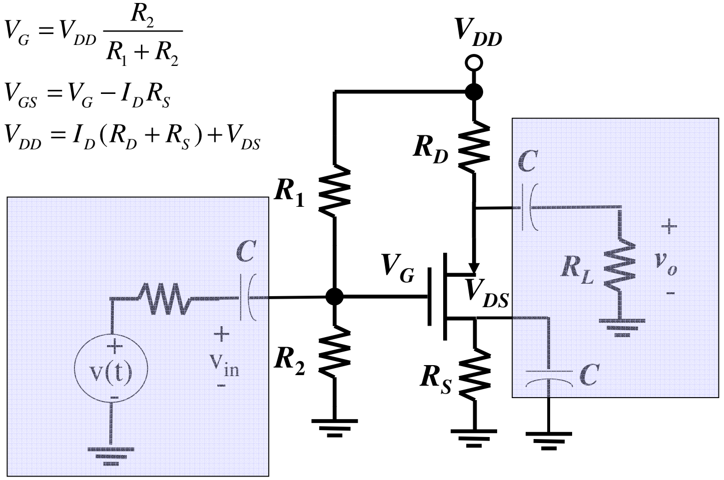


Step 1: find Q point

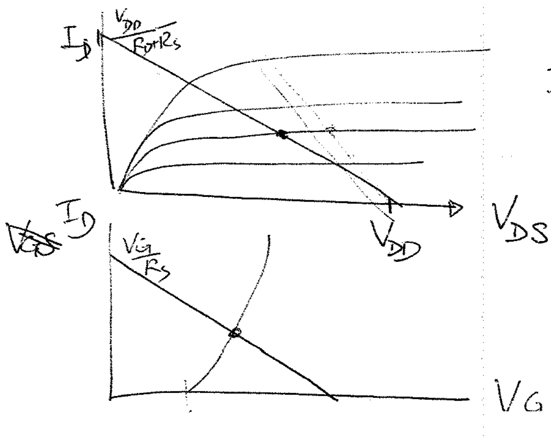
$$V_G = V_{DD}\frac{R_2}{R_1+R_2}$$

$$V_{GS} = V_G - I_D R_S$$

$$V_{DD} = I_D(R_D+R_S)+V_{DS}$$



Load line



$$I_D = \frac{V_{DD}-V_{DS}}{R_D+R_S}$$

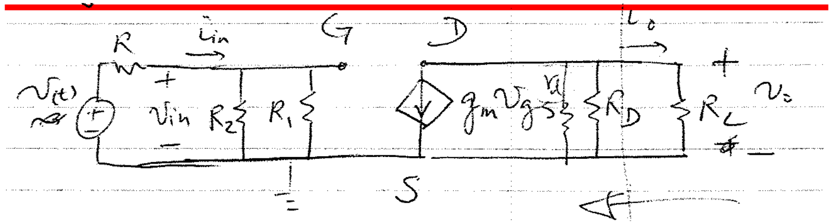
$$I_D = \frac{V_G - V_{GS}}{R_S}$$

$\Downarrow$

g_m.

From load lines, we get I_D → and hence g_m and r_d

Small Signal Model



$$v_g = v_{in}, v_s = 0 \rightarrow v_{gs} = v_{in}$$

$$v_o = \frac{R_L R_D}{R_L+R_D}(-g_m v_{gs})$$

$$A_v = \frac{v_o}{v_{in}} = -g_m\frac{R_L R_D}{R_L+R_D}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_1 R_2}{R_1+R_2}$$

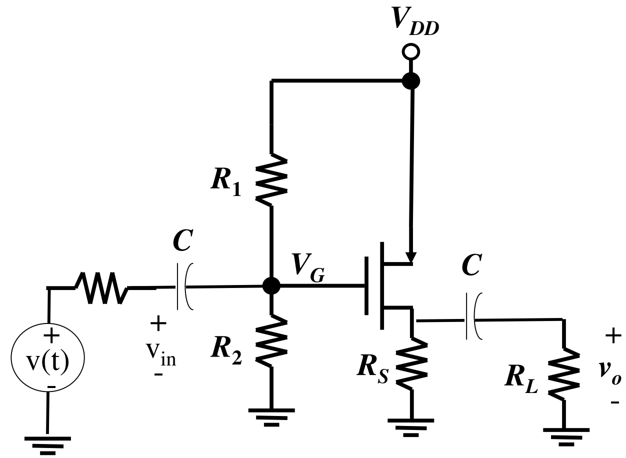
For output impedance R_{out}:

