PRINCIPLES OF GEOPHYSICS

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2018

Free Advice

Group study. is strongly encouraged. Interaction with peers and instructors is very beneficial to your learning and digesting the material. Make good use of it. Discuss the problems with the TA.

Ask questions. It is recommended that you put in at least 2 hours of study for each 1 hour of lecture. It is very useful to read ahead, before the material comes up in the classroom. Feel free to ask questions, make comments or just express your opinions.

Think. The assigned problem sets present an opportunity for you to think about and apply the material presented in the lectures and in some cases to learn new material. Start them early in the week so that you can think about them and exercise your creative potential. *Note that solving a problem does not only consist of inserting numbers into equations and crunching out new numbers.* Strive for 100% on the assignments utilizing all the resources and opportunities available to you.

Expanding your mind. Learning is a process of expanding your **mind**. This is attained by developing new connections between neurons in your brain and strengthening selected neural pathways. Be persistent and don't give up when you come to the inevitable hard sections. For some of you that might be at the beginning, for others the resistance might increase with time. Hang in there.

Don't Copy. Copying the solutions to assigned problems from other sources is not only discouraged. It is not professional, it does not reflect well on you as a future geophyscist, and it is detrimental to your character and professional career. It is an activity that shows disrespect for you, for your classmates, and for your instructors. If you have questions, ask the instructor or the TAs.

If you follow the lectures, keep up with the homework problems, and ask questions to your classmates, TAs and the instructor, you will be on your way to mastering the material. Persistence and motivation are the only prerequisites to success. Make it a game to master new skills and knowledge. Be courageous in facing new challenges.

Measured objectively, what a man can wrest from Truth by passionate striving is utterly infinitesimal. But the striving frees us from the bonds of the self and makes us comrades of those who are the best and the greatest. ***Albert Einstein***

INTRODUCTION

Geophysics is an interdisciplinary physical science concerned with the nature of the earth and its environment and as such seeks to apply the knowledge and techniques of physics, mathematics and chemistry to understand the structure and dynamic behavior of the earth and its environment. The required sequence of Mathematics, Physics and Geophysics courses is designed to provide a basic structure on which to build a program with science electives normally selected from Geology, Astronomy, Oceanography, Mathematics, Physics and Chemistry courses.

Geophysics is the science which deals with investigating the Earth, using the methods and techniques of Physics. The physical properties of earth materials (rocks, air, and water masses) such as density, elasticity, magnetization, and electrical conductivity all allow inference about those materials to be made from measurements of the corresponding physical fields - gravity, seismic waves, magnetic fields, and various kinds of electrical fields. Because Geophysics incorporates the sciences of Physics, Mathematics, Geology (and therefore Chemistry) it is a truly multidisciplinary physical science.

The two great divisions of Geophysics conventionally are labeled as **Exploration Geophysics**, and **Global Geophysics**. In Global Geophysics, we study earthquakes, the main magnetic field, physical oceanography, studies of the Earth's thermal state and meteorology (amongst others!). In Exploration Geophysics, physical principles are applied to the search for, and evaluation of, resources such as oil, gas, minerals, water and building stone. Exploration geophysicists also work in the management of resources and the associated environmental issues. Geophysics contributes to an understanding of the internal structure and evolution of the Earth, earthquakes, the ocean and many other physical phenomena. There are many divisions of geophysics, including oceanography, atmospheric physics, climatology, petroleum geophysics, environmental geophysics, engineering geophysics and mining geophysics.

Geophysical Exploration Techniques

Geophysical methods are divided into two types : **Active** and **Passive**

<u>**Passive methods**</u> (Natural Sources): Incorporate measurements of natural occurring fields or properties of the earth. Ex. SP, Magnetotelluric (MT), Telluric, Gravity, Magnetic.

<u>**Active Methods**</u> (Induced Sources) : A signal is injected into the earth and then measure how the earth respond to the signal. Ex. DC. Resistivity, Seismic Refraction, IP, EM, Mise-A-LA-Masse, GPR.

Common Applications

- oil and gas exploration
- mineral exploration
- diamond exploration (kimberlites)
- hydrogeology
- geotechnical and engineering studies
- tectonic studies
- earthquake hazard assessment
- archaeology

Table 1.1 Geophysical methods and their main applications

Geophysical method	Chapter number	Dependent physical property	Applications (see key below) 1	2	3	4	5	6	7	8	9	10
Gravity	2	Density	**P**	**P**	s	s	s	s	!	!	s	!
Magnetic	3	Susceptibility	**P**	**P**	**P**	s	!	m	!	**P**	**P**	!
Seismic refraction	4,5	Elastic moduli; density	**P**	**P**	m	**P**	s	s	!	!	!	!
Seismic reflection	4,6	Elastic moduli; density	**P**	**P**	m	s	s	m	!	!	!	!
Resistivity	7	Resistivity	m	m	**P**	**P**	**P**	**P**	**P**	s	**P**	m
Spontaneous potential	8	Potential differences	!	!	**P**	m	**P**	m	m	m	!	!
Induced polarization	9	Resistivity; capacitance	m	m	**P**	m	s	m	m	m	m	m
Electromagnetic (EM)	10	Conductance; inductance	s	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	m
EM–VLF	11	Conductance; inductance	m	m	**P**	m	s	s	s	m	m	!
EM – ground penetrating radar	12	Permitivity; conductivity	!	!	m	**P**	**P**	**P**	s	**P**	**P**	**P**
Magneto-telluric	11	Resistivity	s	**P**	**P**	m	m	!	!	!	!	!

P = primary method; s = secondary method; m = may be used but not necessarily the best approach, or has not been developed for this application; (!) = unsuitable

Applications

1 Hydrocarbon exploration (coal, gas, oil)
2 Regional geological studies (over areas of 100s of km^2)
3 Exploration/development of mineral deposits
4 Engineering site investigations
5 Hydrogeological investigations
6 Detection of sub-surface cavities
7 Mapping of leachate and contaminant plumes
8 Location and definition of buried metallic objects
9 Archaeogeophysics
10 Forensic geophysics

CHAPTER 1

FUNDAMENTAL CONSIDERATIONS

- Stress - Strain Relationship

- Elastic Coefficients

- Seismic Waves

- Huygen and Fermat principles

- Snell's Law in Refraction

Problem Set – 1

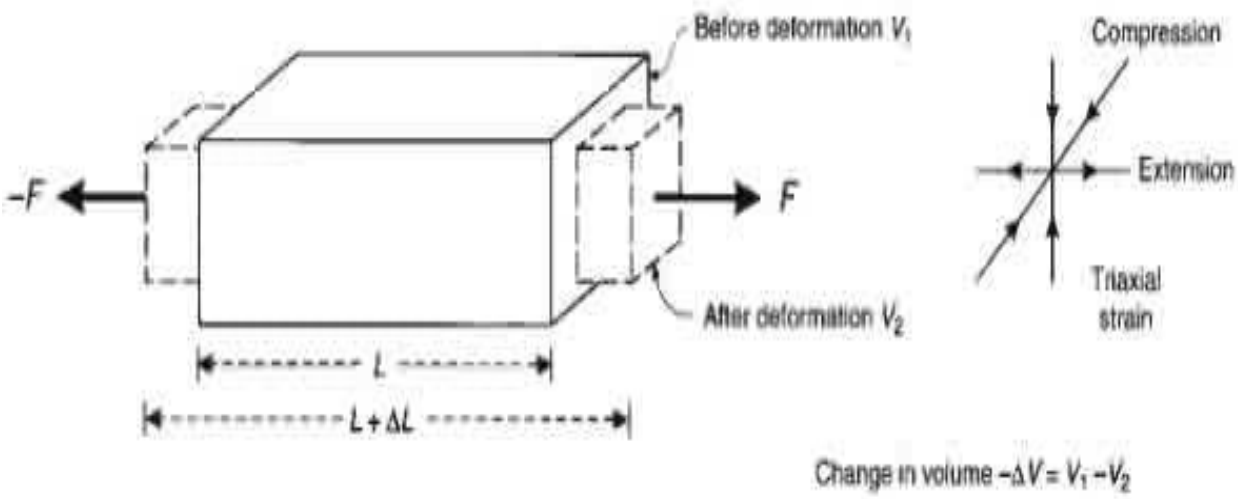
THEORY OF ELASTICITY

Stress is the ratio of applied force F to the area across which it is acts.

Strain is the deformation caused in the body, and is expressed as the ratio of change in length (or volume) to original length (or volume).

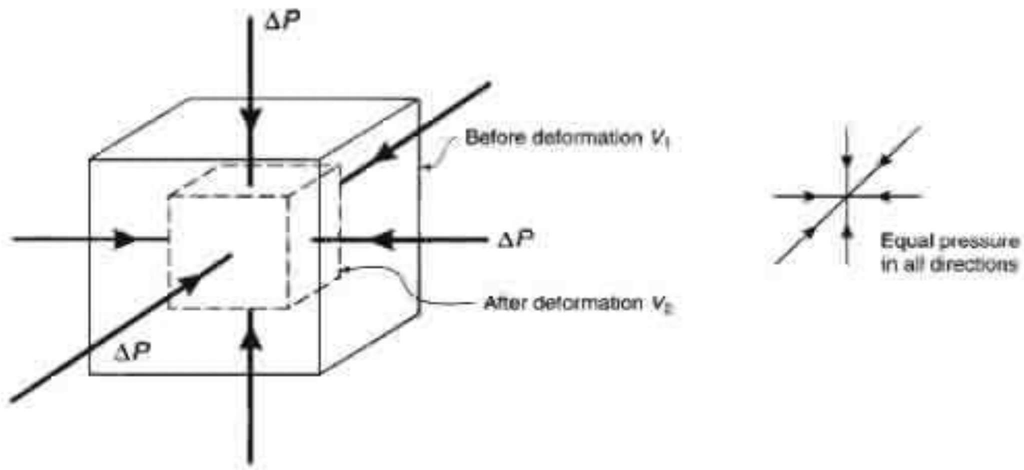
Triaxial Stress

Stresses act along three orthogonal axes, perpendicular to faces of solid, e.g. stretching a bar:



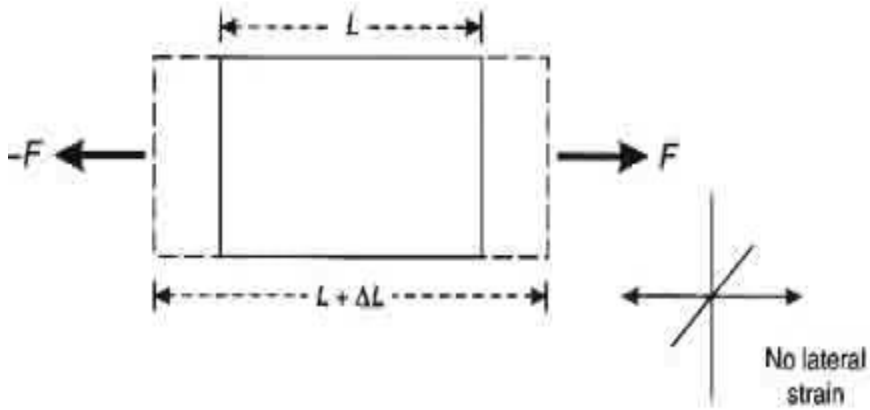
Pressure

Forces act equally in all directions perpendicular to faces of body, e.g. pressure on a cube in water:



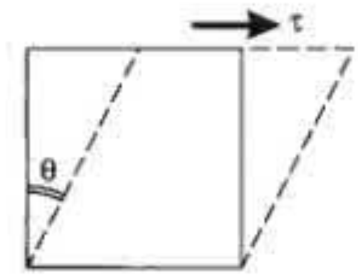
Strain Associated with Seismic Waves Inside a uniform solid, two types of strain can propagate as waves:

Axial Stress : Stresses act in one direction only, e.g. if sides of bar fixed:



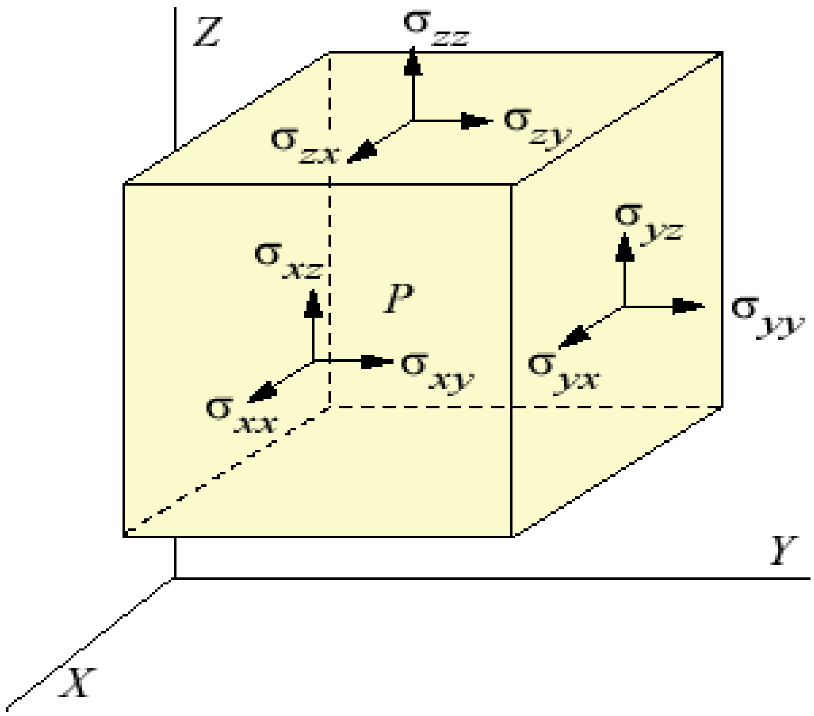
- Change in volume of solid occurs.
- Associated with P wave propagation

Shear Stress : Stresses act parallel to face of solid, e.g. pushing along a table:



- No change in volume.
- Fluids such as water and air cannot support shear stresses.
- Associated with S wave propagation.

Stresses on a solid in 3 dimensions



Stress = Force applied to a body per unit area (s = **F**/*dS*).
The stress can be expressed in two sets of components:
-Normal stress (n), perpendicular to the surface of the body (c.f. pressure) -
Shear stress (t) acting parallel to the surface
of the body For each surface one can define 3 orthogonal
components of stress. The surfaces themselves can be defined as 3 orthogonal components _ definition of **9** components of
stress (direction of the force and direction of the surface on which it acts).

For each body, it is possible to define 3 axes for which shear stresses are zero and only the normal stresses exist (using geometrical transformations). These axes are called the **principal axes**, and the corresponding normal stresses are the **principal stresses**.

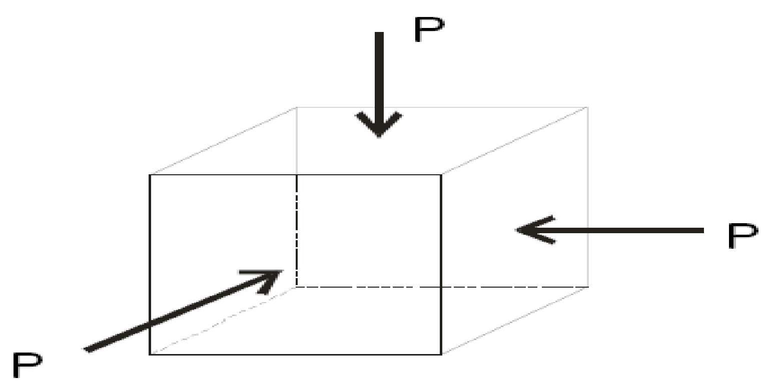
If all 3 principal stresses are equal, the body is subjected to a **pressure** (lithostatic pressure in the case of solid rock).

Pressure = (sum of principal stresses)/3

Conventions for directions:

Stresses towards the *interior*: **compression**

Stresses towards the *exterior*: **tension** (extension, dilatation)

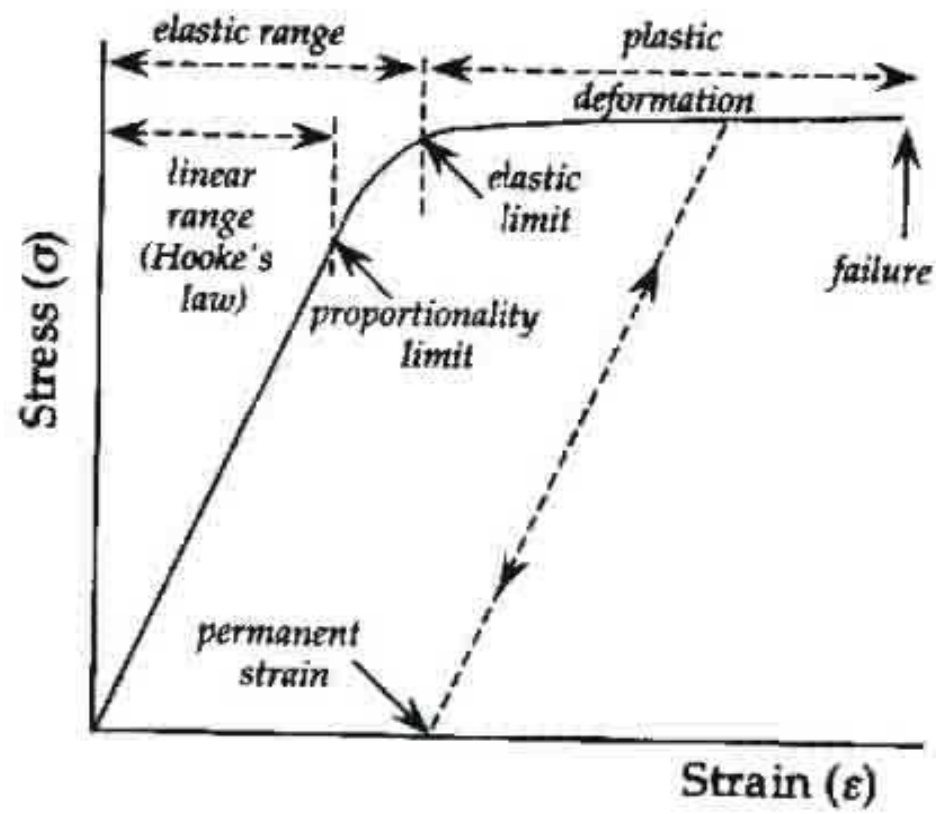


$$\sigma_{ij} \neq 0 \text{ if } i = j$$
$$\sigma_{ij} = 0 \text{ if } i \neq j$$
$$(\sigma_{11} + \sigma_{22} + \sigma_{33}) / 3 = -\mathbf{P}$$

Hooke's Law

Hooke's Law essentially states that stress is proportional to strain.

- At low to moderate strains: Hooke's Law applies and a solid body is said to behave <u>elastically</u>, i.e. will return to original form when stress removed.



- At high strains: the <u>elastic limit</u> is exceeded and a body deforms in a <u>plastic</u> or <u>ductile</u> manner: it is unable to return to its original shape, being permanently strained, or damaged.

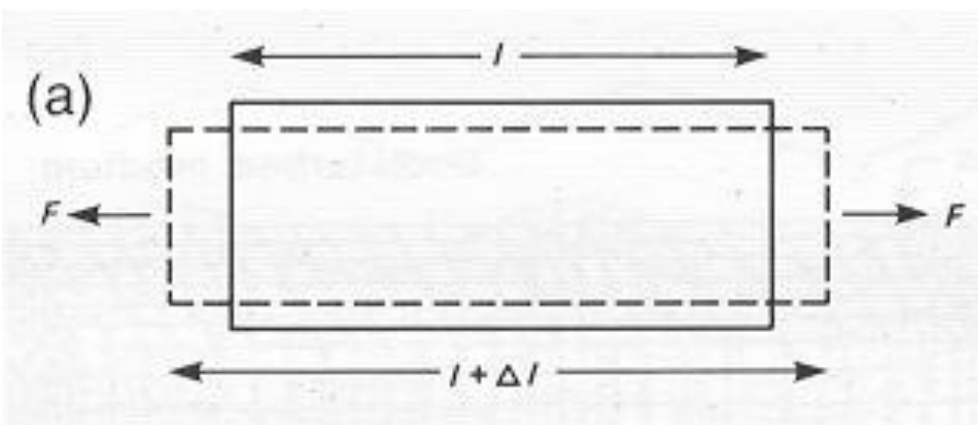
- At very high strains: a solid will fracture, e.g. in earthquake faulting.

Constant of proportionality is called the modulus, and is ratio of stress to strain, e.g. Young's modulus in triaxial strain.

Elastic Moduli (Constants)

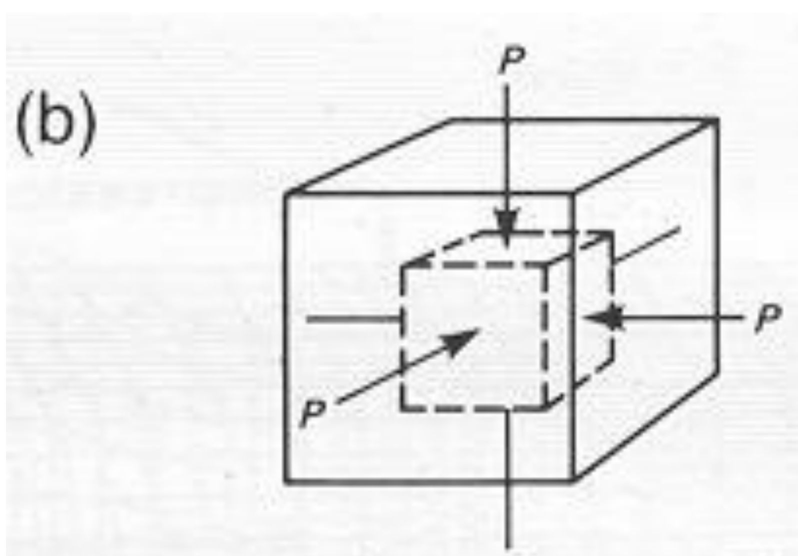
(a) Young's Modulus (*E*) – longitudinal strain proportional to longitudinal stress.