1. Turn off all independent sources.
2. Take away load impedance R_L

$$v_{in} = 0, v_{gs} = 0, g_m v_{gs} = 0$$

$$R_{out} = \frac{r_d R_D}{r_d + R_D}$$

Source Follower

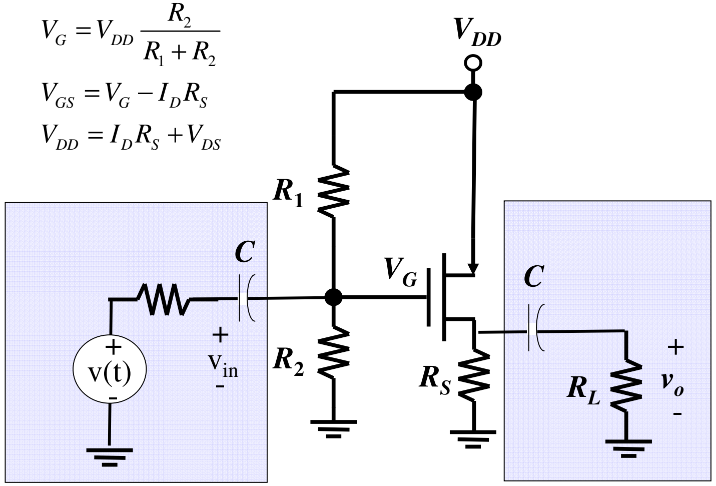


Step 1: find Q point

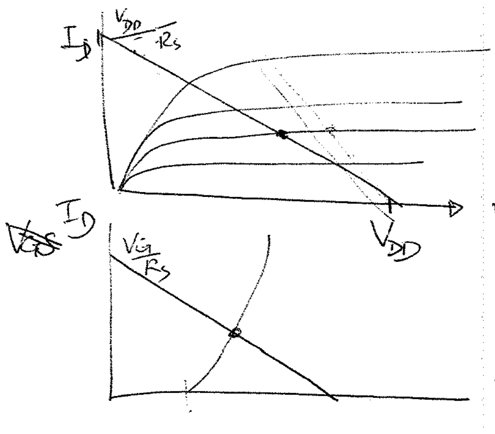
$$V_G = V_{DD}\frac{R_2}{R_1+R_2}$$

$$V_{GS} = V_G - I_D R_S$$

$$V_{DD} = I_D R_S + V_{DS}$$



Load line



$$I_D = \frac{V_{DD}-V_{DS}}{R_S}$$

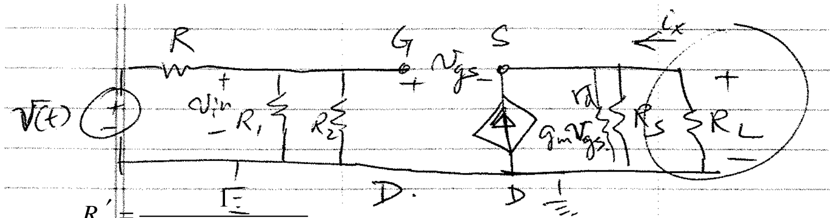
$$I_D = \frac{V_G-V_{GS}}{R_S}$$

$\Downarrow$

g_m.

From load lines, we get I_D → and hence g_m and r_d

Small Signal Model



$$R_L' = \frac{1}{r_d^{-1}+R_S^{-1}+R_L^{-1}}$$

$$v_{gs} = v_{in} - v_o$$

$$v_o = g_m v_{gs} R_L'$$

$$v_{in} = v_{gs}(1+g_m R_L')$$

$$A_v = \frac{v_o}{v_{in}} = \frac{g_m R_L'}{1+g_m R_L'}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{R_1 R_2}{R_1+R_2}$$

For output impedance R_{out}:

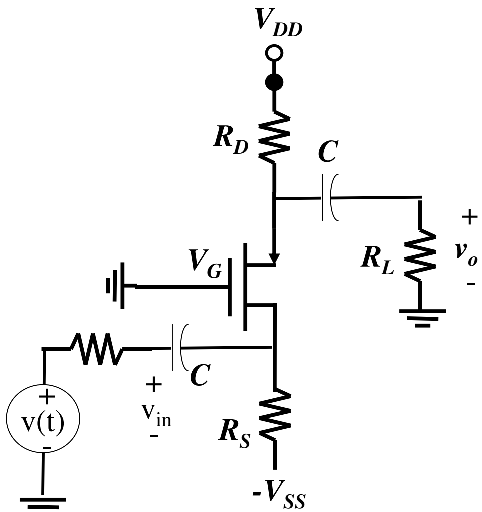
1. Turn off all independent sources.
2. Take away R_L
3. Add V_x and find i_x

$$v_x = v_s, v_g = 0, v_{gs} = -v_x$$

$$R_s' = \frac{r_d R_s}{r_d + R_s}, i_x = \frac{v_x}{R_s'} - g_m(-v_x) = v_x\left(R_s'^{-1} + g_m\right)$$

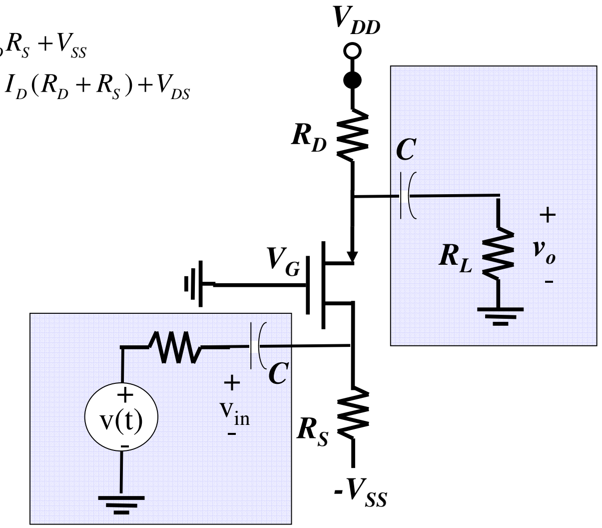
$$R_{out} = \frac{1}{g_m + r_d^{-1} + R_s^{-1}}$$

Common Gate Amplifier



Step 1: find Q point

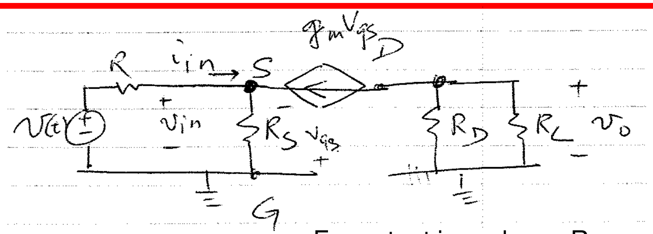
$$V_{GS} = 0 - I_D R_S + V_{SS}$$
$$V_{DD} + V_{SS} = I_D (R_D + R_S) + V_{DS}$$



Load line

The only difference in all three circuits are the intercepts at the axes.
Again from load lines, we get I_D → and hence g_m and r_d

Small Signal Model



$$R_L' = \frac{1}{R_L^{-1} + R_D^{-1}}$$
$$v_{gs} = -v_{in}$$
$$v_o = -g_m v_{gs} R_L'$$
$$A_v = \frac{v_o}{v_{in}} = g_m R_L'$$
$$i_{in} = -(g_m v_{gs} + \frac{v_{gs}}{R_s})$$
$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{g_m + R_s^{-1}}$$

For output impedance R_{out}:

1. Turn off all independent sources.
2. Take away R_L
3. Add V_x and find i_x

$$R' = \frac{RR_s}{R + R_s}$$
$$i_x = \frac{v_x}{R_D} + g_m v_{gs}$$
$$v_{gs} = -g_m v_{gs} R', \ \ but \ \ g_m R' \neq 1 \therefore v_{gs} = 0$$
$$R_{out} = R_D$$

Week 15

- OUTLINE
  - Need for Input Controlled Pull-Up
  - CMOS Inverter Analysis
  - CMOS Voltage Transfer Characteristic
  - Combinatorial logic circuits
  - Logic
  - Binary representations
  - Combinatorial logic circuits
- Reading
  - Chap 7-7.5
  - Supplementary Notes Chapter 4

Digital Circuits – Introduction

- Analog: signal amplitude is continuous with time.
- Digital: signal amplitude is represented by a restricted set of discrete numbers.
  - Binary: only two values are allowed to represent the signal: High or low (i.e. logic 1 or 0).
- Digital word:
  - Each binary digit is called a bit
  - A series of bits form a word
    - Byte is a word consisting of 8-bits
- Advantages of digital signal
  - Digital signal is more resilient to noise→ can more easily differentiate high (1) and low (0)
- Transmission
  - Parallel transmission over a bus containing n wires.
    - Faster but short distance (internal to a computer or chip)
  - Serial transmission (transmit bits sequentially)
    - Longer distance

Analog *vs.* Digital Signals

- **Most** (but not all) **observables are analog**

  think of analog vs. digital watches

but the most convenient way to represent & transmit information electronically is to use digital signals

think of telephony

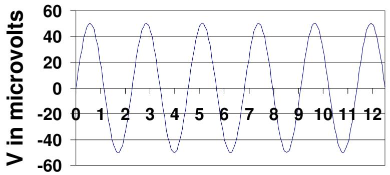
→ Analog-to-digital (A/D) & digital-to-analog (D/A) conversion is essential (and nothing new)

think of a piano keyboard

Analog Signal Example: Microphone Voltage

<u>Voltage with normal piano key stroke</u>

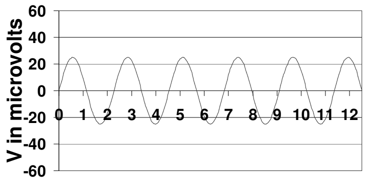
50 microvolt 440 Hz signal



t in milliseconds

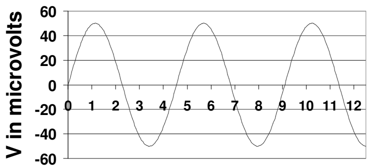
<u>Voltage with soft pedal applied</u>

25 microvolt 440 Hz signal



t in milliseconds

50 microvolt 220 Hz signal



t in milliseconds

Analog signal representing piano key A, below middle C (220 Hz)

Digital Signal Representations

Binary numbers can be used to represent any quantity.

We generally have to agree on some sort of "code", and the dynamic range of the signal in order to know the form and the number of binary digits ("bits") required.

<u>Example 1</u>: Voltage signal with maximum value 2 Volts

- Binary two (**10**) could represent a 2 Volt signal.
- To encode the signal to an accuracy of 1 part in 64 (1.5% precision), 6 binary digits ("bits") are needed

<u>Example 2</u>: Sine wave signal of known frequency and maximum amplitude 50 µV; 1 µV "resolution" needed.

Decimal Numbers: Base 10

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Example:

$3271 = (3x10^3) + (2x10^2) + (7x10^1) + (1x10^0)$

This is a four-digit number. The left hand most number (3 in this example) is often referred as the most significant number and the right most the least significant number (1 in this example).

Numbers: positional notation

- Number Base B $\Rightarrow$ B symbols per digit:
  - Base 10 (Decimal): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  - Base 2 (Binary): 0, 1
- Number representation:
  - $d_{31}d_{30} \ldots d_1d_0$ is a 32 digit number
  - value = $d_{31} \times B^{31} + d_{30} \times B^{30} + \ldots + d_1 \times B^1 + d_0 \times B^0$
- Binary: 0,1 (In binary digits called "bits")

$$11010 = 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$$
$$= 16 + 8 + 2$$
$$= 26$$

  - Here 5 digit binary # turns into a 2 digit decimal #

Hexadecimal Numbers: Base 16

- Hexadecimal:

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
  - Normal digits + 6 more from the alphabet
- Conversion: Binary$\Leftrightarrow$Hex
  - 1 hex digit represents 16 decimal values
  - 4 binary digits represent 16 decimal values
  - $\Rightarrow$1 hex digit replaces 4 binary digits

Digital Signal Representations

Binary numbers can be used to represent any quantity.

We generally have to agree on some sort of "code", and the dynamic range of the signal in order to know the form and the number of binary digits ("bits") required.

Example 1: Voltage signal with maximum value 2 V and minimum of 0 V.

- Binary two (**10**) could represent a 2 Volt signal.
- To encode the signal to an accuracy of 1 part in 64 (1.5% precision), 6 binary digits ("bits") are needed

Example 2: Sine wave signal of known frequency and maximum amplitude 50 μV; 1 μV "resolution" needed.

Resolution

- The size of the smallest element that can be separated from neighboring elements. The term is used to describe imaging systems, the frequency separation achieved by spectrometers, and so on.

Decimal-Binary Conversion

- Decimal to Binary
  - Repeated Division By 2
    - Consider the number 2671.
  - Subtraction – if you know your 2^N values by heart.
- Binary to Decimal conversion

$$110001_2 = 1x2^5 + 1x2^4 + 0x2^3 + 0x2^2 + 0x2^1 + 1x2^0$$
$$= 32_{10} + 16_{10} + 1_{10}$$
$$= 49_{10}$$
$$= 4x10^1 + 9x10^0$$

Example 2 (continued)

Possible digital representation for the sine wave signal:

Analog representation: Amplitude in μV	**Digital representation:** Binary number
1	000001
2	000010
3	000011
4	000100
5	000101
8	001000
16	010000
32	100000
50	**110010**
63	111111

Binary Representation

- N bit can represent 2^N values: typically from 0 to 2^N-1
  - 3-bit word can represent 8 values: e.g. 0, 1, 2, 3, 4, 5, 6, 7
- Conversion
  - Integer to binary
  - Fraction to binary ($13.5_{10}=1101.1_2$ and $0.392_{10}=0.011001_2$)
- Octal and hexadecimal

- Logic gates
  - Combine several logic variable inputs to produce a logic variable output
- Memory
  - Memoryless: output at a given instant depends the input values of that instant.
  - Momory: output depends on previous and present input values.

Boolean algebras

- Algebraic structures
  - "capture the essence" of the logical operations AND, OR and NOT
  - corresponding set for theoretic operations intersection, union and complement
  - named after George Boole, an English mathematician at University College Cork, who first defined them as part of a system of logic in the mid 19th century.
  - Boolean algebra was an attempt to use algebraic techniques to deal with expressions in the propositional calculus.
  - Today, Boolean algebras find many applications in electronic design. They were first applied to switching by Claude Shannon in the 20th century.

Boolean algebras

- The operators of Boolean algebra may be represented in various ways. Often they are simply written as AND, OR and NOT.
- In describing circuits, NAND (NOT AND), NOR (NOT OR) and XOR (eXclusive OR) may also be used.
- Mathematicians often use + for OR and · for AND (since in some ways those operations are analogous to addition and multiplication in other algebraic structures) and represent NOT by a line drawn above the expression being negated.