$\boldsymbol{E = F / A} \div \mathrm{D}\boldsymbol{l / l}$



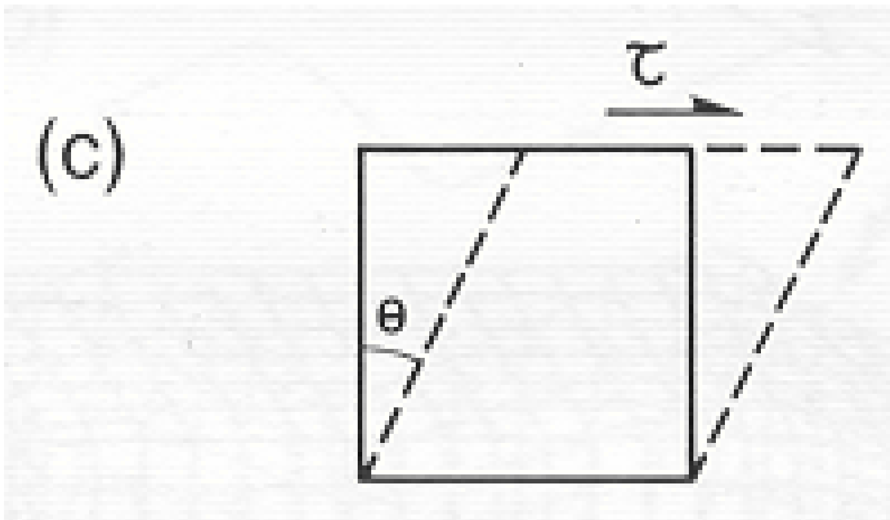
(b) Bulk Modulus (*K*) – describes the change of volume due to a change of pressure.

$\boldsymbol{K = P} \div \mathrm{D}\boldsymbol{V / V}$



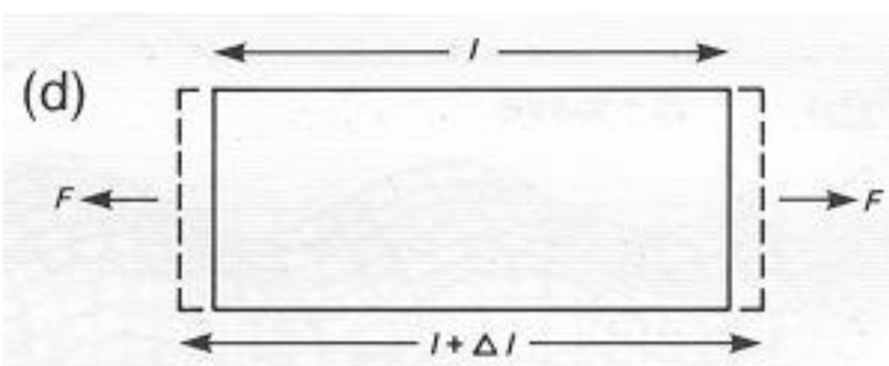
(c) ***Shear Modulus*** (μ) – amount of angular deformation due to the application of a shear stress on one side of the object.

μ ***=*** **t** ***/ tan θ*** Note: μ = 0 in liquids (no rigidity).



(d) Axial modulus (ψ) – response to longitudinal stress, similar to Young's Modulus:

Y ***= F /A ÷*** D***l / l*** except that strain is uniaxial – no transverse strain associated with the application of the longitudinal stress.



Relationships between elastic moduli , Lamé coefficients λ , and μ.

μ = shear modulus (as before)

λ = first Lamé coefficient (no direct physical interpretation)

Young's Modulus: $E = \mu (3\lambda + 2\mu) \div (\lambda + \mu)$

Bulk modulus: $K = \lambda + 2/3\, \mu$

Poisson's Ratio: $\sigma = \lambda / 2 (\lambda + \mu)$

Lamé 1 in terms of Poisson & Young

$\lambda = E\sigma / (1 + \sigma)(1 - 2\sigma)$

Poisson's ratio (Dimensionless ratio) is the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force. Tensile deformation is considered positive and compressive deformation is considered negative. The definition of Poisson's ratio contains a minus sign so that normal materials have a positive ratio

$\sigma = - e_{trans} / e_{longitudinal}$

Poisson solid: material for which $\lambda = \mu$, giving $\sigma = 0.25$

SEISMIC WAVES

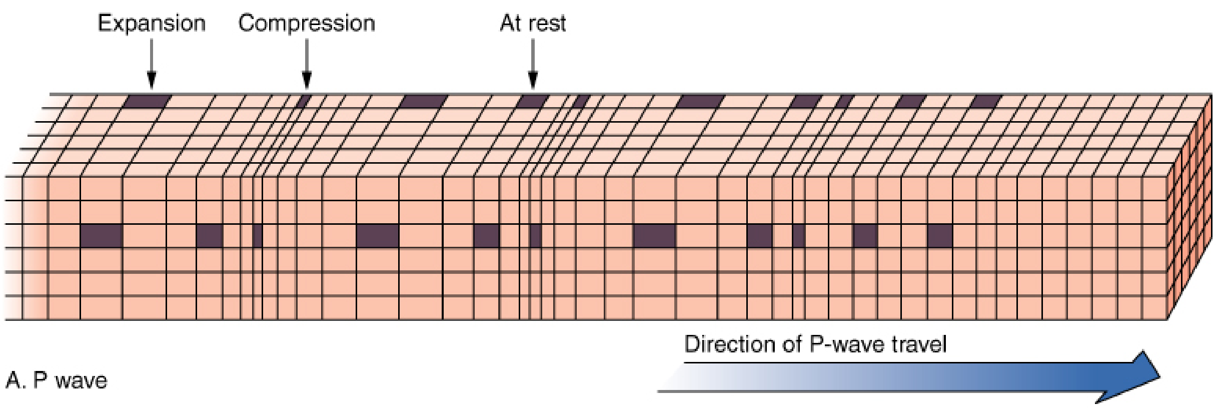
A. Body Waves

Seismic waves are pulses of strain energy that propagate in a solid. Two types of seismic wave can exist inside a uniform solid:

A) P waves (Primary, Compressional, Push-Pull)

Motion of particles in the solid is in direction of wave propagation.

- P waves have highest speed.
- Volumetric change
- Sound is an example of a P wave.



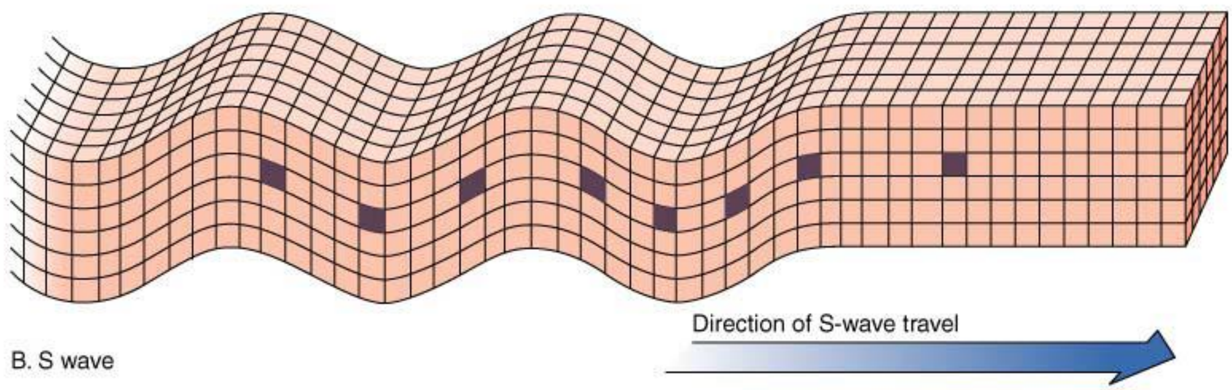
A. P wave

$$V_p = \sqrt{\frac{\Psi}{\rho}} = \sqrt{\frac{K + 4/3\,\mu}{\rho}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

B. S waves (Secondary, Shear, Shake)

Particle motion is in plane perpendicular to direction of propagation.

- If particle motion along a line in perpendicular plane, then S wave is said to be plane polarised: SV in vertical plane, SH horizontal.
- No volume change
- S waves cannot exist in fluids like water or air, because the fluid is unable to support shear stresses.



$$\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{1-2\nu}} \qquad \text{and so,} \quad \nu = \frac{V_P^2 - 2V_S^2}{2\left(V_P^2 - V_S^2\right)}$$

v is the poisson's ratio = 0 for a perfect fluid, so S-waves cannot propagate through fluids. Poisson's ratio is theoretically bounded between 0 and 0.5 and for most rocks lies around 0.25, so typically $\boldsymbol{V_P/V_S}$ is about 1.7.

Vp /Vs ratios (hence Poisson's Ratio) can be characteristic of rock type or physical property, e.g.

-Felsic rocks _ lower Poisson's Ratio

-Mafic rocks _ higher Poisson's Ratio
-Partial melt _ very high Poisson's Ratio (S wave speeds are more strongly affected by melt than P waves)

C. Surface Waves

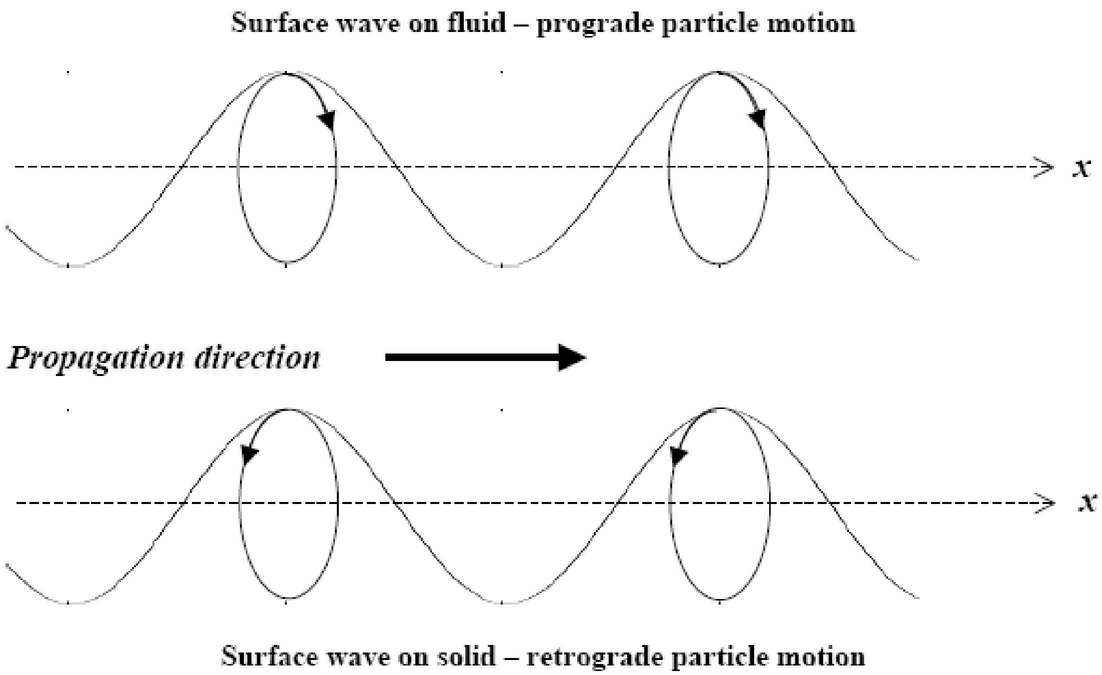
Further a particle moves up and down vertically as well as in the direction of the wave. In detail a particle traces an ellipse with **a *prograde*** rotation as shown in the sketch below. The surface wave on an isotropic half -space is known as a Rayleigh wave and it is similar in form to the surface wave on a liquid half-space except that the particle motion is ***retrograde***.

1. Rayleigh waves

- Propagate along the surface of Earth
- Amplitude decreases exponentially with depth.
- Near the surface the particle motion is retrograde elliptical.
- Rayleigh wave speed is slightly less than S wave: ~92% V_S.

2. Love waves

- Particle motion is SH, i.e. transverse horizontal
- Dispersive propagation: different frequencies travel at different velocities, but usually faster than Rayleigh waves.



The velocity of a Rayleigh wave, V_R, is tied to the S-wave velocity and Poisson's ratio (and hence to V_P) through the solution to the following equation (White, 1983)

$$\left(2-\left(V_R/V_S\right)^2\right)^2-4\left(1-\left(V_R/V_P\right)^2\right)^{\frac{1}{2}}\left(1-\left(V_R/V_S\right)^2\right)^{\frac{1}{2}}=0$$

For typical values of Poisson's ratio the Rayleigh wave velocity varies only from 0.91 to 0.93 V_S.

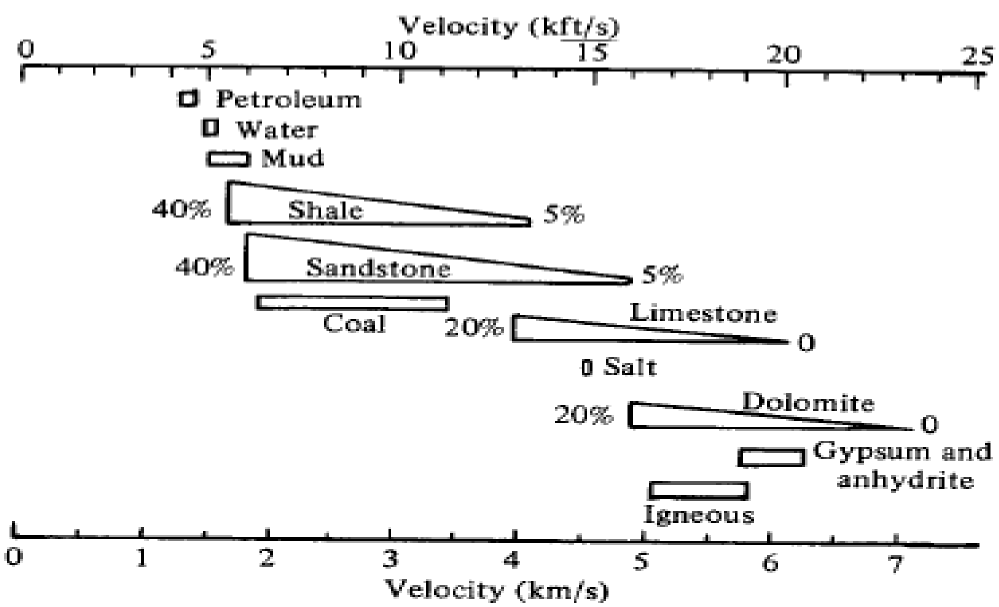
Since it is observed that velocity generally increases with depth, Rayleigh waves of high frequency penetrate to shallow depth and have *low* velocity whereas Rayleigh waves of low frequency penetrate to greater depth and have *high* velocity. The change in velocity with frequency is known as ***dispersion.*** For a typical surface source of Rayleigh waves the initiating disturbance has a broad frequency spectrum so the observed surface motion at some distance from the source is spread out in time, with low frequencies coming first and high frequencies coming later.

Seismic velocity, attenuation and rock properties

Rock properties that affect seismic velocity

- Porosity
- Lithification
- Pressure
- Fluid saturation

- Velocity in unconsolidated near surface soils (the weathered layer)
- Attenuation



Generally, the velocities depend on the elastic modulii and density via:

$$V_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \qquad \text{and} \qquad V_S = \sqrt{\frac{\mu}{\rho}}$$

Rock properties that affect seismic velocity

1) Porosity. A very rough rule due to Wyllie is the so called time average relationship:

$$\frac{1}{V_{bulk}} = \frac{\phi}{V_{fluid}} + \frac{1-\phi}{V_{matrix}}$$

where φ is the porosity.

This is not based on any convincing theory but is roughly right when the effective pressure is high and the rock is fully saturated.

2) Lithification.

Also known as cementation. The degree to which grains in a sedimentary rock are cemented together by post depositional, usually chemical, processes, has a strong effect on the modulii. By filling pore space with minerals of higher density than the fluid it replaces the bulk density is also increased. The combination of porosity reduction and lithification causes the observed increase of velocity with depth of burial and age.

3) Pressure.

In general velocity rises with increasing confining pressure and then levels off to a "terminal velocity" when the effective pressure is high. Finally even at depth, as the pore pressure increases above hydrostatic, the effective pressure decreases as does the velocity. Over pressured zones can be detected in a sedimentary sequence by their anomalously low velocities.

4) Fluid saturation.

From theoretical and empirical studies it is found that the compressional wave velocity decreases with decreasing fluid saturation. As the fraction of gas in the pores increases, K and hence velocity decreases. Less intuitive is the fact that V_s also decreases with an increase in gas content.

Attenuation

It is observed that seismic waves decrease in amplitude due to spherical spreading and due to mechanical or other loss mechanisms in the rock units that the wave passes through.

The attenuation for a sinusoidal propagating wave is defined formally as the energy loss per cycle (wave length) Δ E/E where E is the energy content of the wave.

Mathematically, the propagating wave $A = A_0 e^{i\omega t - ikx}$, get an added damping term $e^{-\alpha x}$, so the solution becomes $A = A_0 e^{i\omega t - ikx} e^{-\alpha x}$

[We can apply this to the definition of attenuation Δ E/E by substituting A^2 for the energy at two points at distance λ (the wavelength) apart and we find $\frac{\Delta E}{E} = 2\alpha\lambda$

Experimentally it is found that the attenuation coefficient α ▪depends on frequency and that there is little dispersion. In fact to a good approximation attenuation can be described by $A = A_0 e^{-\beta f x}$. With x in meters and f in Hertz, a typical shale has a $\beta = 10^{-4}$. So at one Hertz the amplitude falls to A_0/e at 10 km. But at 1000 Hz it falls to A_0/e in 10 m. The attenuation may be as much as 10 times greater in unconsolidated sediments. So attenuation increases rapidly with decreasing wavelength.

Example : Consider attenuation is an unconsolidated medium with a velocity of 250 m/sec and a frequency of 1000 Hz. Then, λ = 0.25 m, and α = a^3 ×256. The wave would fall to 1/e of its initial amplitude when a = 157 m.

Constraints on Seismic Velocity

Seismic velocities vary with mineral content, lithology, porosity, pore fluid saturation, pore pressure, and to some extent temperature.
In igneous rocks with minimal porosity, seismic velocity increases with increasing mafic mineral content.
In sedimentary rocks, effects of porosity and grain cementation are more important, and seismic velocity relationships are complex.
Various empirical relationships have been estimated from either measurements on cores or field observations:

1) P wave velocity as function of age and depth

$$V = 1.47(ZT)^{1/6} \text{ km/s}$$

where Z is depth in km and T is geological age in millions of years (Faust, 1951).

2) Time-average equation

$$\frac{1}{V} = \frac{\phi}{V_f} + \frac{1-\phi}{V_m}$$

where ▪ is porosity, V_f and V_m are P wave velocities of pore fluid and rock matrix respectively (Wyllie, 1958).

- Usually $V_f \approx 1500$ m/s, while V_m depends on lithology.
- If the velocities of pore fluid and matrix known, then porosity can be estimated from the measured P wave velocity.

Waves and Rays

Considering a source at point O on a homogeneous half-space, the resulting seismic P-wave propagates such that all the points of constant phase lie on a hemisphere centered at O. The surface of all points of the same phase is called **a wave front** although the inferred meaning is that the phase in question is associated with some identifiable first arrival of the wave. A more rigorous definition is that the wave front is the surface of all equal travel times from the source. A cross section of such a hemispherical spreading wave is shown in the following cartoon for two successive time steps, t_1 and $2t_1$.

The vector perpendicular to a wave front is defined as a ***ray*.** The ray in the cartoon is directed along a radius from the source, but this is not always the case. Rays are useful for describing what happens to waves when they pass through an interface

Huygen's Principle

Every point on a wavefront can be considered a secondary source of spherical waves, and the position of the wavefront after a given time is the envelope of these secondary wavefronts.

- Treat all the points on a wavefront as point sources that generate secondary spherical wavefronts ('wavelets').
- Useful for understanding reflection, refraction and diffraction of seismic waves.