Boolean Algebra

- NOT operation (inverter)

$$A \bullet \overline{A} = 0$$

$$A + \overline{A} = 1$$

- AND operation

$$A \bullet A = A$$

$$A \bullet 1 = A$$

$$A \bullet 0 = 0$$

$$A \bullet B = B \bullet A$$

$$(A \bullet B) \bullet C = A \bullet (B \bullet C)$$

- OR operation

$$A + A = A$$

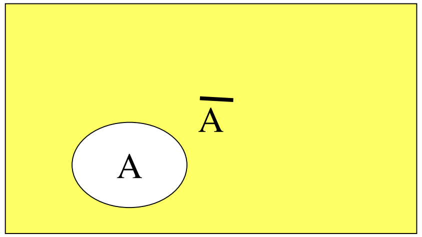
$$A + 1 = 1$$

$$A + 0 = A$$

$$A + B = B + A$$

$$(A + B) + C = A + (B + C)$$

Graphic Representation

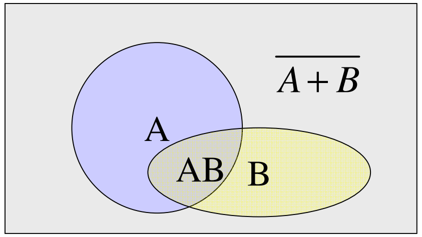


$$A\bullet\overline{A}=0$$

$$A+\overline{A}=1$$

Full square = complete set =1
Yellow part = NOT(A) =$\overline{\text{A}}$
White circle = A

Graphic Representation



$$A\oplus B=A\overline{B}+\overline{A}B=(A+B)\bullet(\overline{A}+\overline{B})=\overline{A\bullet B+\overline{A+B}}$$

Exclusive OR=yellow and blue part – intersection/overlap part
=exactly when only one of the input is true

Boolean Algebra

- Distributive Property

  $$A\bullet(B+C)=A\bullet B+A\bullet C$$

  $$(A+B)\bullet C=(A+B)\bullet(A+C)$$

- De Morgan's laws

  $$\overline{A+B}=\overline{A}\bullet\overline{B}$$

  $$\overline{A\bullet B}=\overline{A}+\overline{B}$$

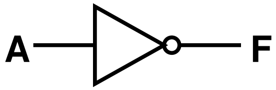
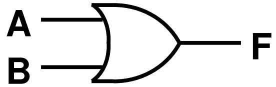
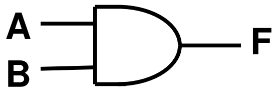
- An excellent web site to visit
  – http://en.wikipedia.org/wiki/Boolean_algebra

Examples

$$F = A\bullet\overline{B}\bullet C + A\bullet B\bullet C + (C+D)\bullet(\overline{D}+E)$$

$$F = C\bullet(A+\overline{D}+E) + D\bullet E$$

Logic Functions, Symbols, & Notation

NAME	SYMBOL	NOTATION	TRUTH TABLE
"NOT"		$F = \overline{A}$	see below
"OR"		$F = A+B$	see below
"AND"		$F = A \cdot B$	see below

"NOT":

A	F
0	1
1	0

"OR":

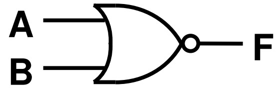
A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

"AND":

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

Logic Functions, Symbols, & Notation 2

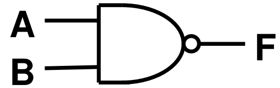
"NOR"



$F = \overline{A+B}$

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

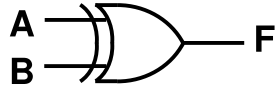
"NAND"



$F = \overline{A \cdot B}$

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

"XOR"
(exclusive OR)

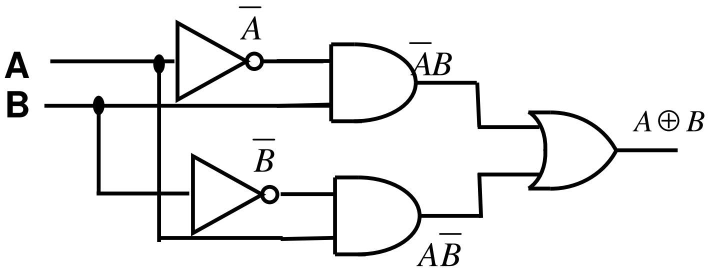


$F = A \oplus B$

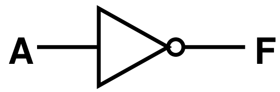
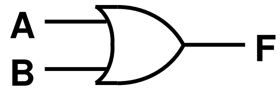
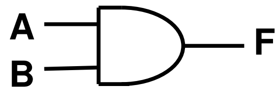
A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

Circuit Realization

$$A \oplus B = A\overline{B} + \overline{A}B = (A+B)\cdot(\overline{A}+\overline{B}) = \overline{A \cdot B + \overline{A+B}}$$



Logic Functions, Symbols, & Notation

NAME	SYMBOL	NOTATION	TRUTH TABLE
"NOT"		$F = \overline{A}$	see below
"OR"		$F = A+B$	see below
"AND"		$F = A \cdot B$	see below

"NOT":

A	F
0	1
1	0

"OR":

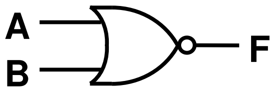
A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

"AND":

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

Logic Functions, Symbols, & Notation 2

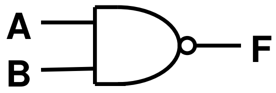
"NOR"



$F = \overline{A+B}$

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

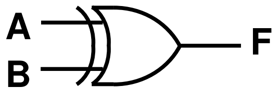
"NAND"



$F = \overline{A \bullet B}$

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

"XOR"
(exclusive OR)



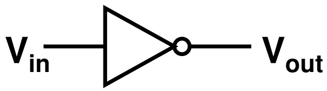
$F = A \oplus B$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

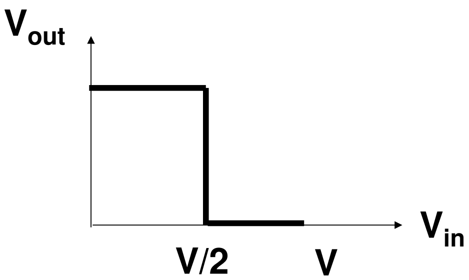
Fan in/Fan out

- Complex digital operations are formed with a variety of gates interconnected to yield the desired logic function.
- Sometimes a number of inputs are connected to one gate input and output of a gate may be connected to a number of gates.
- Fan-in: the maximum number of logic gates that can be connected at the input of a gate without altering its performance.
- Fan-out: the maximum number of logic gates that can be connected to the output of a gate without altering its performance.
- Typical fan-in and fan-out numbers are 3.

Inverter = NOT Gate

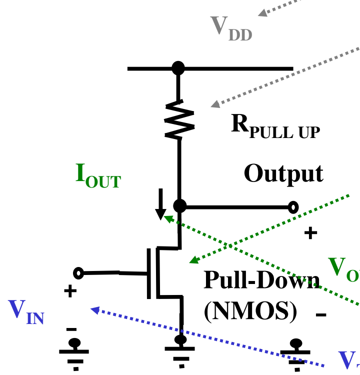


Ideal Transfer Characteristics



Terminology for a Logic Circuit

V_{DD} = Power supply voltage (D is from Drain) we do not draw the symbol.



Pull-Up Network = Set of devices used to carry current from the power supply to the output node to charge the output node to the power supply voltage.

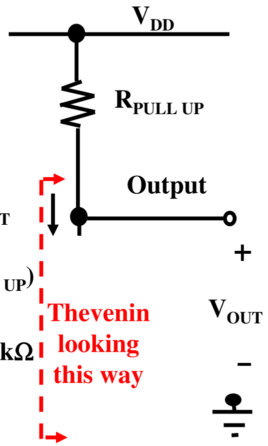
Pull-Down Network = Set of devices used to carry current from the output node to ground to discharge the output node to ground.

I_{OUT} = Current for the device under study.

V_{TD} = Threshold Voltage value of V_{IN} at which the Pull-Down (NMOS transistor) begins to conduct.

$V_{OUT\text{-}SAT\text{-}D}$ = Value of V_{OUT} beyond which the current $I_{OUT\text{-}D}$ saturates at the (drain) current saturation value $I_{OUT\text{-}SAT\text{-}D}$.

Thevenin Model For Pull-Up Device



$V_{THEVENIN} = V_{DD}$

$I_{OUT\ SHORT\ CIRCUIT} = (V_{DD}/R_{PULL\ UP})$

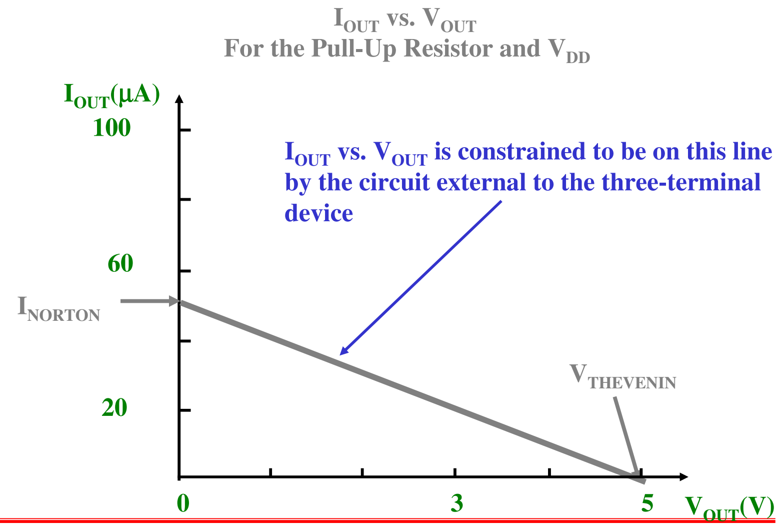
Example:

$V_{DD} = 5V$ and $R_{PULL\ UP} = 100k\Omega$

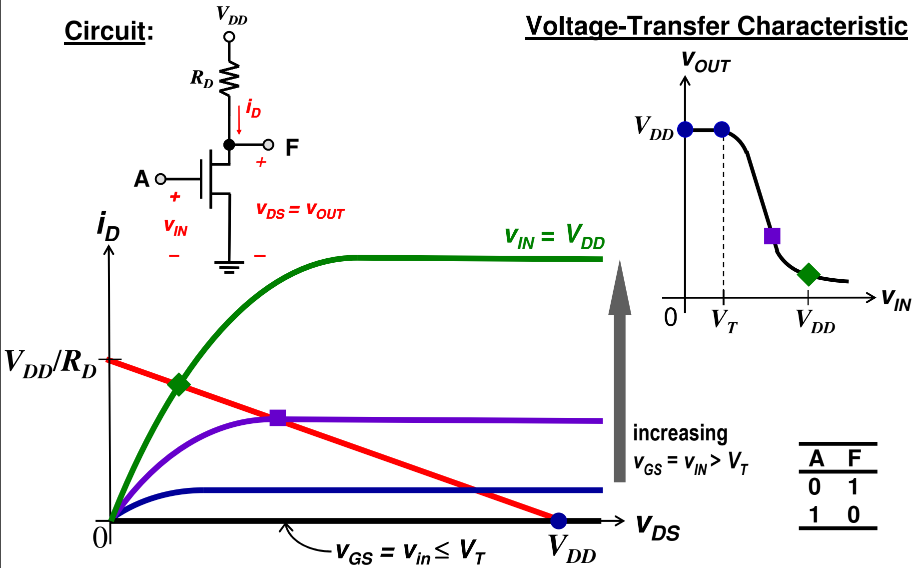
$V_{THEVENIN} = 5V$

$I_{OUT\ SHORT\ CIRCUIT} = 50\ \mu A$

Load Line For Pull-Up Device



NMOS Resistor Pull-Up



A	F
0	1
1	0

Disadvantages of NMOS Logic Gates

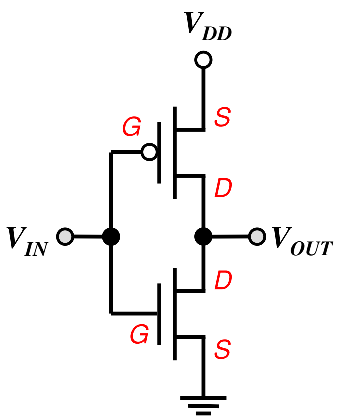
- Large values of $\boldsymbol{R_D}$ are required in order to
  - achieve a low value of V_{OL}
  - keep power consumption low

→ Large resistors are needed, but these take up a lot of space.

- One solution is to replace the resistor with an NMOSFET that is always on.