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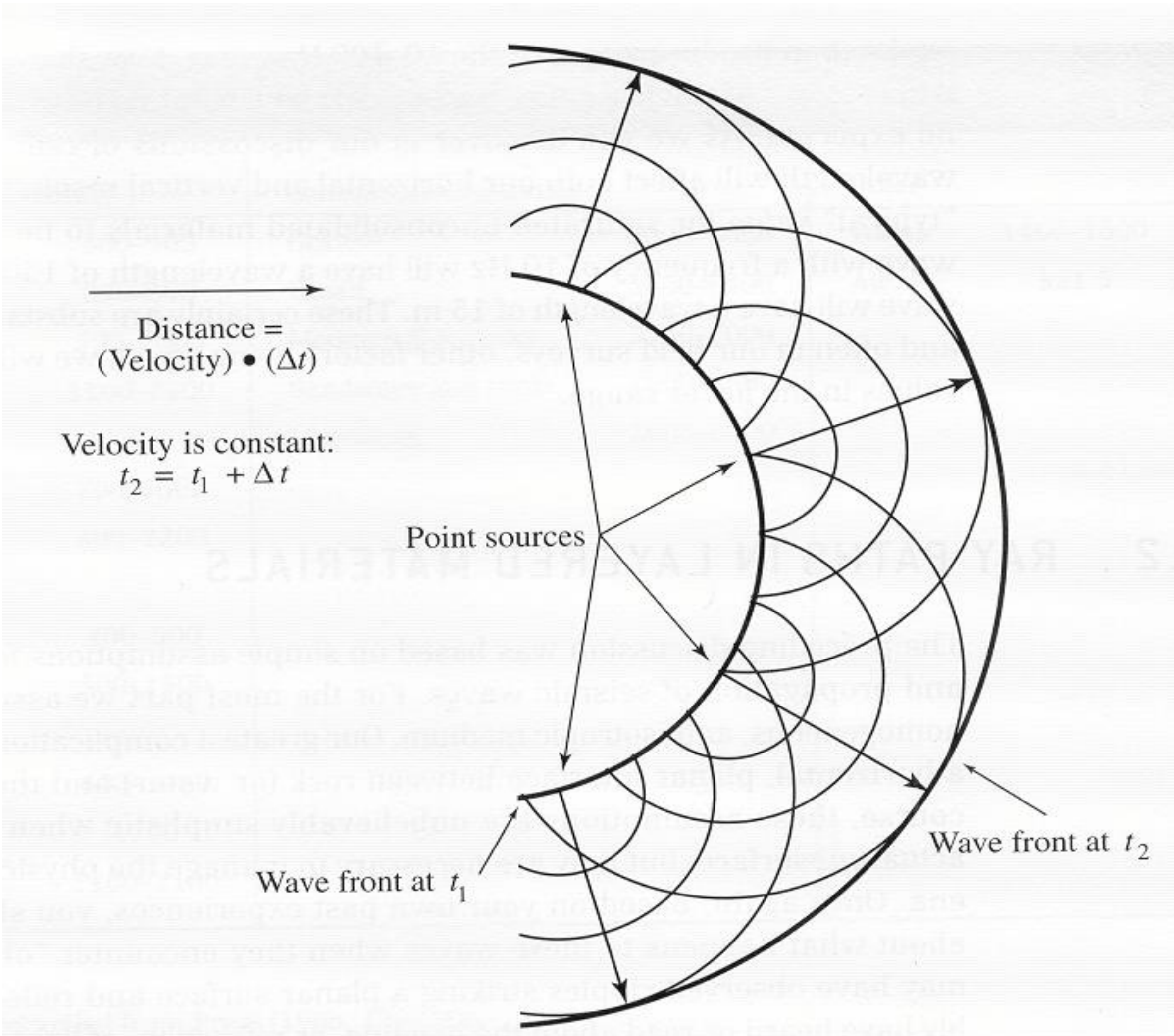


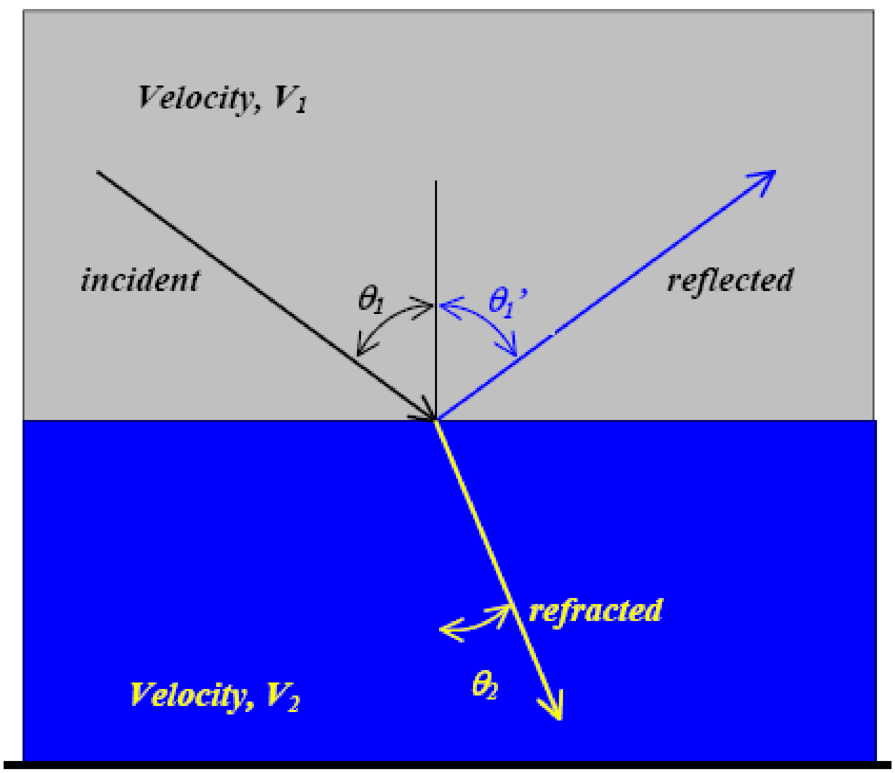
Distance =
(Velocity) • (Δt)
Velocity is constant:
$t_2 = t_1 + \Delta t$
Point sources
Wave front at t_1
Wave front at t_2

•

Snell's Law

A wave incident on a boundary separating two media is reflected back into the first medium and some of the energy is transmitted, or refracted, into the second. The geometry of refraction and reflection is governed by **Snell's Law** which relates the angles of incidence, reflection and refraction to the velocities of the medium.

The cartoon below illustrates the ray geometry for a P-wave incident on the boundary between media of velocity V_1 and V_2. The angles of incidence, reflection and refraction, θ_1, θ_1' and θ_2, respectively are the angles the ray makes with the normal to the interface.



Snell's Law states that:

$$\frac{Sin\,\theta_1}{V_1} = \frac{Sin\,\theta_1'}{V_1} = \frac{Sin\,\theta_2}{V_2}$$

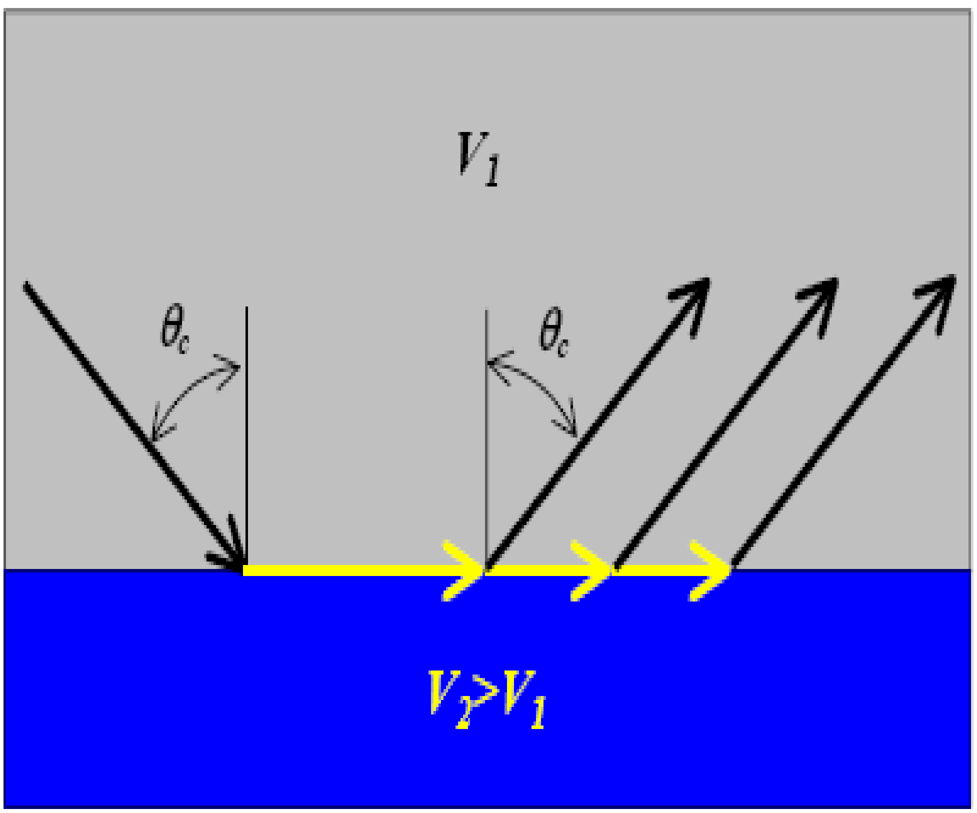
Snell's law requires that the angle of reflection is equal to the angle of incidence.

if $\boldsymbol{V_2}$ is less than $\boldsymbol{V_1}$ the ray is bent towards the normal

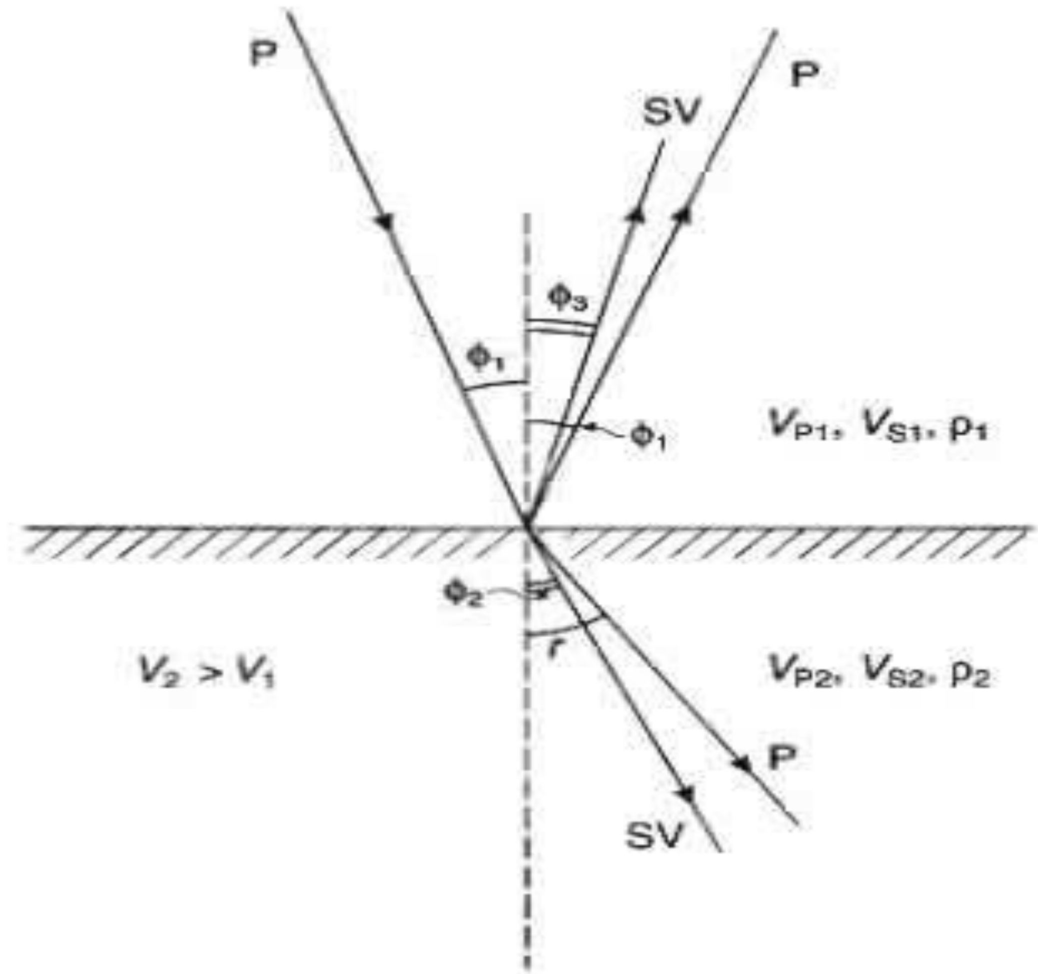
if $\boldsymbol{V_2}$ is greater than $\boldsymbol{V_1}$ the ray is bent away from the normal.

If V_2 is greater than V_1 the angle of refraction is greater than the ngle of incidence. The latter result can lead to a special condition where θ_2 **= 90°** . The angle of incidence for which this occurs is called the ***critical angle***, θ_c.

The critical angle is given by; $\theta_c = Sin^{-1}\left(\frac{V_1}{V_2}\right)$



When a P wave is incident on a boundary, at which elastic properties change, two reflected waves (one P, one S) and two transmitted waves (one P, one S) are generated.



Angles of transmission and reflection of the S waves are less than the P waves. Exact angles of transmission and reflection are given by:

$$\frac{\sin \phi_1}{V_{P1}} = \frac{\sin r}{V_{P2}} = \frac{\sin \phi_3}{V_{S1}} = \frac{\sin \phi_2}{V_{S2}} = p$$

p is known as the ray parameter.

There are two critical angles corresponding to when transmitted P and S waves emerge at 90°.

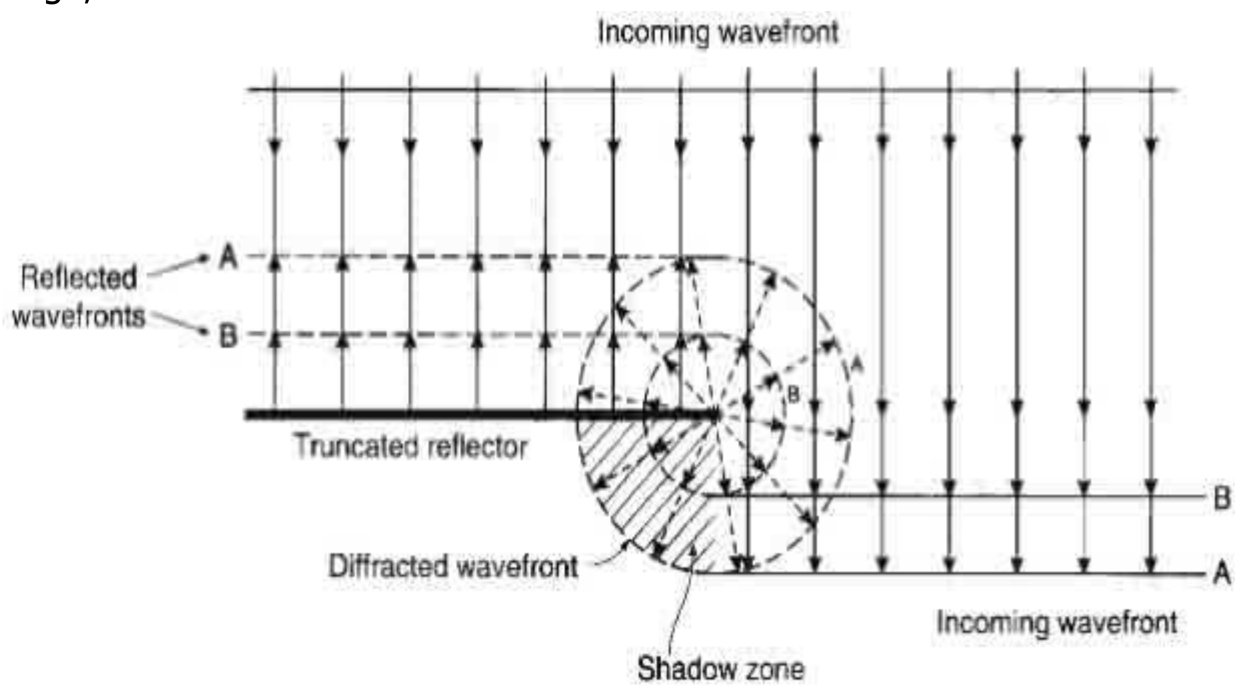
Diffractions

Reflection by Huygen's Principle

When a plane wavefront is incident on a plane boundary, each point of the boundary acts as a secondary source. The superposition of these secondary waves creates the reflection.

Diffraction by Huygen's Principle

If interface truncates abruptly, then secondary waves do not cancel at the edge, and a diffraction is observed.



- This explains how energy can propagate into shadow zones.
- A small scattering object in the subsurface such as a boulder will produce a single diffraction.
- A finite-length interface will produce diffractions from each end, and the interior parts of the arrivals will be opposite polarity.

PROBLEM SET - 1

1. A steel beam is placed vertically in the basement of a building to keep the floor above from sagging. The load on the beam is 5.8×10^4 N and the length of the beam is 2.5 m, and the cross-sectional area of the beam is 7.5×10^{-3} m^2. Find the vertical compression of the beam.

2. A 0.50 m long string, of cross- sectional area 1.0×10^{-6} m^2, has a Young's modulus of 2.0×10^9 Pa. By how much must you stretch a string to obtain a tension of 20.0 N?

3. The upper surface of a cube of gelatin, 5.0 cm on a side, is displaced by 0.64 cm by a tangential force. If the shear modulus of the gelatin is 940 Pa, what is the magnitude of the tangential force?

4. An anchor, made of cast iron of bulk modulus 60.0×10^9 Pa and a volume of 0.230 m^3, is lowered over the side of a ship to the bottom of the harbor where the pressure is greater than sea level pressure by 1.75×10^6 Pa. Find the change in the volume of the anchor.

5. 1. AN ARKOSE HAS A DENSITY OF 2.62 G/CM, A YOUNG MODULUS OF 0.16X10 N/M , AND A POISSON'S RATIO OF 0.29 . TWELVE GEOPHONES ARE ARRANGED ALONG A LINE AT 10 M INTERVAL. THE SHOT POINT IS LOCATED 5 M FROM THE FIRST GEOPHONE IN THE LINE.
CONSTRUCT A GRAPH THAT ILLUSTRATES TIME OF ARRIVAL AGAINST GEOPHONE POSITION FOR THE P-WAVE , S-WAVE AND SURFACE WAVE.

6. THE VOLUM OF ALUMINUM BLOCK WAS PLACED UNDER HYDRAULIC PRESSURE IS 0.4 M^3 A.

A. FIND THE CHANGE IN VOLUM OF THE ALUMINUM WHEN SUBJECTED TO PRESSURE OF 2.1×10^7 N/M^2. THE BULK MODULUS IS 0.85 X 1011 N/M^2 AND THE RIGIDITY MODULUS IS 0.36×10^{11} N/M^2.

B. WHAT IS THE CUBICAL DILATATION B.

C. FIND YOUNG'S MODULUS, COMPRESSIBILITY, AND POISSON'S RATIO.

7. A 15 HZ SEISMIC WAVE TRAVELLING AT *5.5* KM/SEC PROPAGATES FOR 1500 M THROUGH A MEDIUM WITH AN ABSORPTION COEFFICIENT OF 0.3 dB . WHAT IS THE WAVE ATTENUATION IN dB DU E SOLELY TO ABSORPTION.

8. CALCULATE THE AMPLITUDE OF THE REFLECTED AND TRANSMITTED P- AND S-WAVES WHERE THE INCIDENT P-WAVE STRIKE THE INTERFACE FROM A WATER LAYER (P-VELOCITY *=2.5* K/S, 5-VELOCITY =0, DENSITY = 1.20 g/cc) AT 25^0 WHEN THE SEAFLOOR IS:

A. SOFT (P=3K/S, S=1.5 K/S. DENSITY2.O g/cc

B. HARD (**P= 4** K/S, 5= 2.5 K/S, DENSITY =2.5 g/cc

C. REPEAT FOR AN ANGLE OF INCIDENCE 30^0

9. A rock sample is taken to the lab and is subjected to a uniaxial stress (that is, it is stressed in only one direction with the remaining directions free). As a result of the stress, the length of the sample increases by 3% and the width decreases by 1%. What is the ratio of the P wave velocity to the S wave velocity in this sample?

10. A material has a shear modulus of 8.8×10^9 Pa, a bulk modulus of 2.35×10^{10} Pa, a density of 2200 kg/m3 and a quality factor $Q = 100$.
a) What are the P-wave velocity and S-wave velocity of the medium, in units of km/s
b) If a P-wave of frequency 10 Hz has a displacement amplitude of 1 μm at a distance of 10 m from the source, what would be the wave amplitude at 100 m?

11. Near-surface fresh water in a Lake Superior has been observed to have a P-wave velocity of 1435 meters/sec. Estimate its bulk modulus, assuming it is pure water.

12. A laboratory has determined that the Gabbro has the following properties:
Bulk modulus= 0.952130×10^{12} dyne/cm2
Shear modulus=0.403425×10^{12} dyne/cm2
Density=2.931 gm/cc
Determine the (a) shear wave velocity, the (b) compressional wave velocity, and (c) Poisson's ratio for this rock.

CHAPTER 2

SEISMIC REFRACTION METHOD

- Fundamentals

- Two Horizontal Interfaces

- Dipping Interfaces

- The Non ideal Subsurface

- The Delay-Time Method

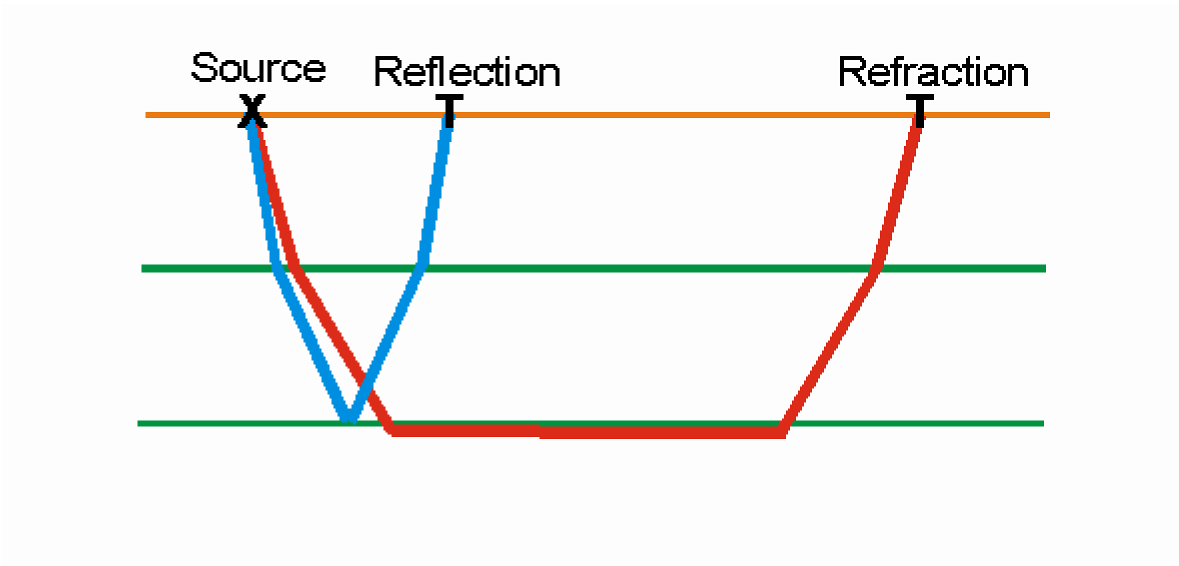
- Field Procedures & Interpretation

Problem Set - 2

Seismic Refraction

A signal, similar to a sound pulse, is transmitted into the Earth. The signal recorded at the surface can be used to infer subsurface properties. There are two main classes of survey:

- **<u>Seismic Refraction</u>**: the signal returns to the surface by refraction at subsurface interfaces, and is recorded at distances much greater than depth of investigation.