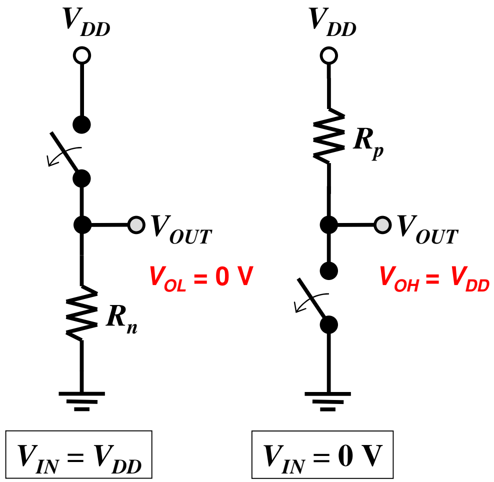
The CMOS Inverter: Intuitive Perspective

CIRCUIT

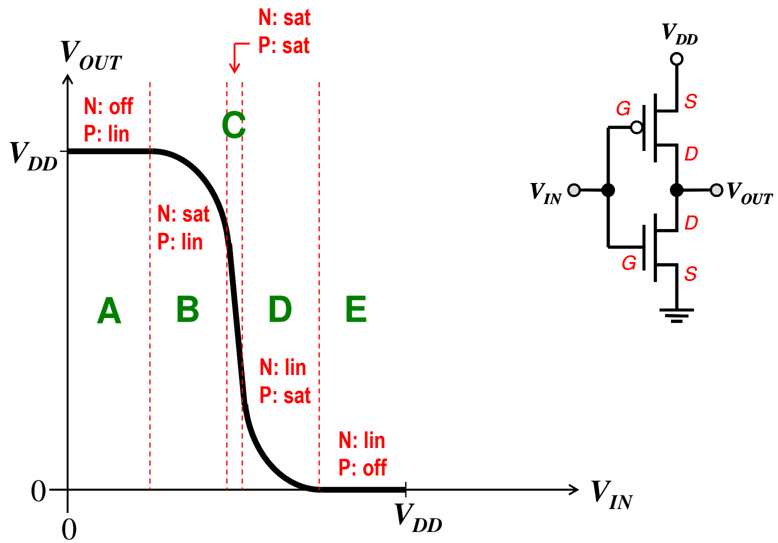


Low static power consumption, since one MOSFET is always off in steady state

SWITCH MODELS



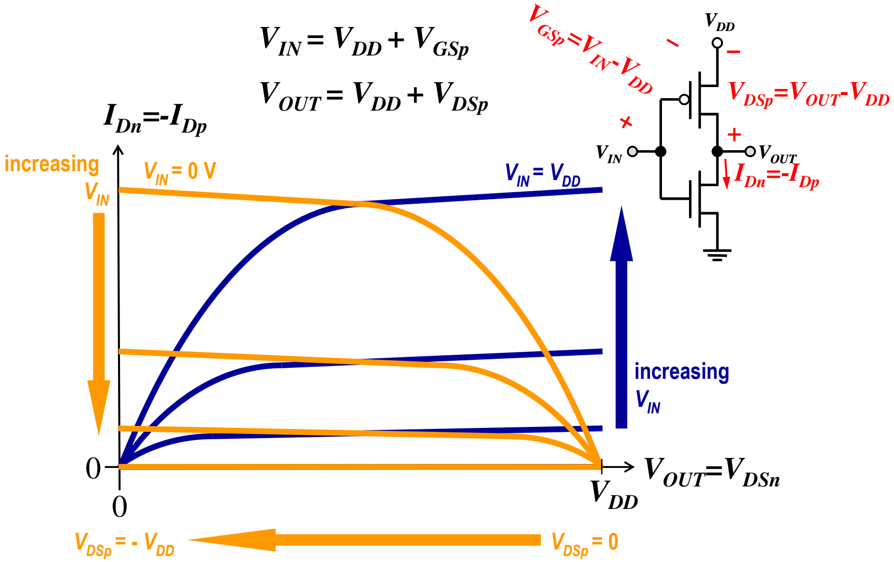
CMOS Inverter Voltage Transfer Characteristic



CMOS Inverter Load-Line Analysis

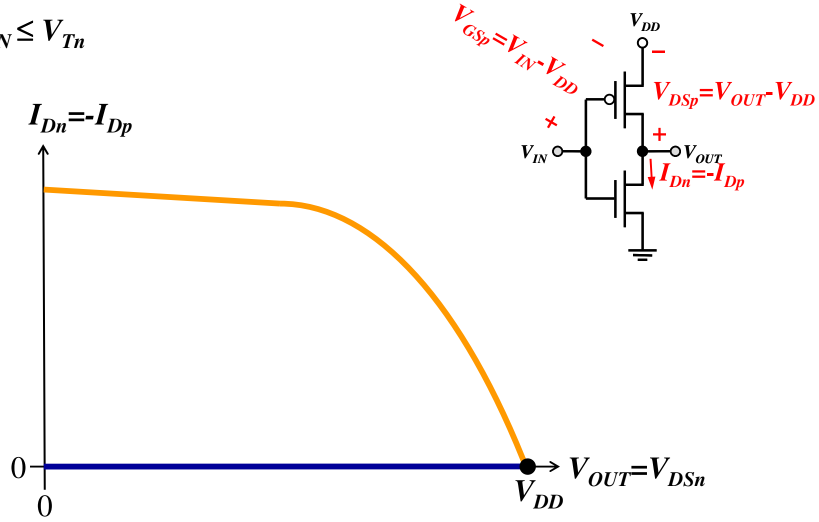
$$V_{IN} = V_{DD} + V_{GSp}$$

$$V_{OUT} = V_{DD} + V_{DSp}$$



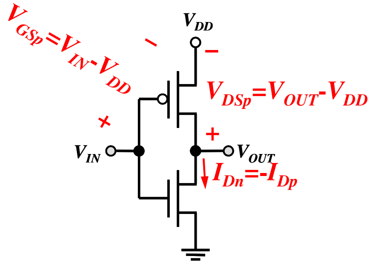
CMOS Inverter Load-Line Analysis: Region A