- **<u>Seismic Reflection</u>**: the seismic signal is reflected back to the surface at layer interfaces, and is recorded at distances less than depth of investigation.

Applications
Seismic Refraction

- Rock competence for engineering applications
- Depth to Bedrock
- Groundwater exploration

- Correction of lateral, near-surface, variations in seismic reflection surveys
- Crustal structure and tectonics

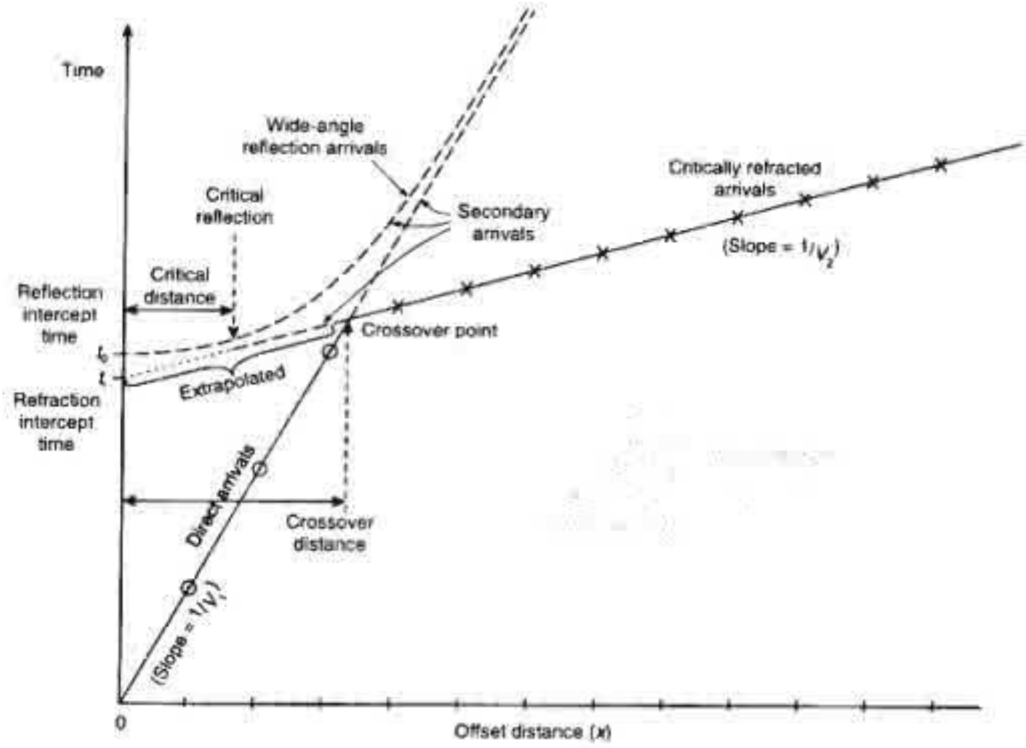
Seismic Reflection

- Detection of subsurface cavities
- Shallow stratigraphy
- Site surveys for offshore installations
- Hydrocarbon exploration
- Crustal structure and tectonics

Travel Time Curves

Analysis of seismic refraction data is primarily based on interpretation of critical refraction travel times. Plots of seismic arrival times vs. source-receiver offset are called travel time curves.
Example
Travel time curves for three arrivals shown previously:

- Direct arrival from source to receiver in top layer
- Critical refraction along top of second layer
- Reflection from top of second layer



Critical Distance

Offset at which critical refraction first appears.

- Critical refraction has same travel time as reflection
- Angle of reflection same as critical angle

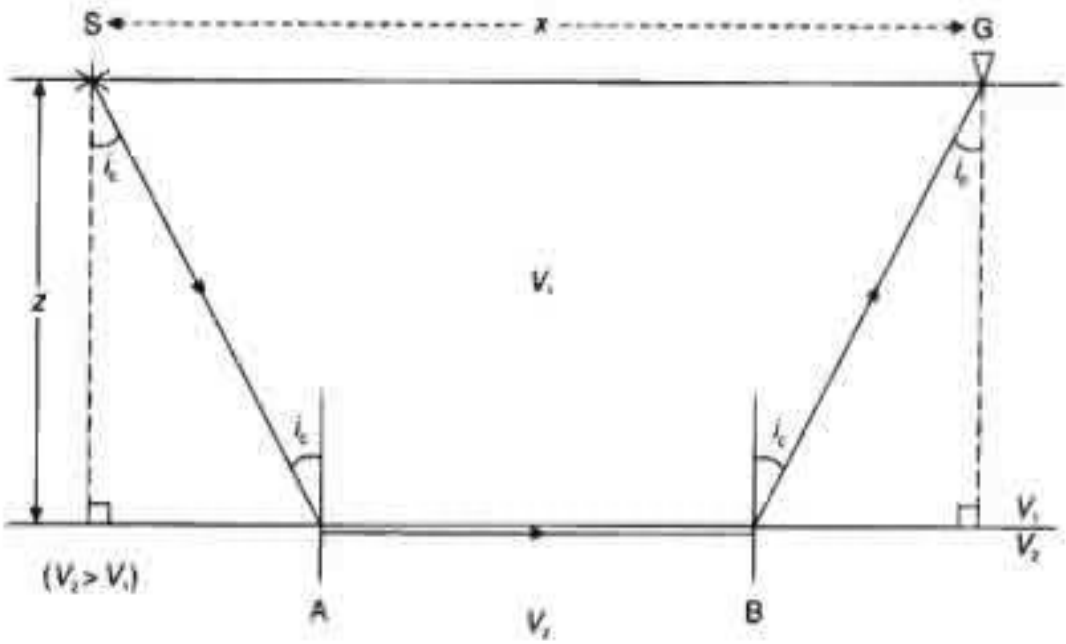
Crossover Distance

Offset at which critical refraction becomes first arrival.

Field Surveying

Usually we analyze P wave refraction data, but S wave data occasionally recorded

1. Horizontal Interfaces: Two Layers



For critical refraction at top of second layer, total travel time from source S to receiver G is given by:

$$T_{SG} = T_{SA} + T_{AB} + T_{BG}$$

Hypoteneuse and horizontal side of end 90°-triangle are:

$Z/\cos i_C$ and respectively.

So, as two end triangles are the same:

$$T_{SG} = \frac{2Z}{V_1 \cos i_c} + \frac{(X - 2Z \tan i_c)}{V_2}$$

$$= \frac{X}{V_2} + \frac{2Z}{V_1 \cos i_c}\left(1 - \frac{V_1}{V_2}\sin i_c\right)$$

At critical angle, Snell's law becomes: $\sin i_c = \frac{V_1}{V_2}$

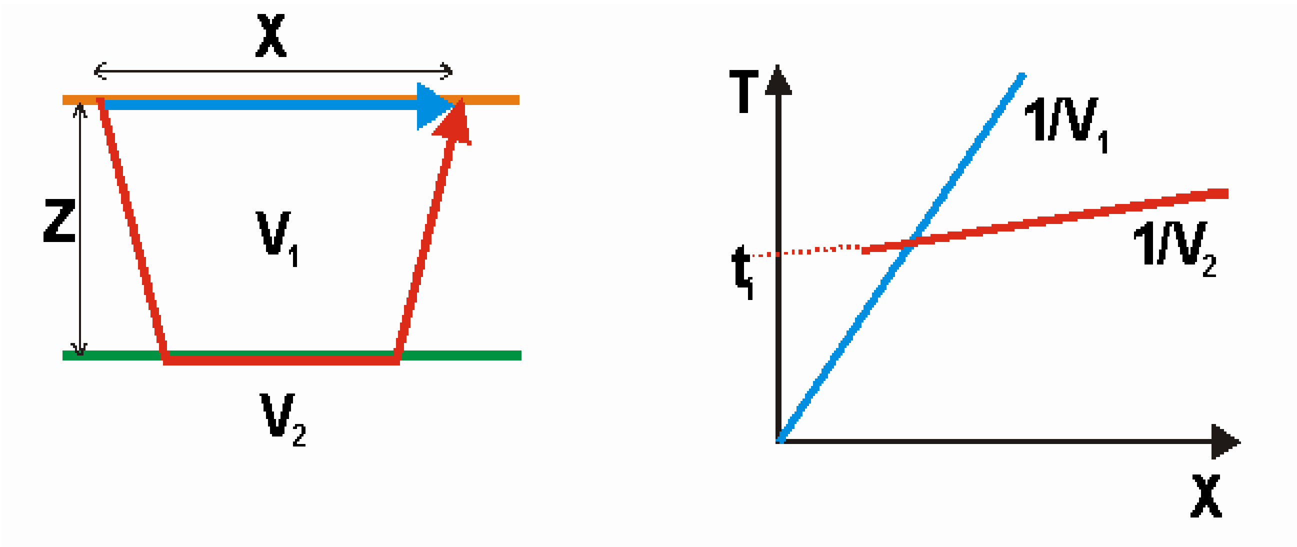
Substituting for V_1/V_2, and using $\cos^2$▪ + $\sin^2$▪ = 1:

$$T_{SG} = \frac{X}{V_2} + \frac{2Z\cos i_c}{V_1}$$

This equation represents a straight line of slope $1/V_2$ and intercept

$$t_i = \frac{2Z\cos i_C}{V_1} = \frac{2Z\left(V_1^2 - V_2^2\right)^{\frac{1}{2}}}{V_1 V_2}$$

Interpretation of Two Layer Case



From travel times of direct arrival and critical refraction, we can find velocities of two layers and depth to interface:

1. Velocity of layer 1 given by slope of direct arrival
2. Velocity of layer 2 given by slope of critical refraction
3. Estimate t_i from plot and solve for Z:

$$Z = \frac{t_i V_1 V_2}{2\sqrt{V_2^2 - V_1^2}}$$

Depth from Crossover Distance

At crossover point, traveltime of direct and refraction are equal:

$$\frac{X_{cross}}{V_1} = \frac{X_{cross}}{V_2} + \frac{2Z\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

Solve for Z to get:

[Depth to interface is always less than half the crossover distance]

$X_{critical}$ = Minimum distance for refraction arrival

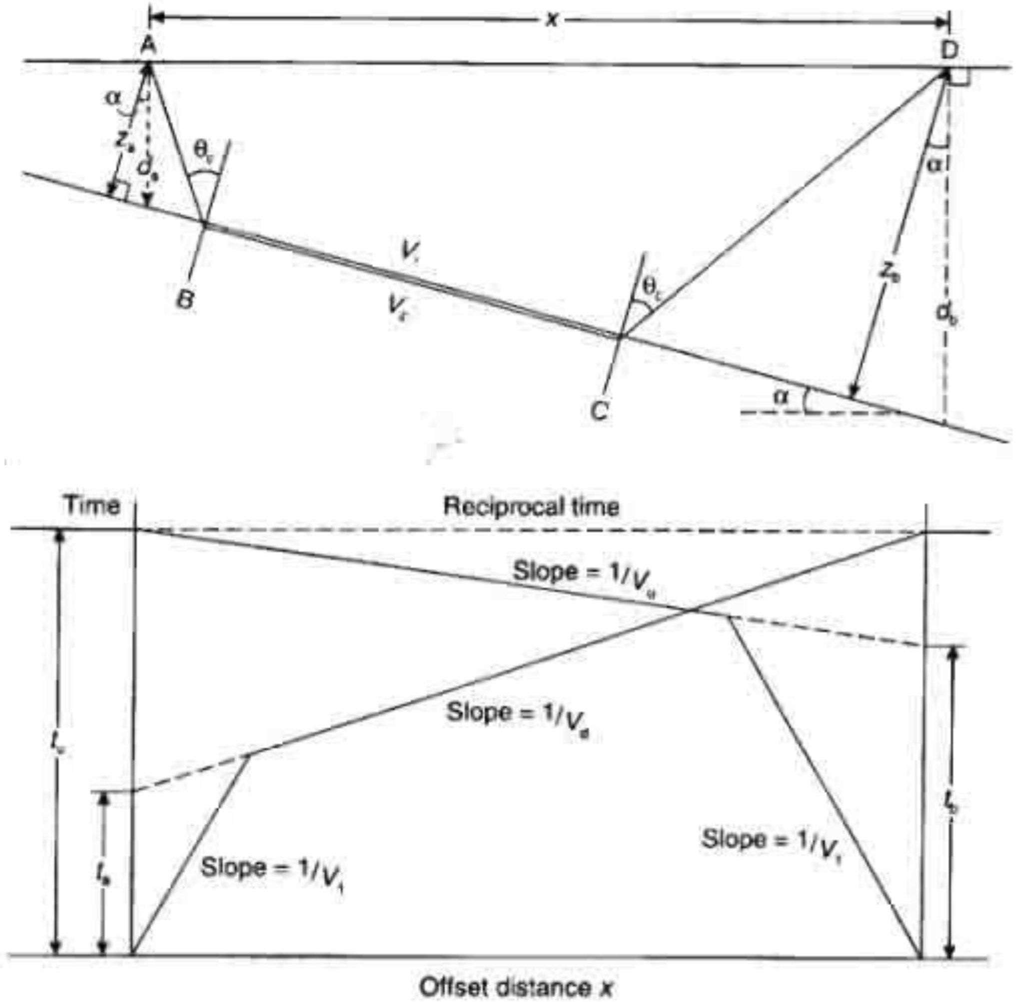
= 2 z_1 tan $_{ic}$

2. Dipping Interfaces : Two Layer Case

When a refractor dips, the slope of the traveltime curve does not represent the "true" layer velocity:

- shooting updip, i.e. geophones are on updip side of shot, apparent refractor velocity is higher
- shooting downdip apparent velocity is lower

To determine both the layer velocity and the interface dip, forward and reverse refraction profiles must be acquired.



Note: Travel times are equal in forward and reverse directions for switched, <u>reciprocal</u>, source/receiver positions.

Geometry is same as horizontal 2-layer case, but rotated through ･, with extra time delay at D. So traveltime is:

$$T_{ABCD} = \frac{X\cos\alpha}{V_2} + \frac{(Z_a + Z_b)\cos i_C}{V_1}$$

Formulae for up/downdip times are (not proved here):

$$T_d = \frac{X\sin(\theta_C + \alpha)}{V_1} + \frac{2Z_a\cos\theta_C}{V_1} = \frac{X}{V_d} + t_a$$

$$T_u = \frac{X\sin(\theta_C - \alpha)}{V_1} + \frac{2Z_b\cos\theta_C}{V_1} = \frac{X}{V_u} + t_b$$

where V_u/ V_d and t_u/ t_d are the apparent refractor velocities and intercept times.

$$\theta_C + \alpha = \sin^{-1}\left(\frac{V_1}{V_d}\right); \quad \theta_C - \alpha = \sin^{-1}\left(\frac{V_1}{V_u}\right)$$

Can now solve for dip, depth and velocities:

1) Adding and subtracting, we can solve for interface dip · and critical angle ·C:

$$\theta_c = \frac{1}{2}\left[\sin^{-1}\left(\frac{V_1}{V_d}\right) + \sin^{-1}\left(\frac{V_1}{V_u}\right)\right]; \quad \alpha = \frac{1}{2}\left[\sin^{-1}\left(\frac{V_1}{V_d}\right) - \sin^{-1}\left(\frac{V_1}{V_u}\right)\right]$$

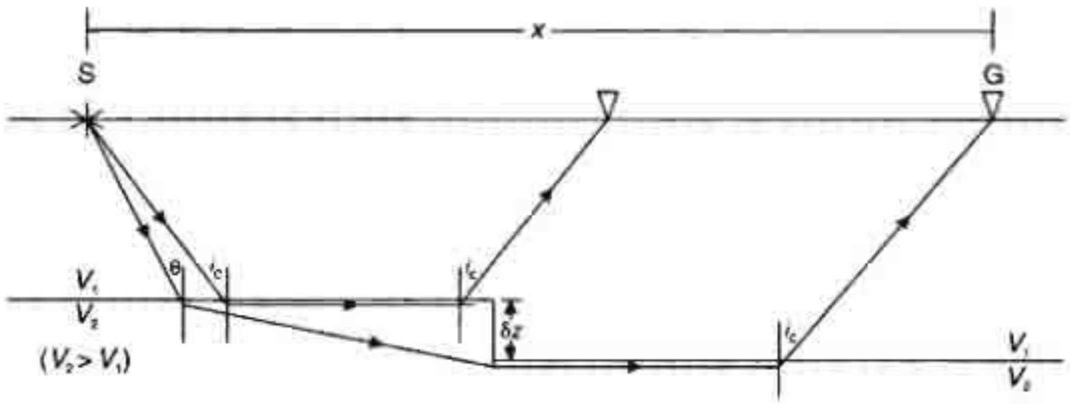
[V_1 is known from direct arrival, and V_u and V_d are estimated from the refraction traveltime curves]

2) Can find layer 2 velocity from Snell's law: $V_2 = V_1 / \sin\theta_C$

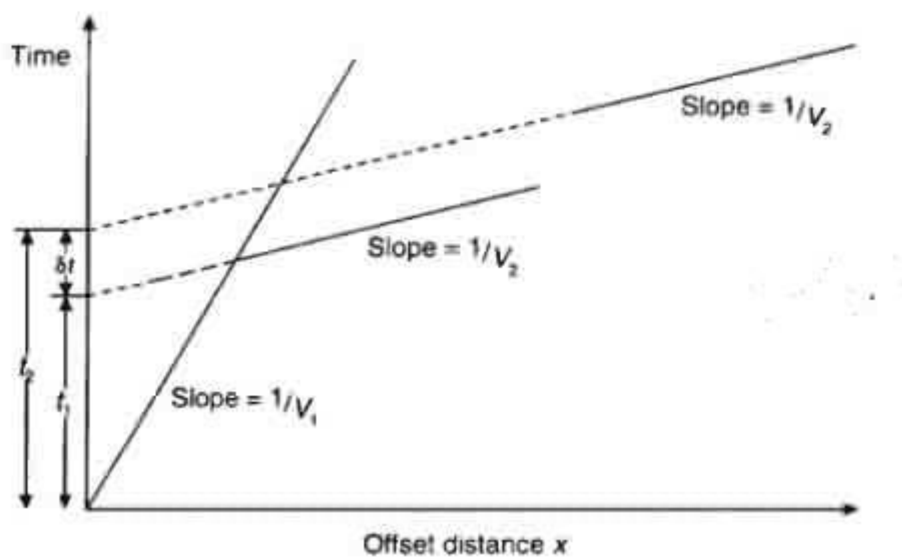
1. Can get slant interface depth from intercept times, and convert to vertical depth at source position:

$$Z_a = \frac{V_1 t_a}{2\cos\theta_C}; \quad d_a = \frac{Z_a}{\cos\alpha} = \frac{V_1 t_a}{2\cos\theta_C\cos\alpha}$$

3. Faulted Planar Interface (Diffraction)



If refractor faulted, then there will be a sharp offset in the travel time curve:



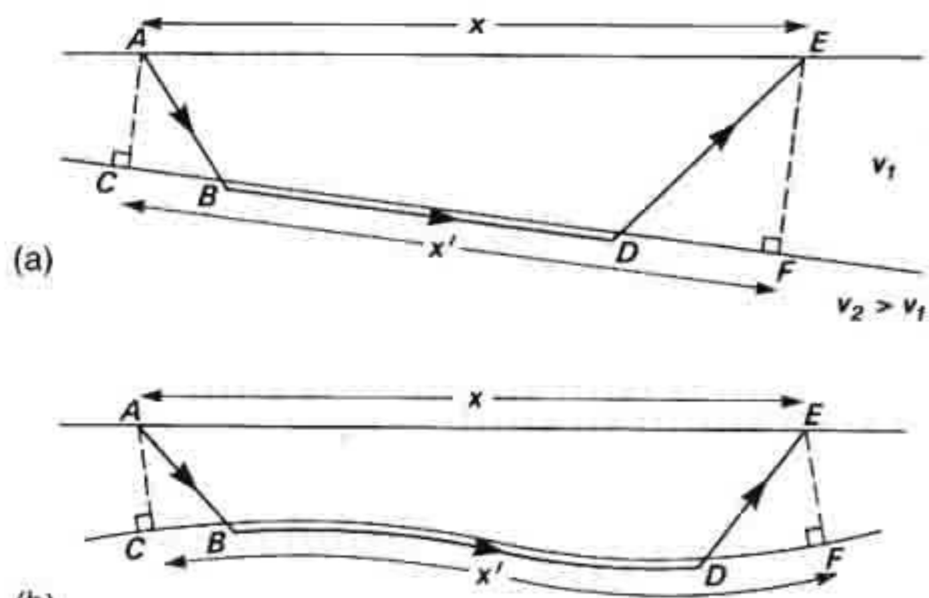
Can estimate throw on fault from offset in curves, i.e. difference between two intercept times, from simple formula:

$$\delta z = \frac{\delta t V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$

Delay Times in Refraction

For irregular travel time curves, e.g. due to bedrock topography or glacial fill, much analysis is based on delay times.
Total Delay Time . Difference in travel time along actual ray path and projection of ray path along refracting interface:

$$\delta t = T_{AE} - T_{CF}$$



$$T_{CF} = \frac{CF}{V_2}$$;

Total delay time is delay time at shot plus delay time at geophone:

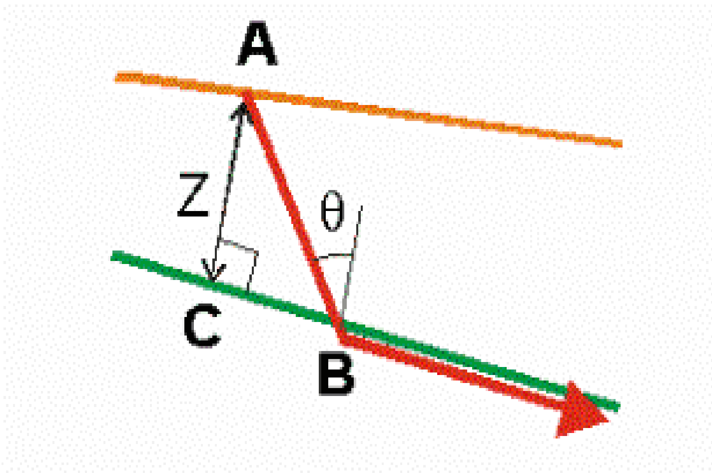
$$\delta = \left(\frac{AB}{V_1} - \frac{CB}{V_2}\right) + \left(\frac{DE}{V_1} - \frac{DF}{V_2}\right) = \delta_S + \delta_G \approx T_{AB} - \frac{x}{V_2}$$

For small dips, can assume $x = x^I$ and:

$$T_{AB} = \frac{x}{V_2} + \delta_S + \delta_G$$

Refractor Depth from Delay Time

If velocities of both layers are known, then refractor depth at point A can be calculated from delay time at point A:



$$\delta_A = \frac{AB}{V_1} - \frac{BC}{V_2}$$

Using the triangle to get lengths in terms of z:

$$\delta_A = \frac{Z}{V_1 \cos\theta} - \frac{Z \tan\theta}{V_2}$$

$$= \frac{Z}{V_1 \cos\theta}\left(1 - \frac{V_1 \sin\theta}{V_2}\right)$$

Using Snell's law to express angles in terms of velocities:

$$\delta t_A = \frac{Z}{V_1\left(1 - \frac{V_1^2}{V_2^2}\right)^{\frac{1}{2}}}\left(1 - \frac{V_1^2}{V_2^2}\right)$$

Simplifying:

$$\delta t_A = \frac{Z\left(V_2^2 - V_1^2\right)^{\frac{1}{2}}}{V_1 V_2}$$

So refractor depth at A is:

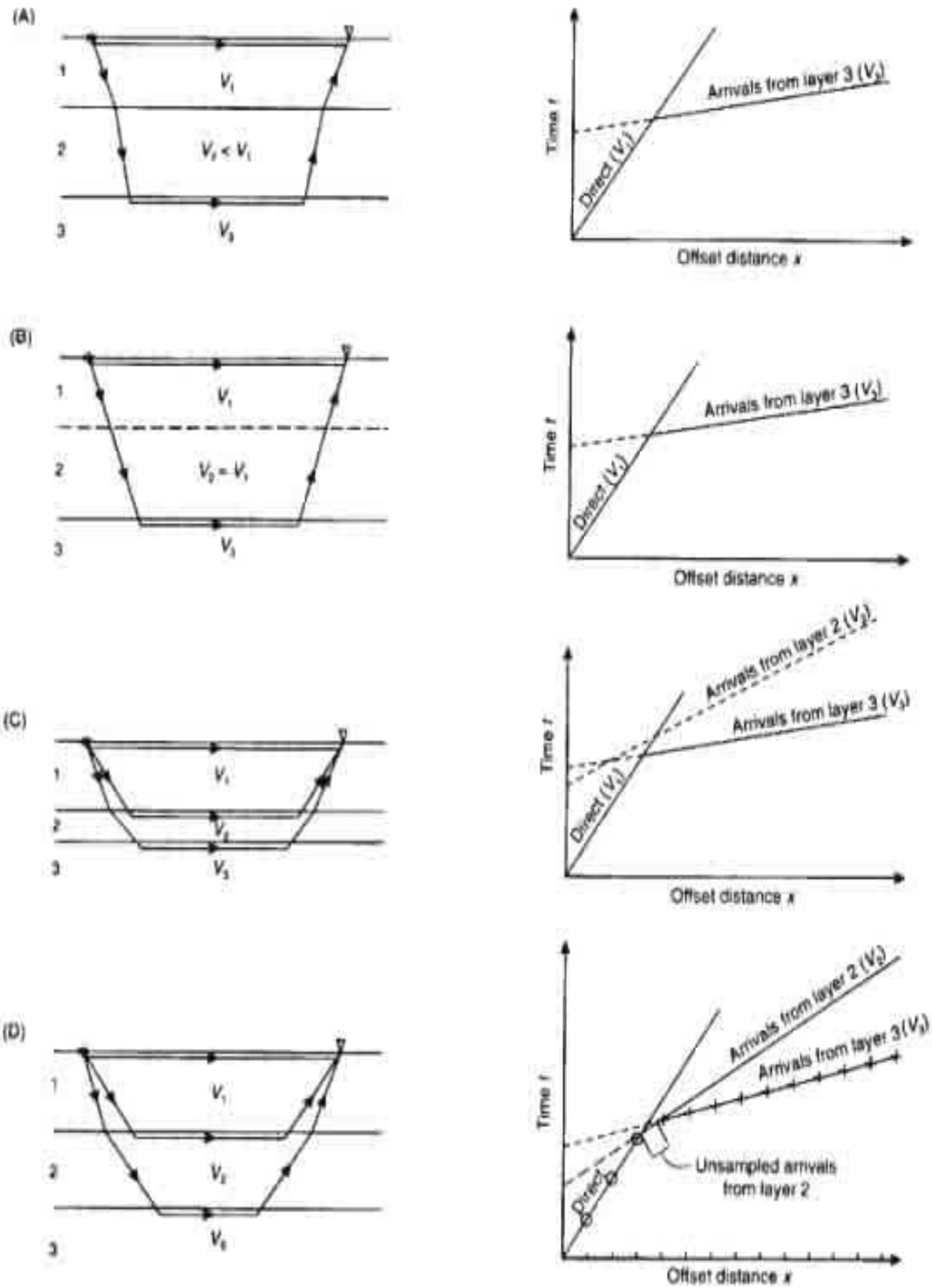
$$Z = \frac{\delta t_A V_1 V_2}{\left(V_2^2 - V_1^2\right)^{\frac{1}{2}}}$$

Blind layer problem

Blind layers occur when there is a **low velocity layer (LVL)**. Head waves only occur for a velocity increase. Thus, there will be **no refraction** from the top of the LVL and this **layer will not be detected** on the time-distance plot. This is shown below.

Hidden layer problem

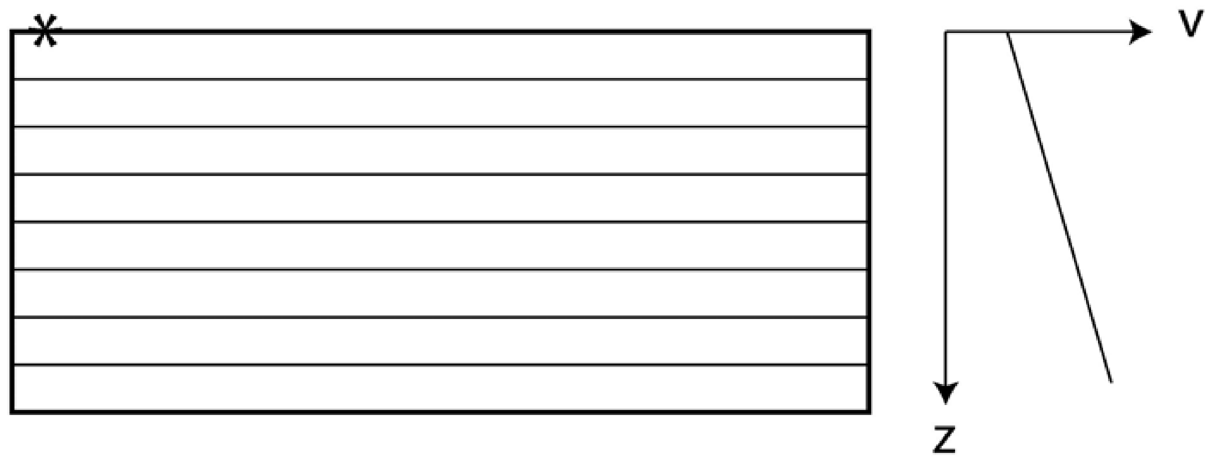
Hidden layers result when there is a velocity increase with layer depth, but the head wave from the top of one layer is **never the first arrival** on a time-distance plot. Head waves from a deeper layer arrive at the detectors before the arrivals from this layer. Two factors can cause hidden layers: 1) the layer is very thin or 2) there is only a small velocity increase at the top of the layer. This is shown below. It is sometimes possible to recognize hidden layers by looking for arrivals after the first arriving energy.



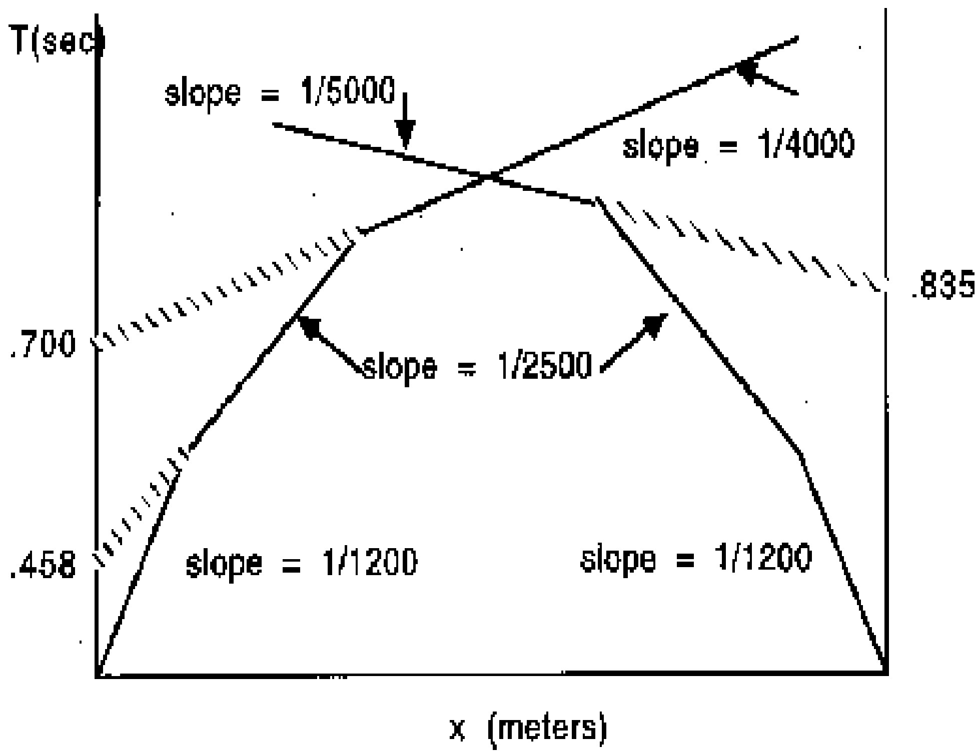
(A)
1
V_1
2
$V_2 < V_1$
3
V_3
Time t
Direct (V_1)
Arrivals from layer 3 (V_3)
Offset distance x
(B)
1
V_1
2
$V_2 = V_1$
3
V_3
Time t
Direct (V_1)
Arrivals from layer 3 (V_3)
Offset distance x
(C)
1
V_1
2
V_2
3
V_3
Time t
Direct (V_1)
Arrivals from layer 2 (V_2)
Arrivals from layer 3 (V_3)
Offset distance x
(D)
1
V_1
2
V_2
3
V_3
Time t
Direct
Arrivals from layer 2 (V_2)
Arrivals from layer 3 (V_3)
Unsampled arrivals
from layer 2
Offset distance x

PROBLEM SET - 2

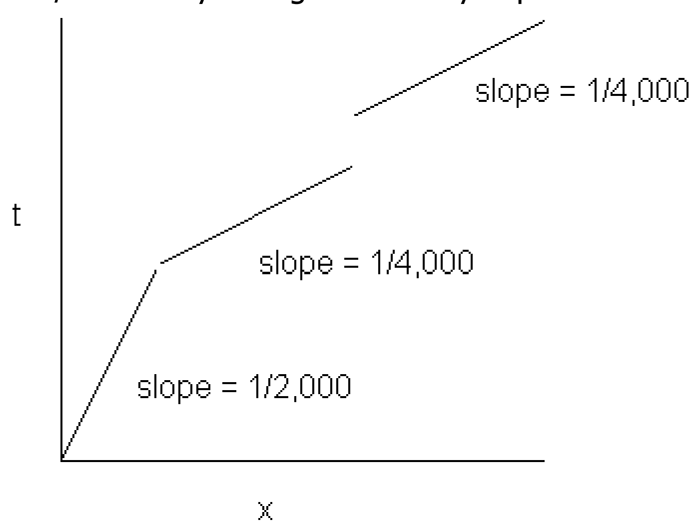
Q1. Consider a case where there is a **continuous velocity increase with depth,** as is commonly observed in the Earth. The propagation of seismic waves can be investigated by assuming that the subsurface is made up of infinitely thin layers of uniform velocity. Sketch the expected ray paths and travel time curve.
Remember that Snell's Law requires that the ray parameter is constant: **P = sinθ/ v**



2. Given the reversed refraction observations (travel time vs. distance curves) shown below, calculate the velocities and depths to the interfaces. Calculate the dip angles of the interfaces.



3. Given the following schematic travel-time curve, describe a subsurface structure and/or velocity changes that may explain them.



4. A seismic refraction survey was conducted along an abandoned railroad grade about 2.5 miles southeast of the town of Osakis. The railroad grade is known to be resting on Pleistocene glacial deposits. A 12 channel system was used with an sledge hammer for an energy source, and the following first break times were picked from the traces.

Distance	**First break time(meters) (milliseconds)**
5	14
15	23
25	30
35	36.75
45	42
55	50
65	54.5
75	61
85	65.5
95	66.5
105	70
115	73.25

A reversed profile yielded essentially identical results, so we can assume horizontal layering. It is assumed that the first and second layers will represent the grade fill and the glacial deposits, respectively.

Plot the travel time relationships and estimate the velocities of the first two layers (in meter/second). What is the thickness of the grade fill?
Based on your velocity estimate for the second layer, do you think the glacial deposits are saturated or unsaturated (below or above the water table)?
Do you have any evidence of bedrock (ie a third layer) below the glacial deposits? If so, what is its velocity (in meters/second) and depth?

5. SUPPOSE THAT A REVERSED REFRACTION SURVEY (USING SHOTS A AND B) INDICATED VELOCITIES V1 = 1500 M/SEC. AND V2 = 2500 M/SEC. FROM SHOT A AND VELOCITIES V1 = 1500 M/SEC AND V2 = 3250 M/SEC. FROM SHOT B.
FIND THE DIP OF REFRACTOR. WHAT WOULD BE THE CHANGES IN VELOCITIES IF THE FEFRACTOR HAD A SLOPE 10 DEGREES LARGER THAN THE ONE YOU COMPUTED.

6. Construct a travel time curve to a distance of 120 m for a structure with an 10 m thick soil layer with P-wave velocity 500 m/s over a saturated layer 20 m thick with P-wave velocity 1500 m/s over bedrock with velocity 3000 m/s. From the graph, what are the 2 crossover distances?

7. Construct a travel time curve for the following data for both the forward and reverse profiles. Evaluate slopes and intercept times, and use those values to determine the subsurface structure for the first 2 layers, and then estimate the properties of the bottom layer. Offsets are in meters and times are in milliseconds.

Offset x	5	10	15	20	25	30	35	40	45	50	55	60
Forward	8.3	16.7	25.0	33.3	41.7	50.0	58.3	66.7	72.5	75.1	77.7	80.3
Reverse	8.3	16.7	25.0	33.3	40.7	44.3	48.0	51.6	55.3	58.9	62.6	66.3

8. The following data are from a profile over a buried steep fault scarp underlying alluvium. Use the data to determine velocities for the alluvium and bedrock and the throw and approximate position of the buried fault step. Offsets are in meters and times are in milliseconds.

Offset x	5	10	15	20	25	30	35	40	45	50	55	60
Forward	3.6	7.1	10.7	14.3	17.9	21.4	23.0	24.0	24.9	25.8	26.7	27.7
Reverse	3.6	7.1	10.7	14.3	17.9	21.4	25.0	28.6	30.4	25.8	26.7	27.7

9. FROM SNELL'S LAW, CALCULATE THE CHANGE IN DIRECTION OF A SEISMIC WAVE WHEN IT REFRACTED FROM A SANDSTONE STRATUM (V = 4000 M/SEC AND 350 M THICK) INTO A LIMESTONE STRATUM (V=6000 M/SEC) FOR EACH OF THE FOLLOWING ANGLES OF INCIDENCE 0, 15, 25, 35, 45, AND 60.

WHAT IS THE CRITICAL ANGLE.
WHAT IS THE MINIMUM DISTANCE FOR REFRACTION ARRIVALS
WHAT IS THE CROSS- OVER DISTANCE
COMPARE THE VALUES OF X_{CO} WITH THE THICKNESS OF SANDSTONE LAYER
WHAT IS THE NMO AND REFLECTION TRAVEL TIME

CHAPTER 3

SEISMIC REFLECTION METHOD

- A Single Subsurface Interface
- Analysis of Arrival Times
- Normal Move out
- Determining of Velocity & Thickness
- Dipping Interface
- Common Field Procedures
- Velocity Analysis
- Applications in Petroleum exploration

Problem Set - 3

SEISMIC REFLECTION

1. Reflection at normal incidence

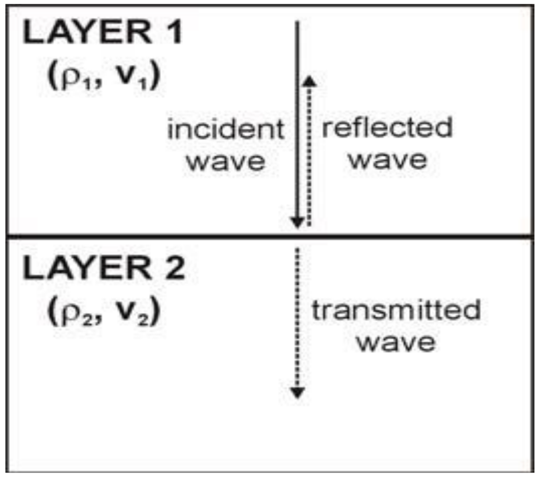
Consider two horizontal layers that have different seismic velocities (v) and densities (ρ). A P-wave with amplitude Ai travels downward through the upper layer and encounters the interface between the layers at 90° (normal angle). This produces two new P-waves: a **reflected** P-wave that travels upward through Layer 1 and a **transmitted** P-wave that enters Layer 2.
The **reflection co-efficient** is the ratio of the amplitudes of the reflected and incident waves**:** **R = Ar/Ai**
Similarly, the **transmission co-efficient** is the ratio of the amplitudes of the transmitted and incident waves: **T = At/Ai**
The amount of energy that is partitioned into transmission and reflection depend on **the angle** between the incident wave and interface and on the **acoustic impedance (Z)** of each layer:
Z1 = ρ1v1 and Z2 = ρ2v2

For **normal incident waves**, it can be shown that:

$$R = \rho_2 v_2 - \rho_1 v_1 / \rho_2 v_2 + \rho_1 v_1 = Z_2 - Z_1 / Z_2 + Z_1$$

$$T = 2 \rho_1 v_1 / \rho_2 v_2 + \rho_1 v_1 = 2Z_1 / Z_2 + Z_1$$

These are the **Zoeppritz equations**. There are also more complicated forms of the Zoeppritz equations that can be used for any angle of incidence.



These equations show that the reflection and transmission coefficients depend on the **difference in impedance** between the two layers.

• if Z1 = Z2, there is no reflection. All energy is transmitted into the second layer. This does not mean that ρ1=ρ2 and v1= v2! All that matters is that $\boldsymbol{\rho_1 v_1 = \rho_2 v_2}$.

• **R** can have a value of +1 to -1. **R** will be negative when $Z_1 > Z_2$. A negative value means that there will be a phase change of 180° in the phase of the reflected wave (a peak becomes a trough). This is called a **negative polarity reflection**.

• **T** is always positive – transmitted waves have the same phase as the incident wave. **T** can be larger than 1.

• Reflection coefficients for the Earth are generally less than ±0.2, with maximum values of ±0.5. Most energy is transmitted, not reflected.

Case 1: An increase in velocity with depth

A 600 m thick layer of sandstone overlies a granite basement with a higher velocity. A seismic wave is generated at the surface and travels vertically downward. At the sandstonegranite interface, the incident wave is split into a reflected wave and transmitted wave. The amplitude of the reflected and transmitted waves (Ar and At) can be calculated from the Zoeppritz equations. Assume that Ai = 1.0 and that there is no geometrical spreading, attenuation, or scattering. Velocity and density are constant within each layer.