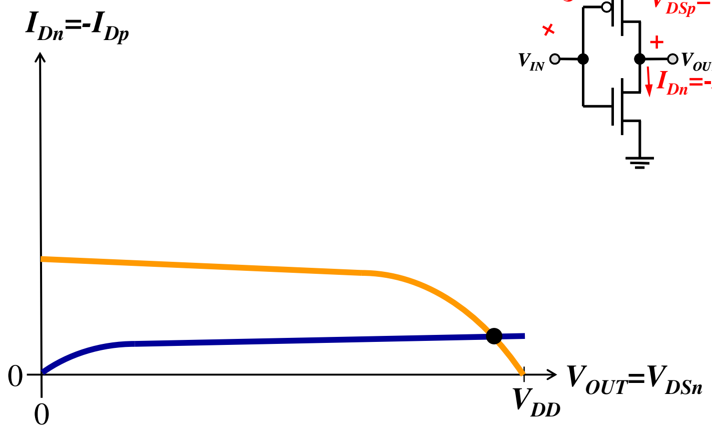
$V_{IN} \leq V_{Tn}$



CMOS Inverter Load-Line Analysis: Region B

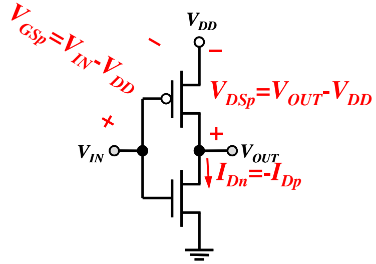
$V_{DD}/2 > V_{IN} > V_{Tn}$

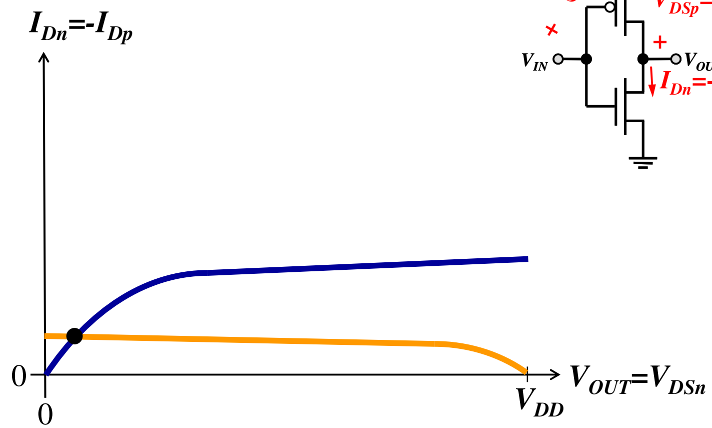




CMOS Inverter Load-Line Analysis: Region D

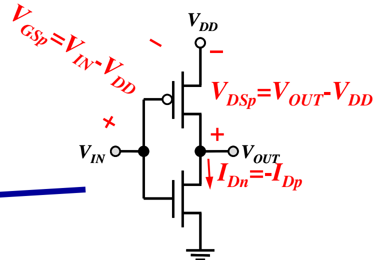
$V_{DD} - |V_{Tp}| > V_{IN} > V_{DD}/2$

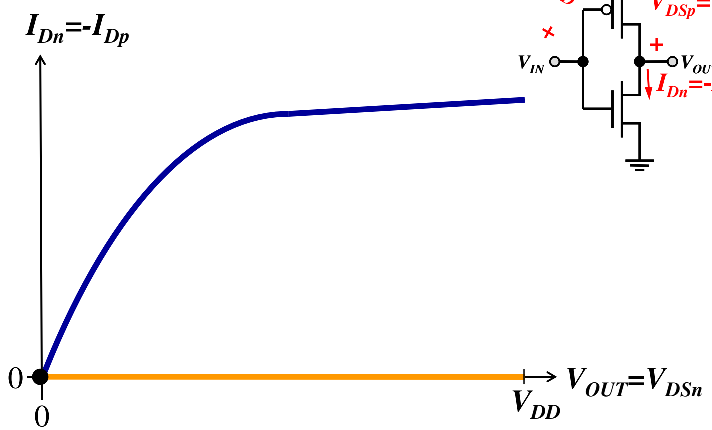




CMOS Inverter Load-Line Analysis: Region E

$V_{IN} > V_{DD} - |V_{Tp}|$

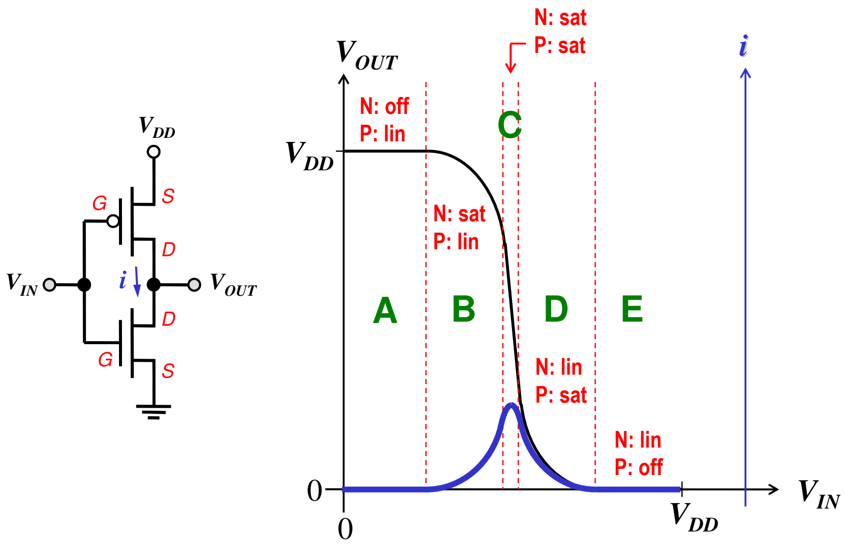




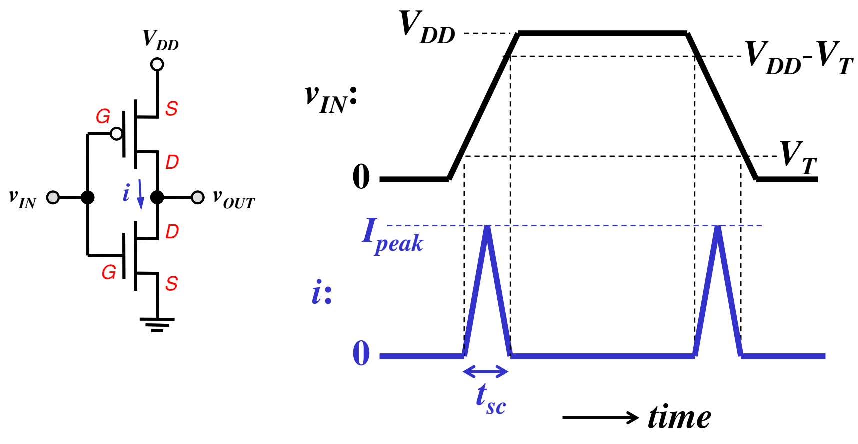
Features of CMOS Digital Circuits

- The output is always connected to V_{DD} or **GND** in steady state
  - → Full logic swing; large noise margins
  - → Logic levels are not dependent upon the relative sizes of the devices ("ratioless")
- There is no direct path between V_{DD} and **GND** in steady state
  - → no static power dissipation

The CMOS Inverter: Current Flow during Switching



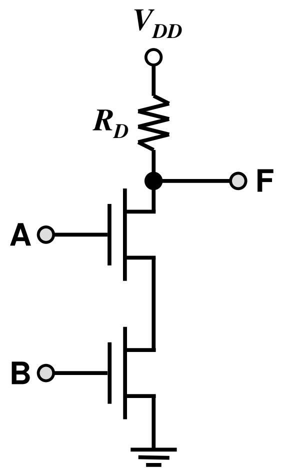
Power Dissipation due to Direct-Path Current



Energy consumed per switching period: $E_{dp} = t_{sc} V_{DD} I_{peak}$

NMOS NAND Gate

- Output is low only if both inputs are high

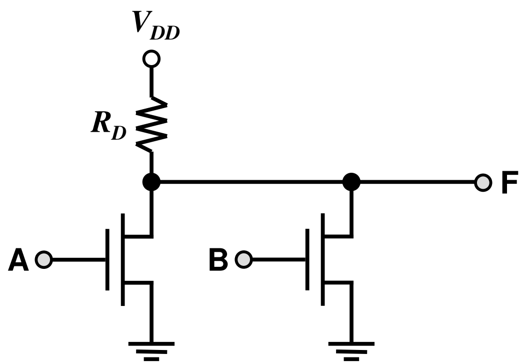


Truth Table

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

NMOS NOR Gate

- Output is low if either input is high

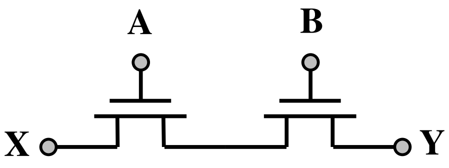


Truth Table

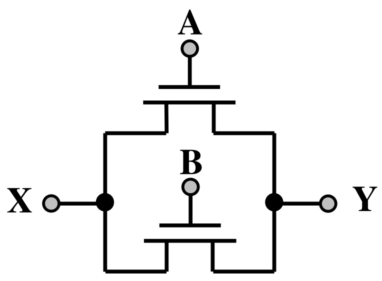
A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

N-Channel MOSFET Operation

An NMOSFET is a closed switch when the input is high



$Y = X$ if A and B

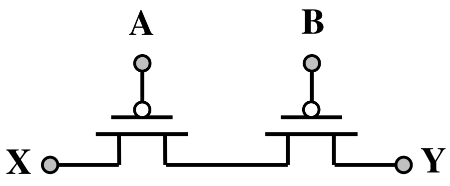


$Y = X$ if A or B

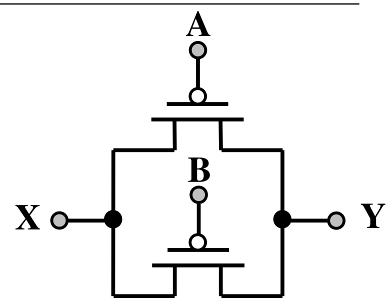
NMOSFETs pass a "strong" **0** but a "weak" **1**

P-Channel MOSFET Operation

A PMOSFET is a closed switch when the input is low



$Y = X$ if $\overline{A}$ and $\overline{B}$
$= (\overline{A + B})$

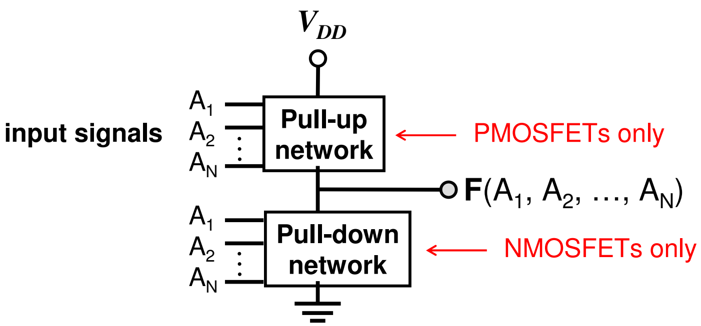


$Y = X$ if $\overline{A}$ or $\overline{B}$
$= (\overline{AB})$

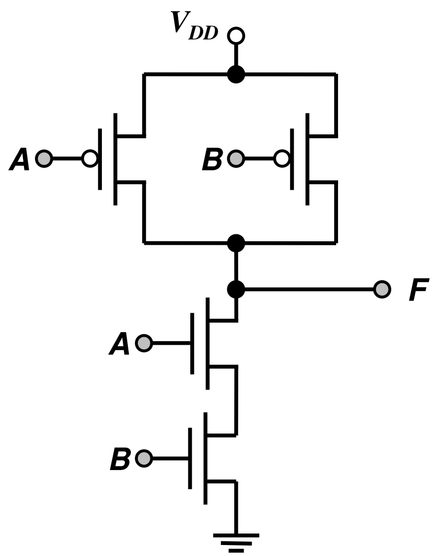
PMOSFETs pass a "strong" **1** but a "weak" **0**

Pull-Down and Pull-Up Devices

- In CMOS logic gates, NMOSFETs are used to connect the output to **GND**, whereas PMOSFETs are used to connect the output to V_{DD}.
  - An NMOSFET functions as a ***pull-down device*** when it is turned on (gate voltage = V_{DD})
  - A PMOSFET functions as a ***pull-up device*** when it is turned on (gate voltage = ***GND***)

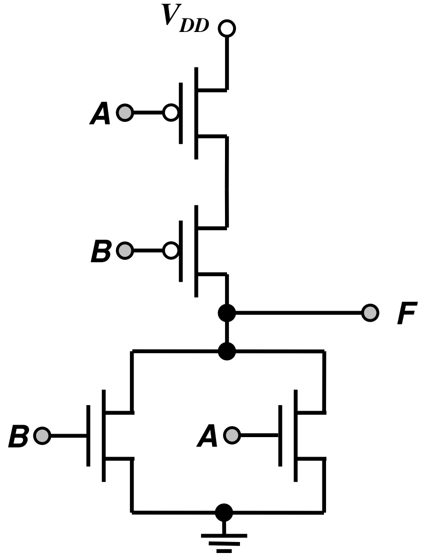


CMOS NAND Gate



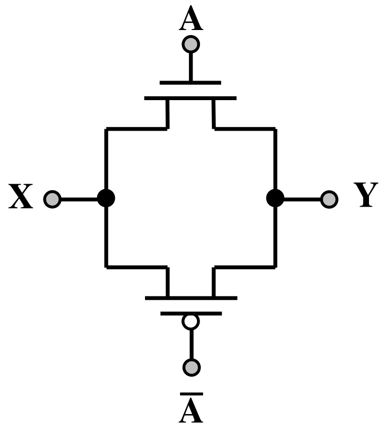
A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

CMOS NOR Gate



A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

CMOS Pass Gate



$Y = X$ if A