First, calculate the impedance of each layer:

$Z1 = \rho_1 v_1 = 2700 \times 4.1 = 11{,}070$ (kg km s^{-1} m^{-3})

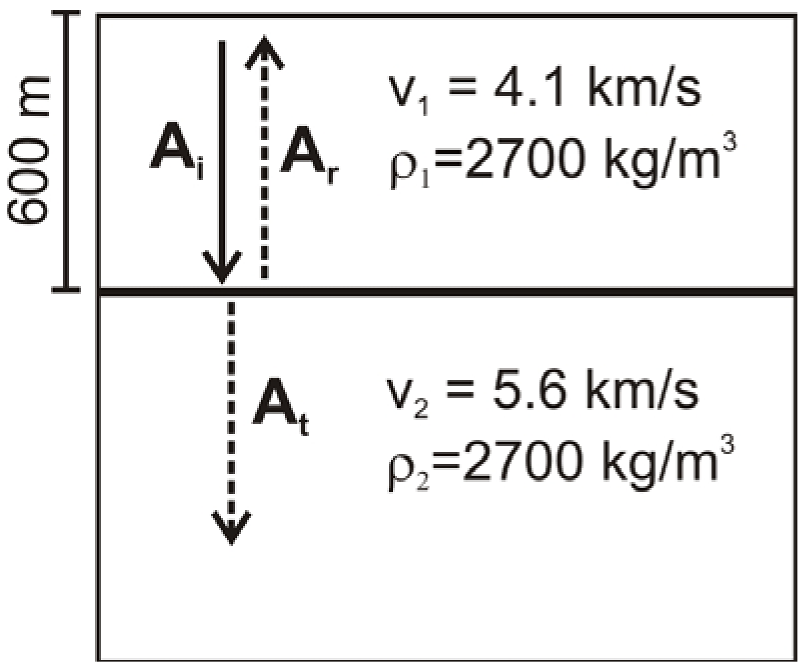
$Z2 = \rho 2v2 = 2700 \times 5.6 = 15{,}120$ (kg km s^{-1} m^{-3})

The reflection and transmission co-efficients are then:

R = 0.15 , T = = 0.85

The amplitude of the two waves are:

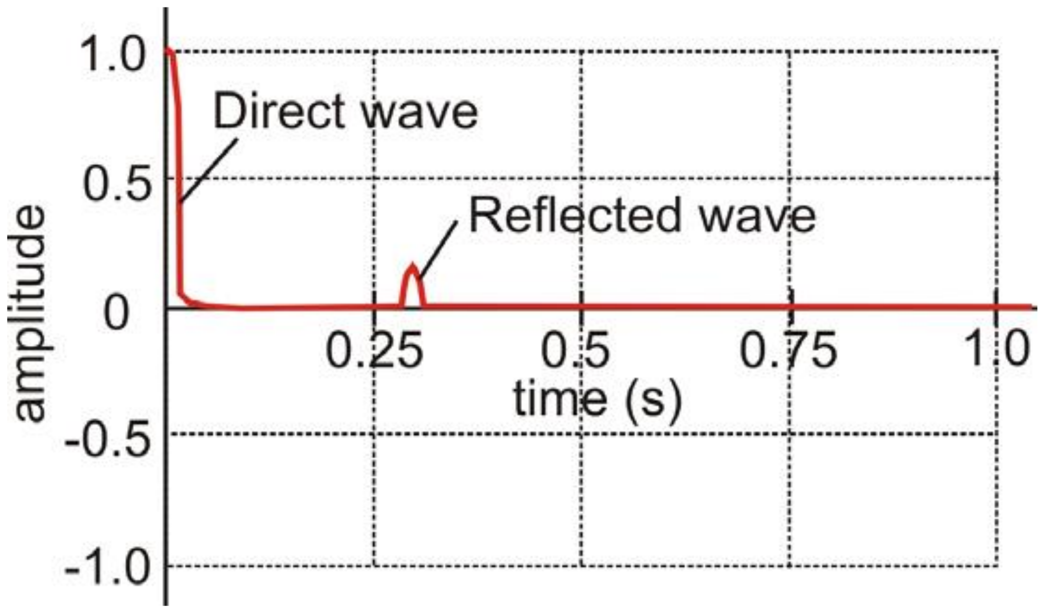
A_r = R x A_i = 0.15 **A_t = T x A_i = 0.85**



Consider a seismic survey configuration where you have a seismic source and a receiver on the ground next to each other. The receiver will record seismic waves that travel directly between the source and receiver, as well as seismic waves that are reflected upward to the surface. In this case, only one reflected wave will be recorded. The time at which it arrives at the receiver is:

Time = 2 x 600 / 4100 = 0.29s

The seismic record will look like this:



More than two layers:

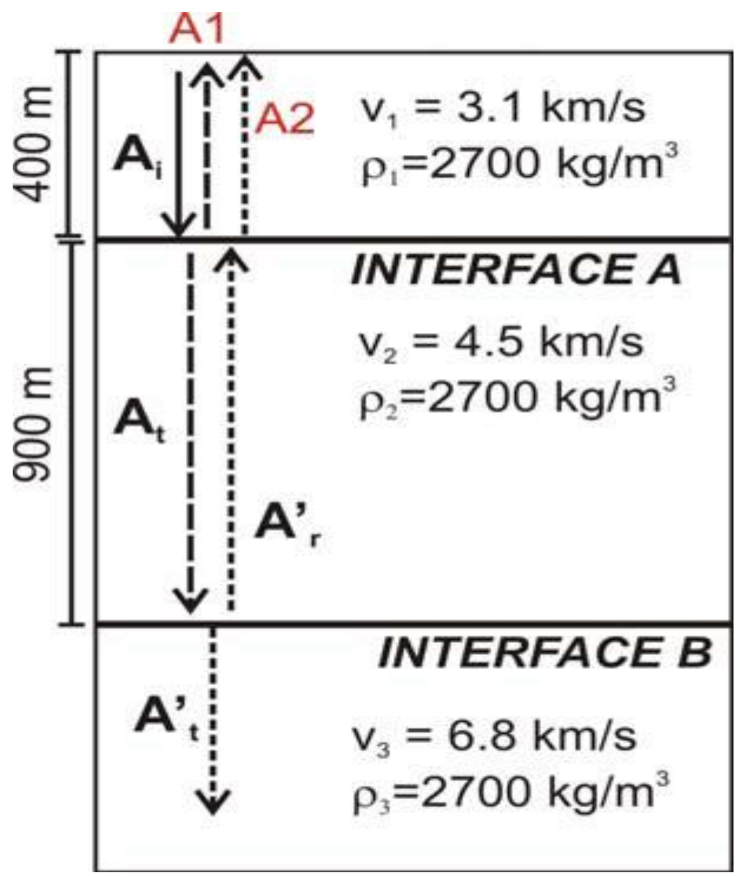
The same Zoeppritz equations can be applied to models with more than two layers.

Case 1: Velocity increase with depth

In this case, there will be two reflected waves that are recorded by the seismic station (also called **arrivals**).

A1 is the wave that is reflected from Interface A.

A2 is the wave that is transmitted through Interface A, reflected from Interface B, transmitted through Interface A, and then recorded at the surface



Arrival A1:

Amplitude: given by the reflection co-efficient at interface A (RA):
$Z1 = 2700 \times 3.1 = 8370$ $Z2 = 2700 \times 4.5 = 12150$
Therefore: $R_A = 0.18$,   Arrival time: 0.26s

Arrival A2:

To calculate the arrival time and amplitude of A2, we need to consider all the interfaces that it has encountered between the source and receiver.

1. Transmitted through Interface A.

The amplitude of the transmitted wave at A is **T_A = 0.82**

2. Reflected at interface B.

The reflection amplitude depends on:

• The amplitude of the incident wave. The incident wave is the one that was transmitted through Interface A: **TA = 0.82**

• Reflection co-efficient from Layer 2 to Layer 3 **(RB)**

Z2 = 12150 Z3 = 2700 × 6.8 = 18360

$\mathbf{R_B = 0.2}$

Therefore, the amplitude of the reflected wave is**: TA × RB = 0.16**

3. Transmitted through Interface A.

The amplitude of the transmitted wave is the product of:

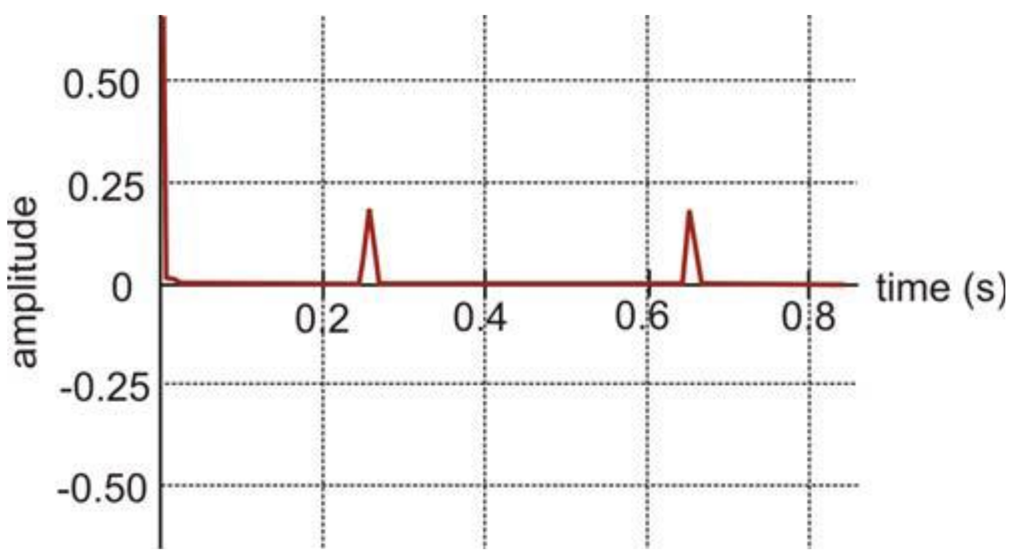
• the amplitude of the reflected wave from Interface **B (= TA × RB =0.16)**

• transmission co-efficient from Layer 2 to Layer 1

Therefore **the final amplitude** of A2 will be: **TA × RB × T'A = 0.16 × 1.18 = 0.19**

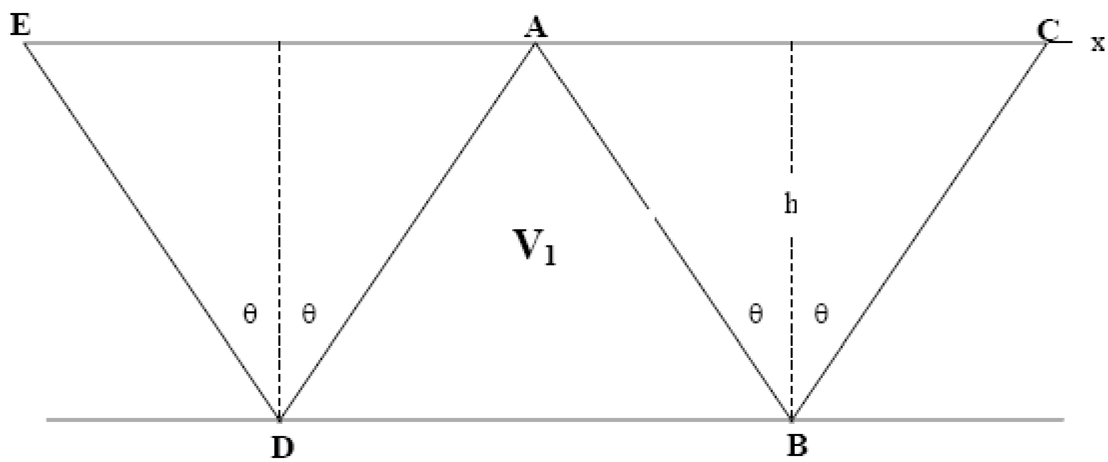
The **total travel time** of A2 will be: **0.13 + 0.40 + 0.13s = 0.66s**

The seismic record will look like:



Reflection time- distance plots

Consider a source (shot point) at point A with geophones spread out along the x-axis on either side of the shot point.



A raypath from A to C or A to E is: $2\sqrt{h^2 + \left(\frac{x}{2}\right)^2}$

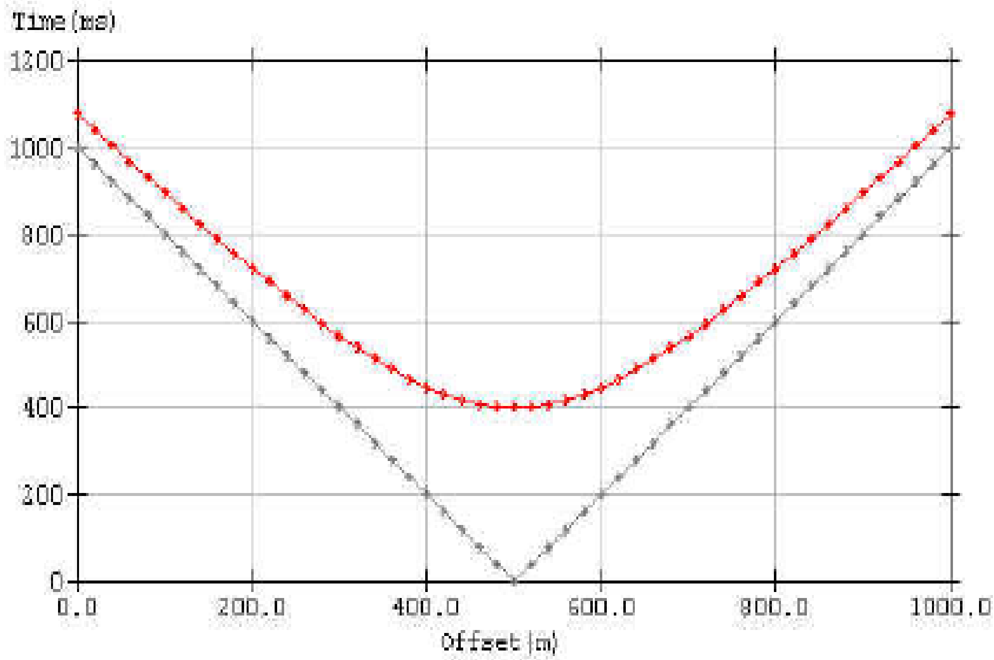
The travel time, t, is the raypath divided by the velocity, V_1, or:

$$t = \frac{\sqrt{x^2 + 4h^2}}{V_1}$$

Rearranging:

$$\frac{V_1^2 t^2}{4h^2} - \frac{x^2}{4h^2} = 1$$

This is the equation of a hyperbola symmetric about the t axis. The travel time plot for the direct wave arrivals and the reflected arrivals are shown in the following plot. The first layer is 100 m thick and its velocity is 500 m/s. The intercept of the reflected arrival on the t axis, t_i, is the two-way zero offset time and for this model is equal to 400ms. At large offsets the hyperbola asymptotes to the direct wave with slope $1/V_1$.



In most seismic reflection surveys the geophones are placed at offsets small compared to the depth of the reflector. Under this condition an approximate expression can be derived via:

$$t^2 = \frac{4h^2}{V_1^2} + \frac{x^2}{V_1^2}$$

which can be rewritten as;

$$t = \frac{2h}{V_1}\left[1 + \left(\frac{x}{2h}\right)^2\right]^{\frac{1}{2}}$$

or since $\frac{2h}{V_1} = t_i$,

$$t = t_i\left[1 + \left(\frac{x}{V_1 t_i}\right)^2\right]^{\frac{1}{2}}$$

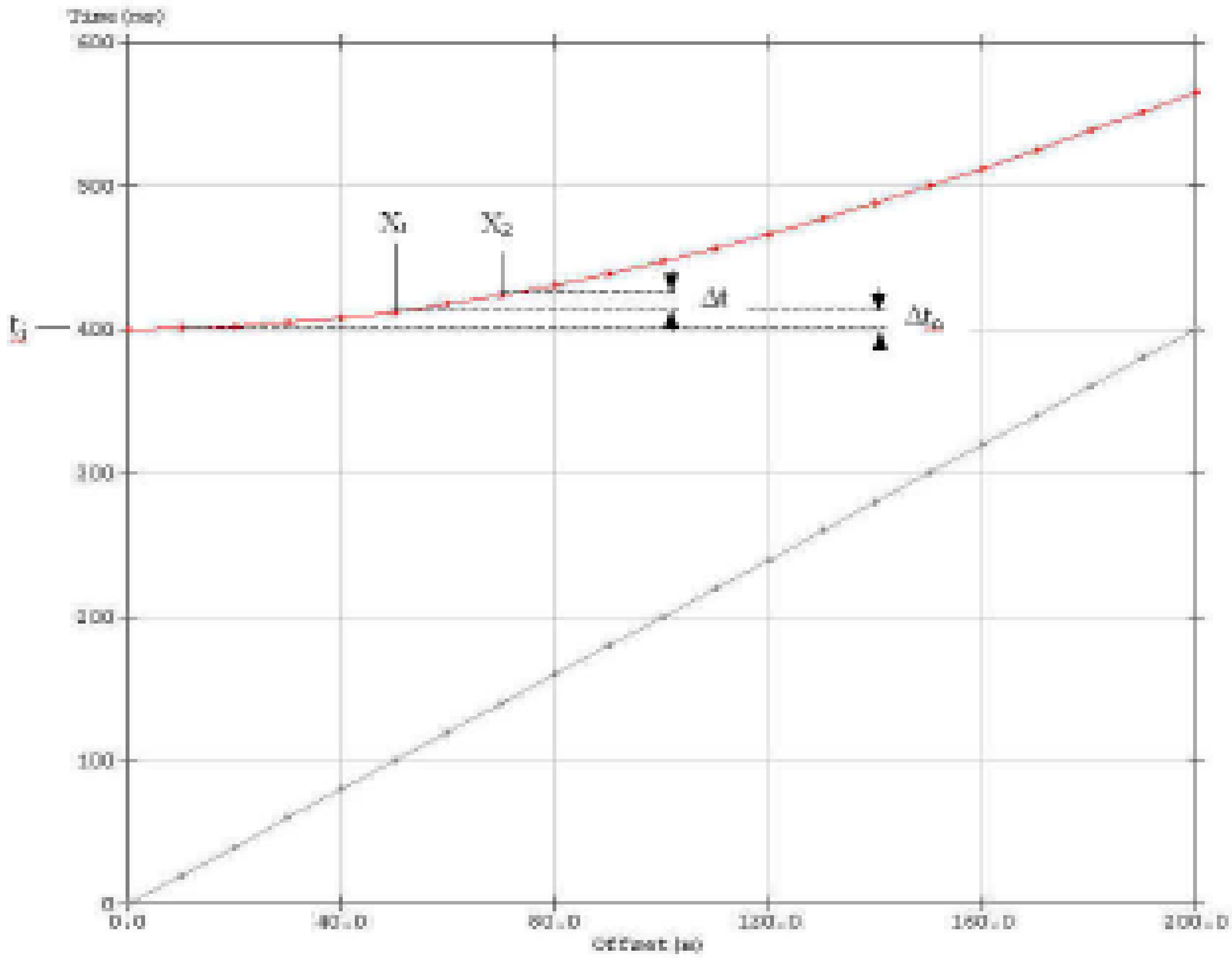
Since $\frac{x}{V_1 t_i}$ is less than 1, the square root can be expanded with the binomial expansion. Keeping only the first term in the expansion the following expression for the travel time is obtained:

$$t = t_i\left[1+\frac{1}{2}\left(\frac{x}{V_1 t_i}\right)^2\right]$$

This is the basic travel time equation that is used as the starting point for the interpretation of most reflection surveys.

Moveout

A useful parameter for characterizing and interpreting reflection arrivals is the moveout, the difference in travel times to two offset distances. The following expanded plot of one side of the hyperbola of the previous reflection plot shows the moveout, Δt, for two small offsets.



Using the small offset travel time expression for x_1 and x_2 yields the following expression for the moveout:

$$\Delta t = \frac{x_2^2 - x_1^2}{2V_1^2 t_i}$$

The normal moveout (NMO), Δt_n, is a special term used for the moveout when x_1 is zero. The NMO for an offset x is then:

$$NMO, \Delta t_n, = \frac{x^2}{2V_1^2 t_i}$$

With the value of the intercept time, t_i, the velocity is determined via:

$$V_1 = \frac{x}{(2t_i \Delta t_n)^{1/2}}$$

and the depth is then determined by:

$$h = \frac{V_1 t_i}{2}$$

For a given offset the NMO decreases as the reflector depth increases and/or as the velocity increases.

In a layered medium the velocity obtained from the NMO of a deep reflector is an average of the intervening layer velocities. Dix (1955) found that the root-mean-square velocity defined by:

$$V_{rms} = \left(\frac{\sum_1^n V_i^2 t_i}{\sum_1^n t_i} \right)^{\frac{1}{2}}$$

where V_i is the velocity in layer i and t_i is the travel time in layer i is the best average to use.

In interpretation the NMO's for successive reflections are used to obtain the average velocity to each reflector. Assuming these are the V_{rms} velocities defined above then Dix (1955) showed that the velocity in the layer bounded by the n^{th} and $n\text{-}1^{th}$ layer is given by:

$$V_n = \left(\frac{V_{rms_n}^2 t_n - V_{rms_{n-1}}^2 t_{n-1}}{t_n - t_{n-1}} \right)^{\frac{1}{2}}$$

Dip moveout

If the interface is dipping as in the figure below the up-dip and down-dip travel times are changed by an amount dependant on the dip angle θ. The time-distance plot is still a hyperbola but the axis of symmetry is shifted up-dip by 2h sinθ. (Shown by the dashed line in the figure. Note also that the depth is still the perpendicular distance from the interface to the shot point). The binomial expansion for the travel time for small offsets becomes:

$$t = t_i \left(1 + \frac{x^2 + 4xh\sin\theta}{2V^2 t_i^2} \right)$$

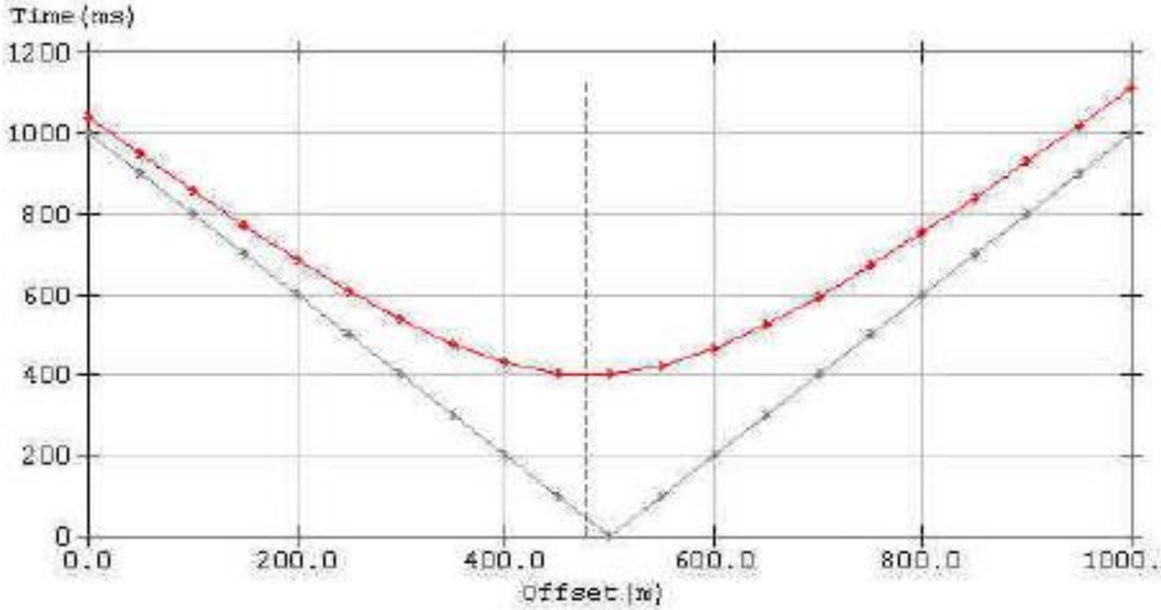
For geophones offset a distance x up-dip and down-dip, the dip moveout is defined as:

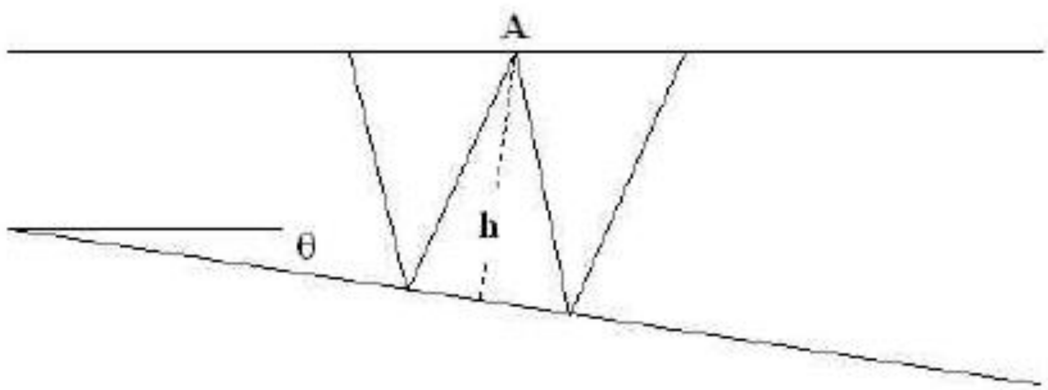
$$dip\ moveout = \Delta t_d = t_{+x} - t_{-x} = \frac{2x\sin\theta}{V}$$

For small dips when $\theta \approx \theta$ the dip moveout yields the dip via;

$$\theta \approx \frac{V\Delta t_d}{2x}$$

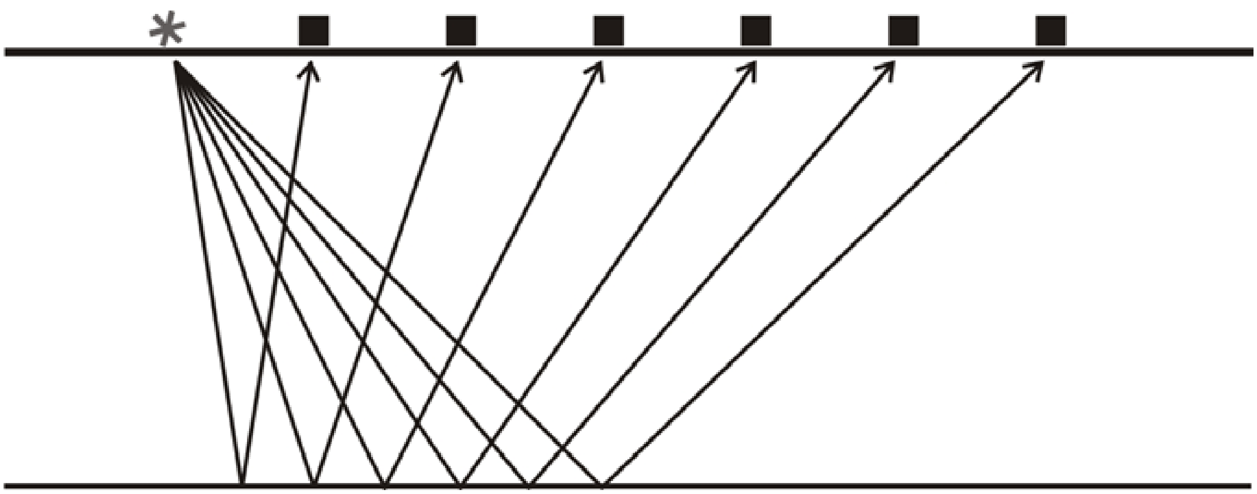
The velocity can be obtained with sufficient accuracy by averaging the velocities obtained in the usual manner from the up-dip and down-dip NMO's.



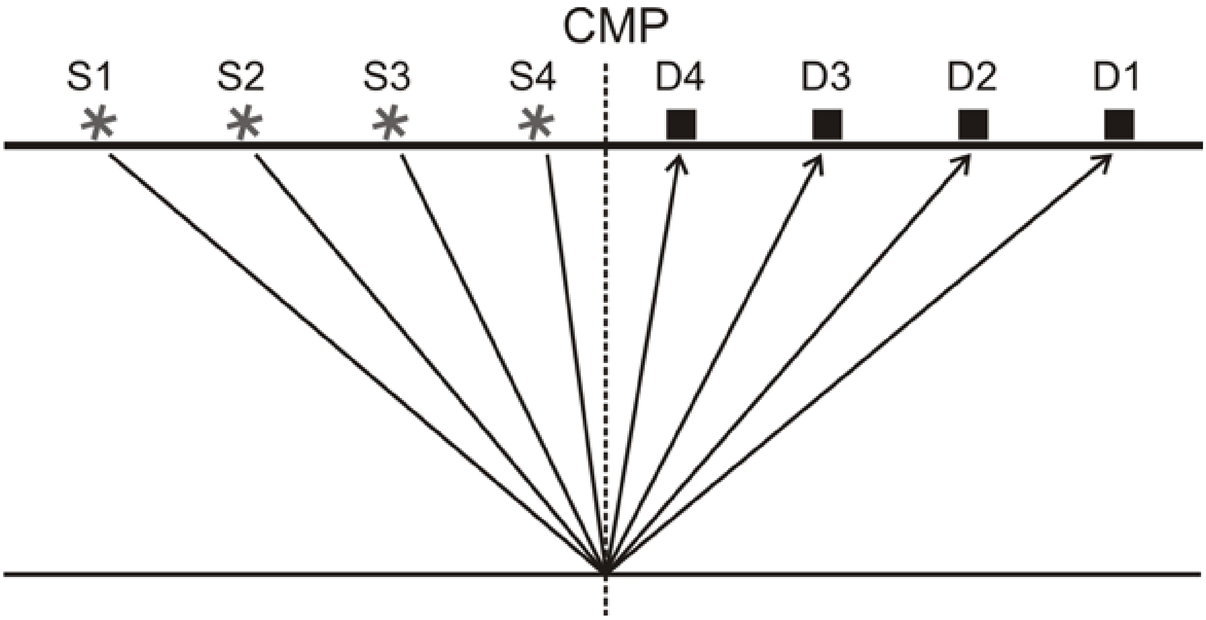


Common Mid-Point Gathers

There are two disadvantages to using only a shot gather for analysis:
(1) Reflections tend to have a low amplitude (generally less than 20% of the incident wave amplitude). This means that noise in the seismic data can obscure reflections.
(2) Each reflection occurs at a different point on the interface. The analysis of shot gathers assumes uniform horizontal layers. If there are significant lateral variations in structure, this assumption is no longer valid and the analysis can result in errors.



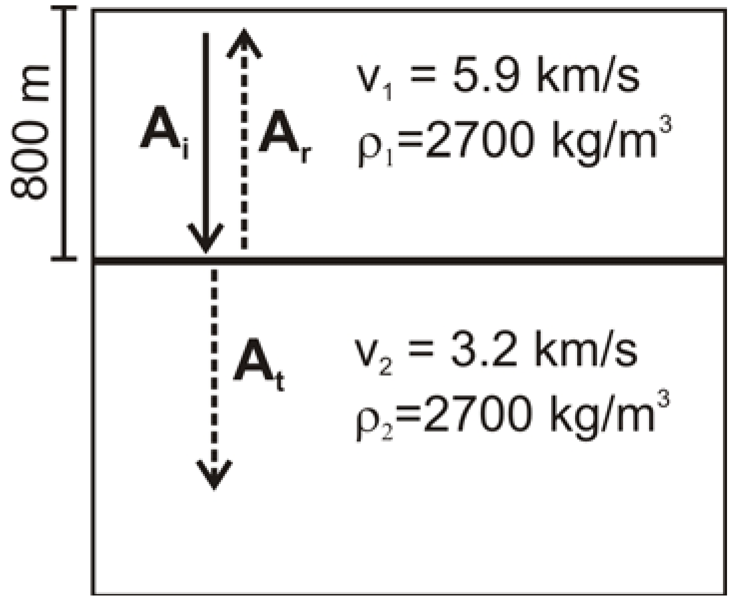
These problems can be overcome by using shots from a number of different points and multiple detectors. The source and detectors are moved in between shots.
From the complete data set, a subset of traces are chosen that have a common reflection point. The reflection point is taken to be halfway in between the shot and the detector.
• this is valid for areas with horizontal layering
• if the reflection occurs at a dipping interface, this is an approximation, but does not introduce large errors



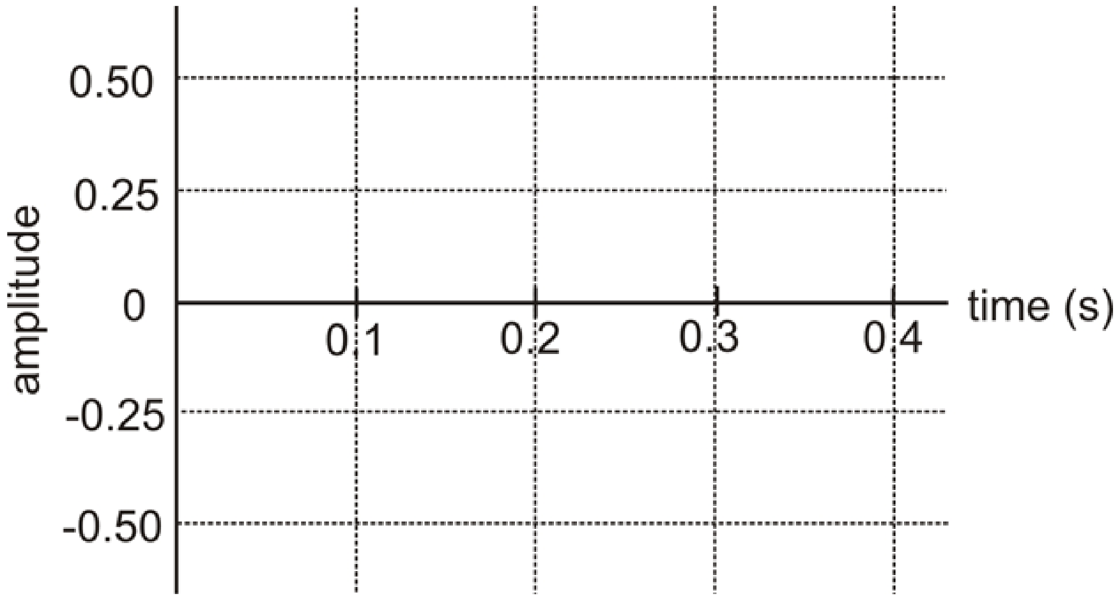
The collection of traces with the same reflection point is called a **common mid-point gather (CMP gather)** or common depth point (CDP) gather.

PROBLEM SET - 3

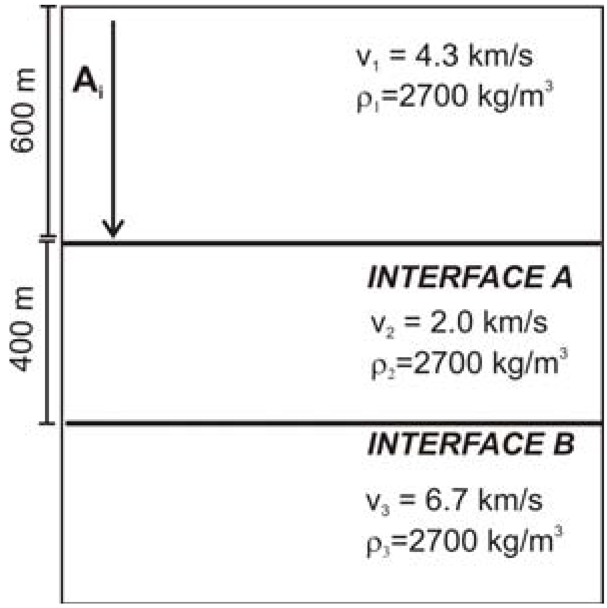
1. Consider a high velocity layer that overlies a low velocity layer.



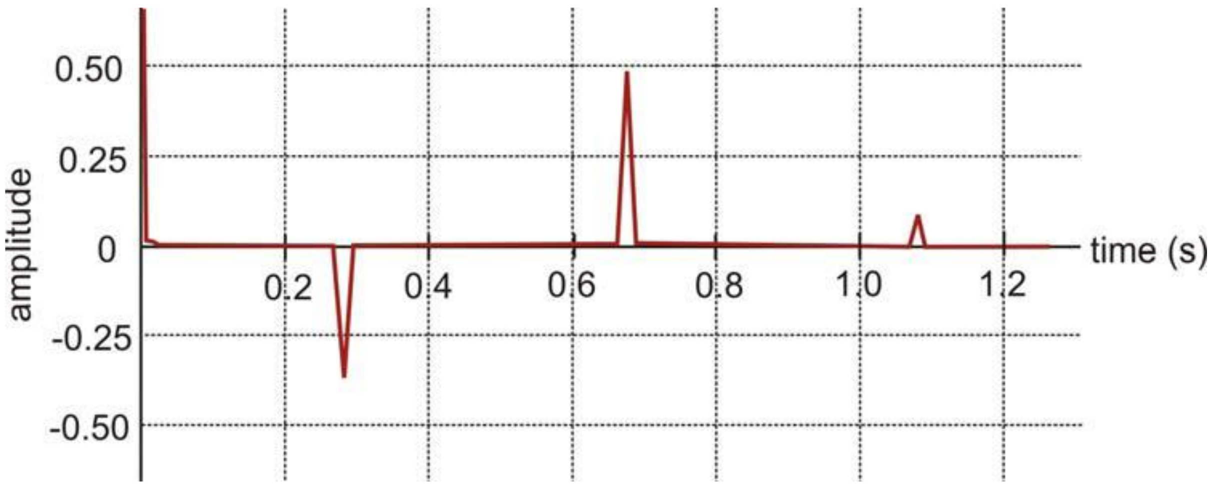
A. What are the amplitudes of the reflected and transmitted waves?
B. At what time will the reflected wave arrive back at the surface? What is its polarity?
C. What will the seismic record at the surface look like?



2. Consider a case with a low velocity layer. This could represent a gas- filled layer within high velocity rocks. What are the first three arrivals (after the direct P- wave)?



Note : The corresponding seismic record will be:



3. CALCULATE THE REFLECTION AND TRANSMISSION COEFFICIENTS FOR NORMAL INCIDENCE ON EACH BOUNDARY WITH THE FOLLOWING MATERIAL PROPERTY CONTRASTS. WHAT PERCENT OF THE ENERGY IS REFLECTED AND TRANSMITTED AT EACH INTERFACE.

	VELOCITIES FT/S.	**Density**
VERY STRONG REFLECTOR	11000 & 15000	NONE
GOOD REFLECTOR	14000 & 15000	NONE
WEAK REFLECTOR	8000 & 8200	NONE
SOFT OCEAN BOTTOM	NONE	1.0 & 2.0
HARD OCEAN BOTTOM	5000 & 10000	1.0 & 2.5
WEATHERED OVER UNWEATH. MATERIAL	1600 & 6000	1.6 & 2.0
SAND WITH 30% POROSITY OVER THE SAME SAND WITH GAS-FILLED PORES.	6000 & 9000	1.4 & 2.4

4. Seismic waves with a dominant frequency of 50 Hz travel through sediments with a velocity of 3000 m/s.

- What is the smallest layer thickness that will be detected?
- If the deepest reflector is at a depth of 2000 m, what is the size of the smallest horizontal feature that will be detected?
- What is the largest geophone spacing that should be used in the survey?

CHAPTER 4

EARTHQUAKE SEISMOLOGY

- Definition and Historical review
- Classification of Earthquakes
- Earthquakes : Where and Why
- Causes of Earthquakes
- Earthquake Epicenter & Hypocenter
- Magnitude & Intensity

EARTHQUAKE SEISMOLOGY

COMPOSITION AND STRUCTURE OF THE EARTH

1. **Crust.** The outermost rock layer, divided into continental and oceanic crust

a. **Continental Crust** (averages about 35 km thick; 60 km in mountain ranges; diagram shows range of 20-70 km) ***Granitic composition***

b. **Oceanic Crust** (5 - 12 km thick; diagram shows 7-10 km average)

Basaltic composition. Oceanic crust has layered structure (ophiolite complex) consisting of the following:

1. Pillow basalts,
2. " Sheeted dikes " - interconnected basaltic dikes
3. Gabbro

2. **Mantle (2885 km thick).** The mantle stretches from the below the crust to 2900 km below the surface. The upper part is partially molten and the lower part is very dense. The main mantle rock is peridotite.

Lithosphere = outermost 100 km of Earth . Consists of the crust plus the outermost part of the mantle. Divided into tectonic or lithospheric **plates** that cover surface of Earth

Asthenosphere = low velocity zone at 100 - 250 km depth in Earth (seismic wave velocity decreases). Rocks are at or near melting point.

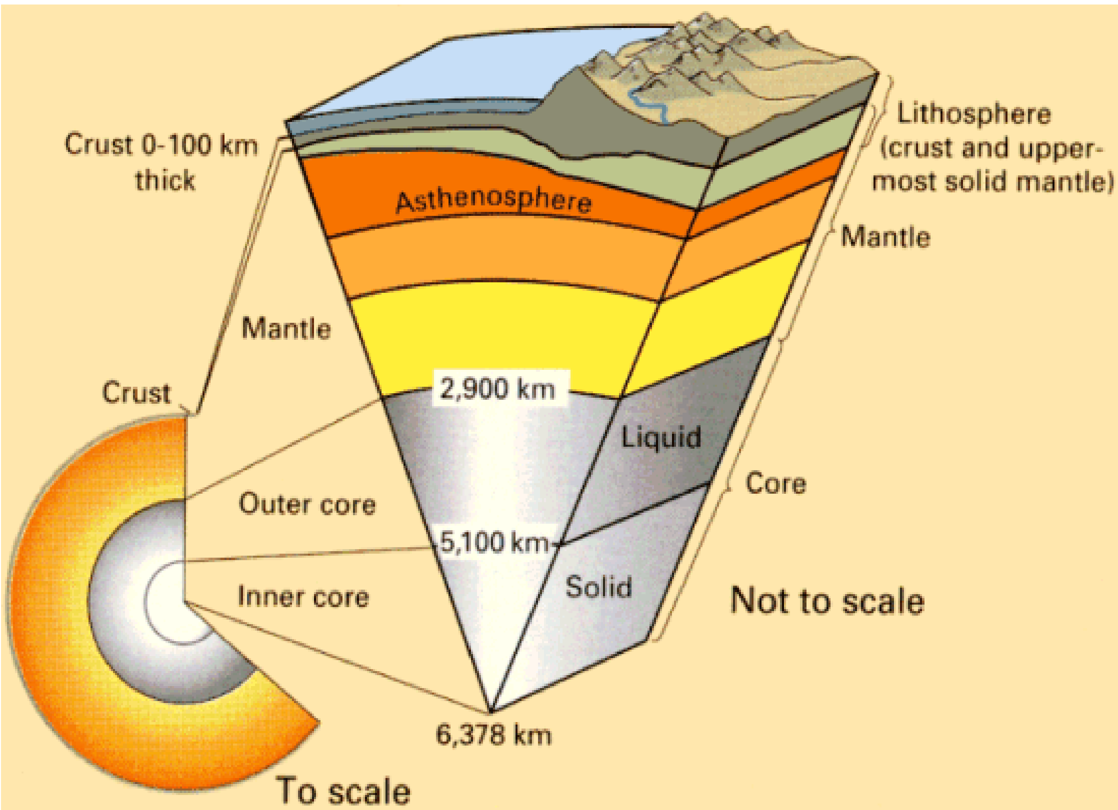
3. Outer core (2270 km thick)

S-waves cannot pass through outer core, therefore we know the outer core is liquid (molten).

Composition: Molten **Fe** (85%) with some **Ni**, based on studies of composition of meteorites. Core may also contain lighter elements such as Si, S, C, or O.

Convection in liquid outer core plus spin of solid inner core generates Earth's magnetic field. **Magnetic field** is also evidence for a dominantly iron core.

4. **Inner core (1216 km radius). Solid Fe (85%) with some Ni** - based on studies of meteorites



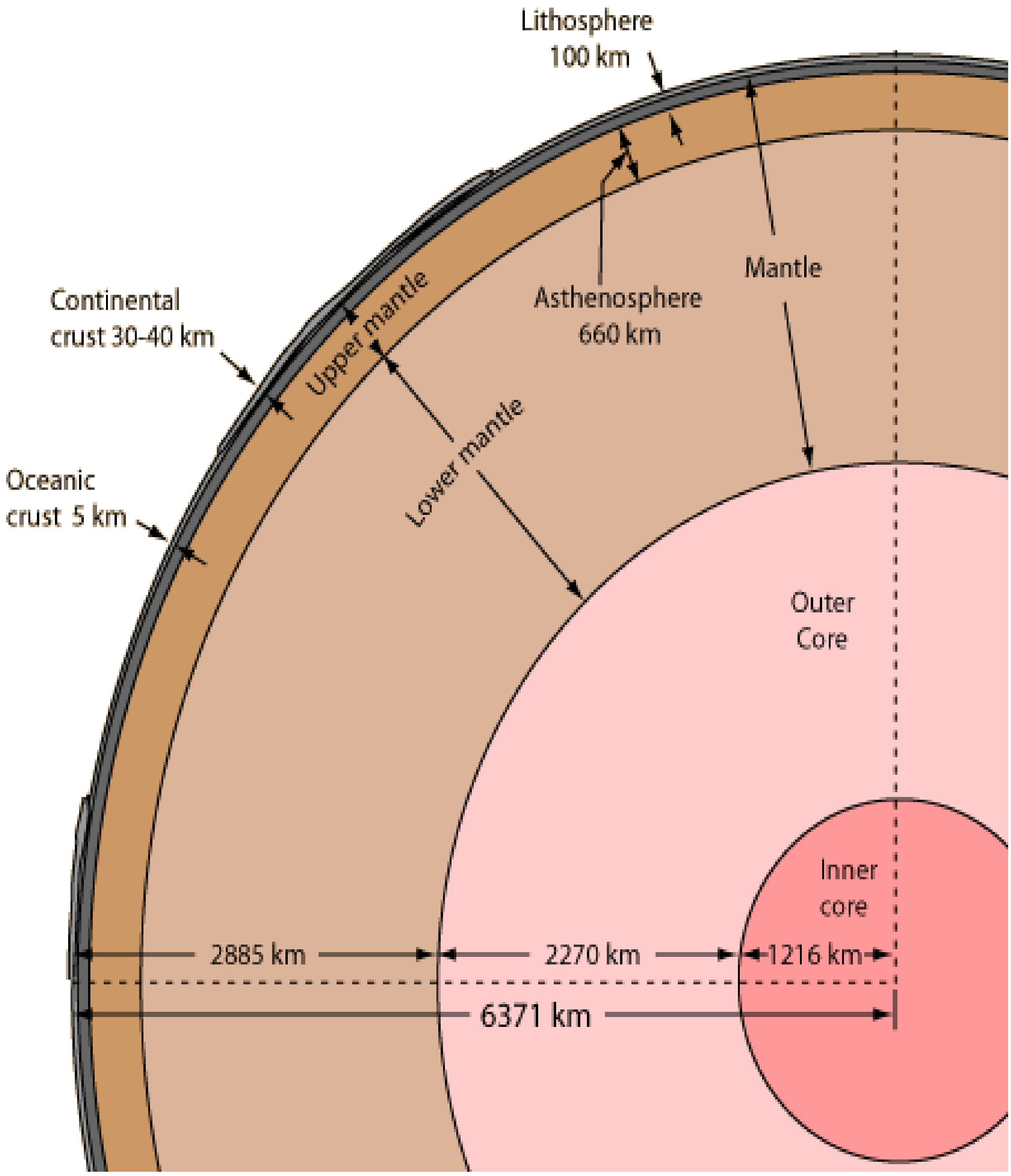
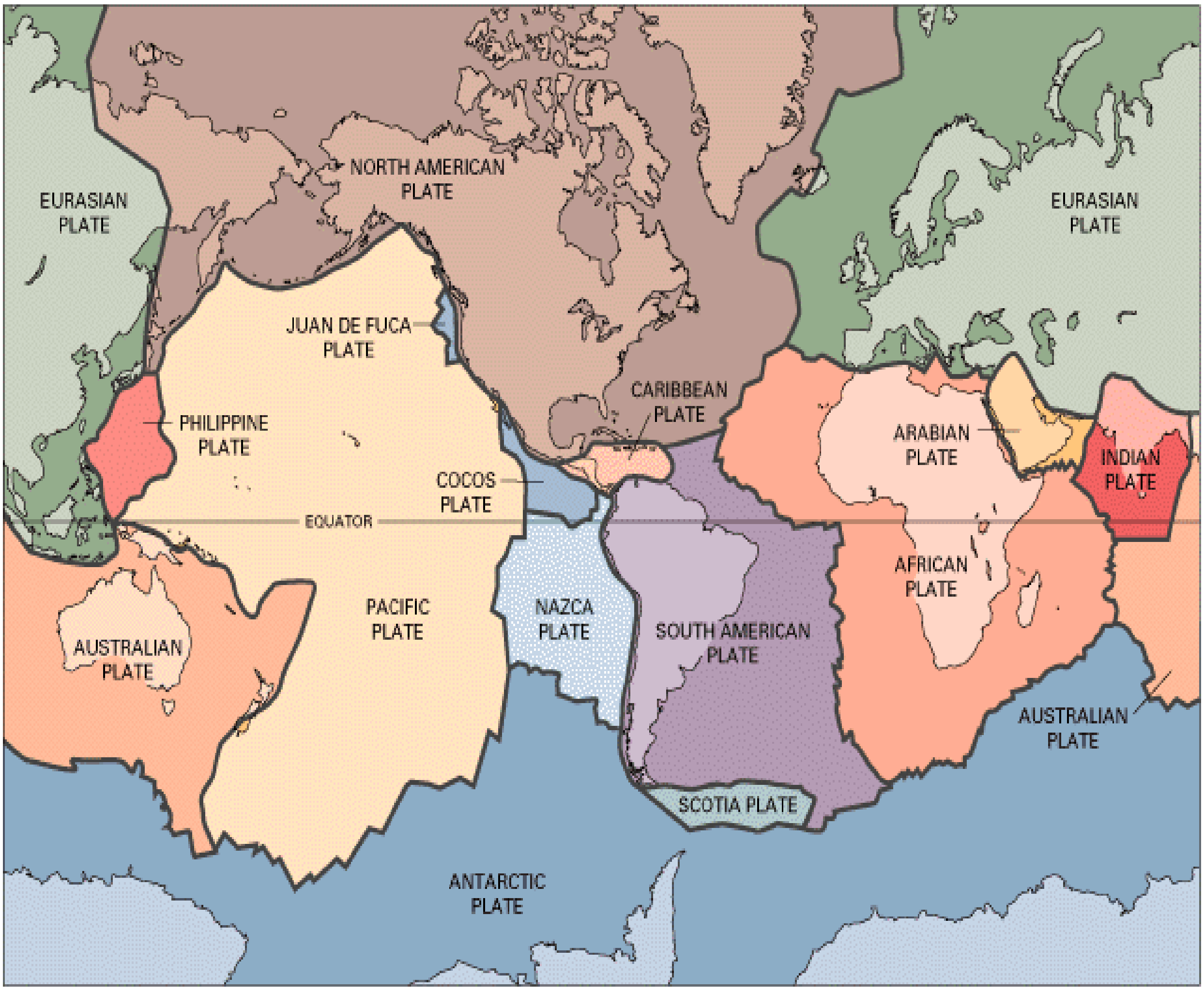

Lithosphere
100 km
Continental
crust 30-40 km
Oceanic
crust 5 km
Upper mantle
Lower mantle
Asthenosphere
660 km
Mantle
Outer
Core
Inner
core
2885 km
2270 km
1216 km
6371 km

Plate Tectonics

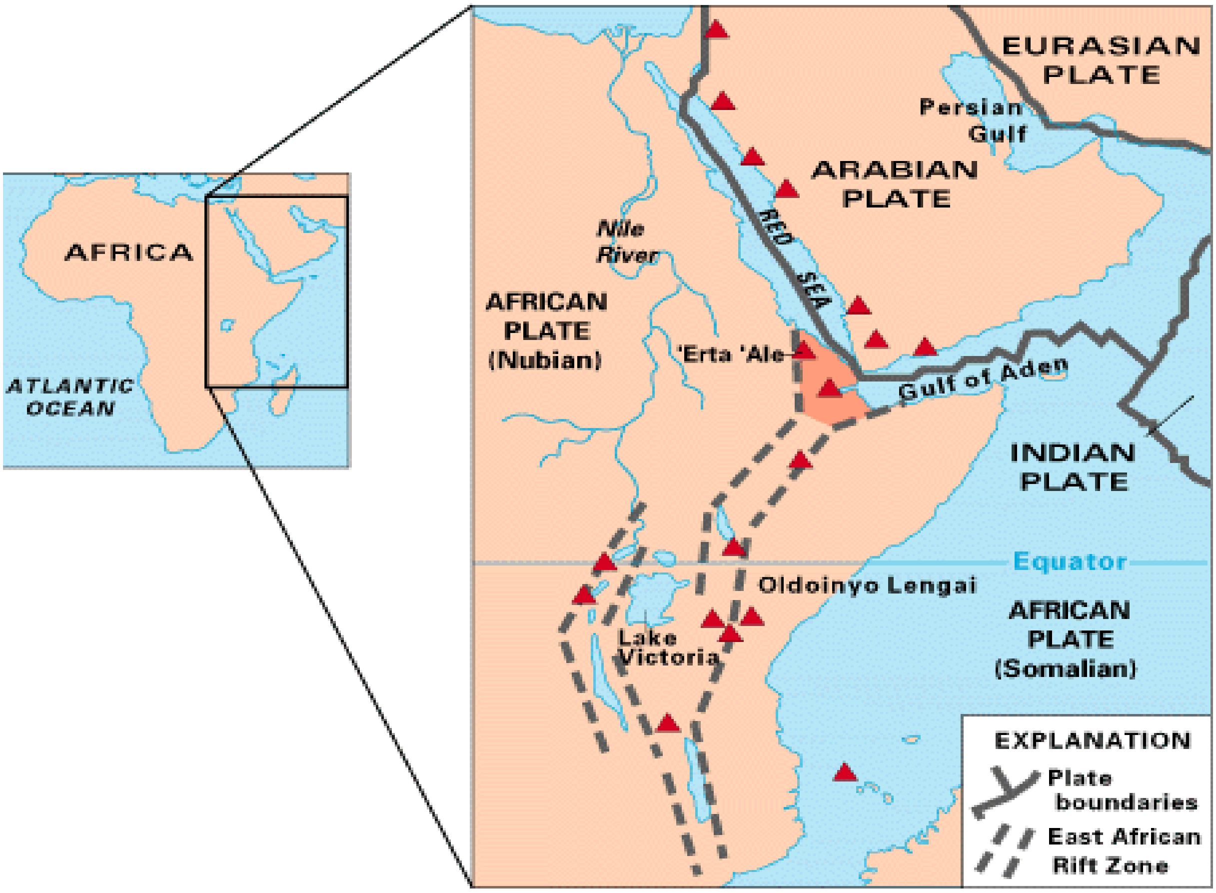
The Earth's lithosphere is broken up into 6 major plates and about 14 minor ones. Oceanic plates are 50-100 km thick. Continental plates are 100-250 km thick. Tectonic plates can include both continental and oceanic areas. Six major plates are:

1. Indian-Australian
2. Pacific
3. American (N. and S)
4. Antarctic
5. African
6. Eurasian



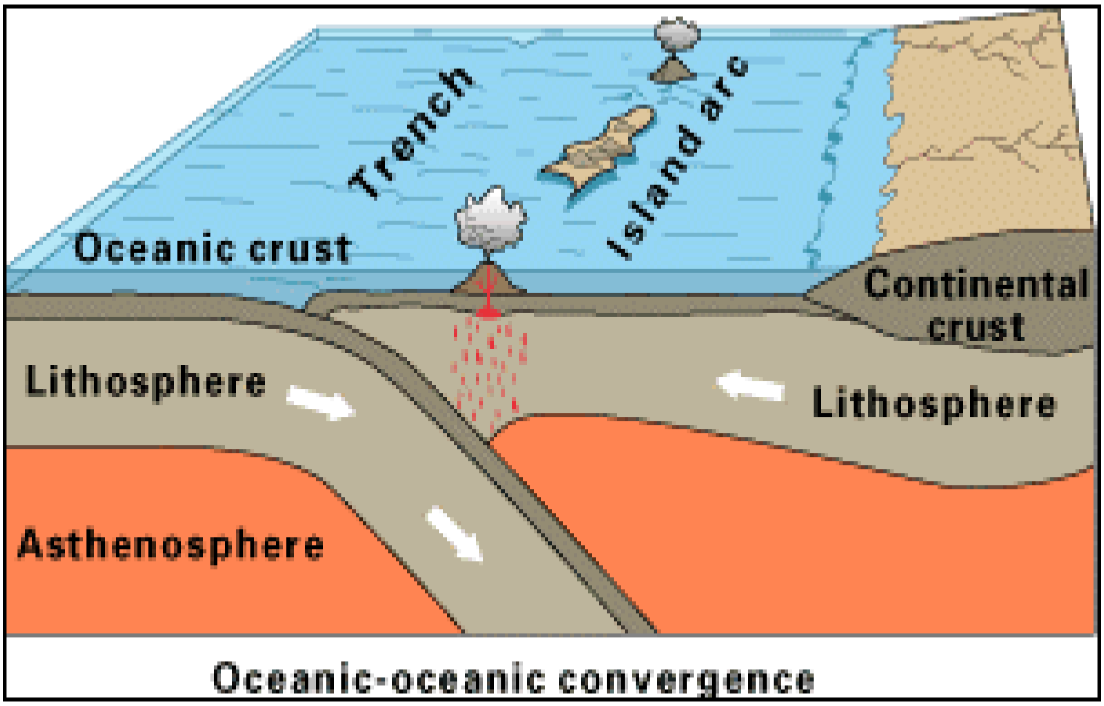
- **Plate Boundaries** - Tectonic plates interact in various ways as they move across the asthenosphere, producing volcanoes, earthquakes and mountain systems. There are 3 primary types of Tectonic Plate boundaries: Divergent boundaries; Covergent boundaries; and Transform.

- **<u>Divergent</u>** - Plates move away from one another, creating a tensional environment. Characterized by shallow-focus earthquakes and volcanism. Release of pressure causes partial melting of mantle peridotite and produces basaltic magma. Magma rises to surface and forms new oceanic crust. Occur in oceanic crust (oceanic ridges) and in continental crust (rift valleys). Continental rift valleys may eventually flood to form a new ocean basin.

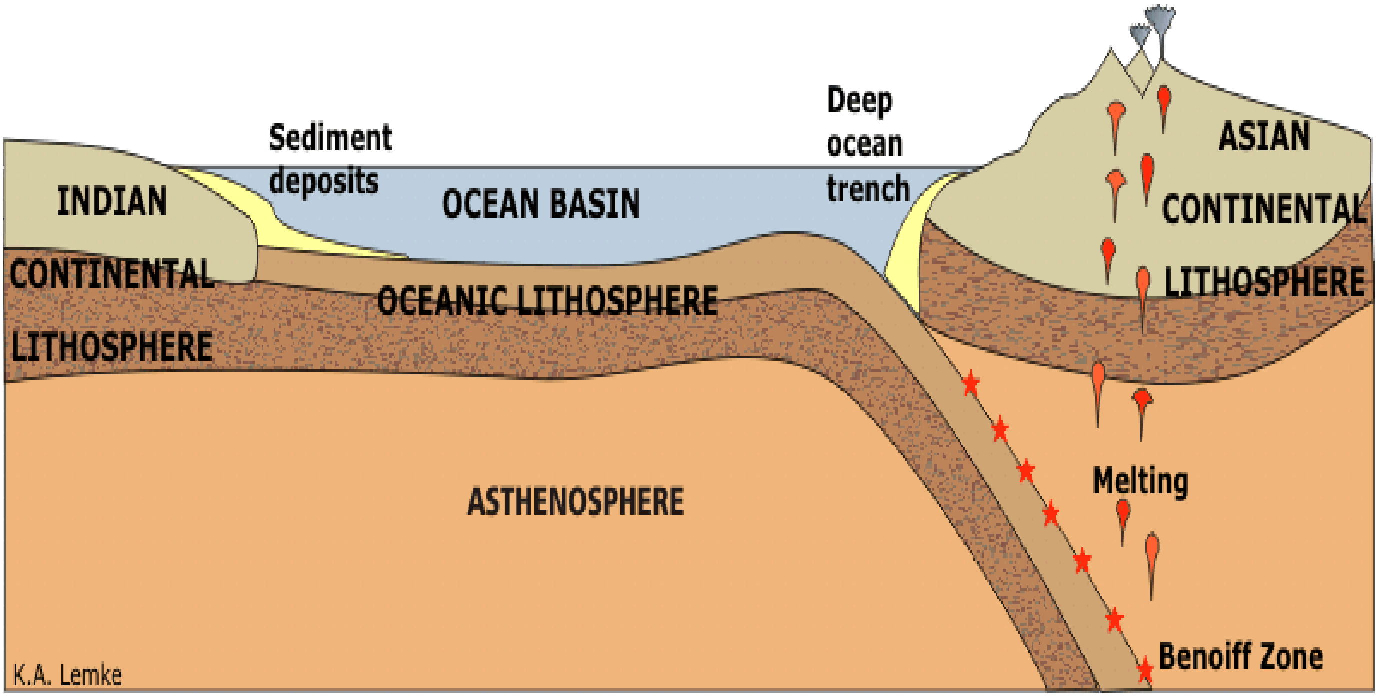


- **Convergent** - Plates move toward one another, creating a compressional environment. Characterized by deformation, volcanism, metamorphism, mountain building, seismicity, and important mineral deposits. Three possible kinds of convergent boundaries:

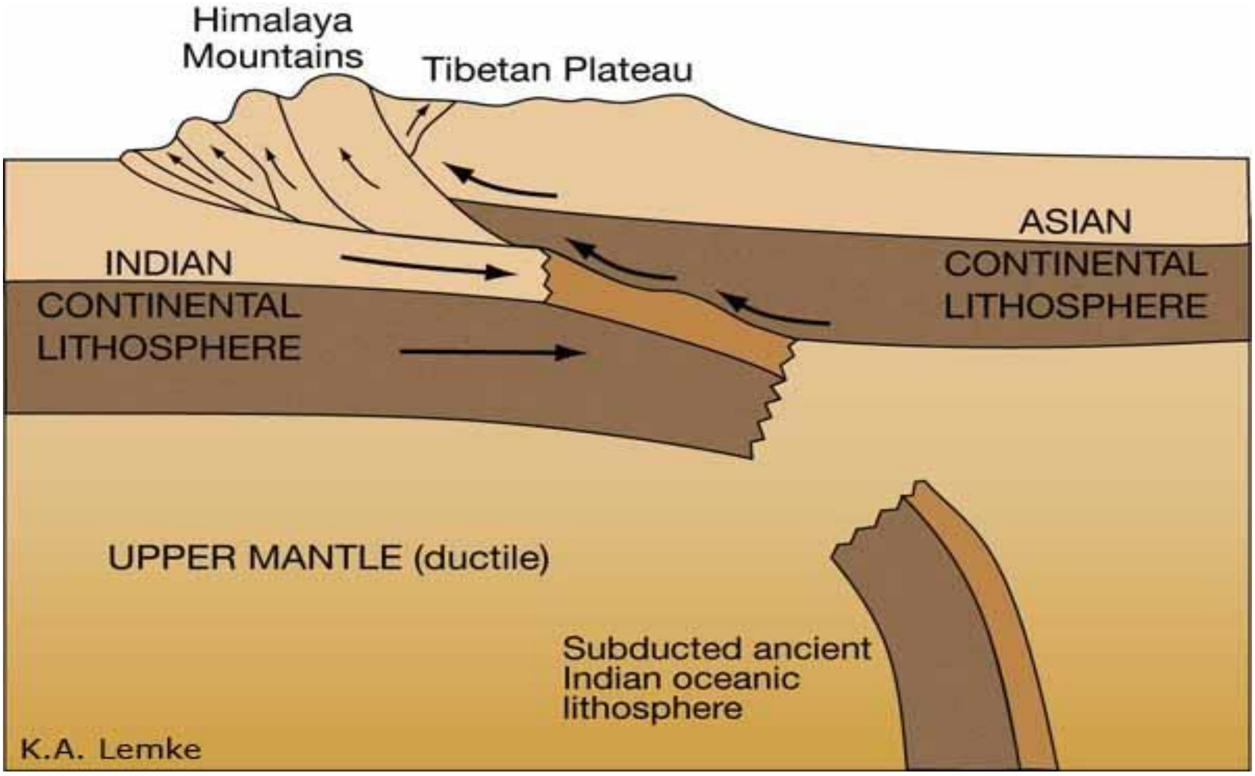
    1. **Oceanic-Oceanic Boundary** - One plate is subducted, initiating andesitic ocean floor volcanism on the other. Can eventually form an island arc volcanic island chain with an adjacent deep ocean trench. Characterized by a progression from shallow to deep focus earthquakes from the trench toward the island arc (Benioff zone). May also form a back-arc basin if subduction rate is faster than forward motion of overriding plate.



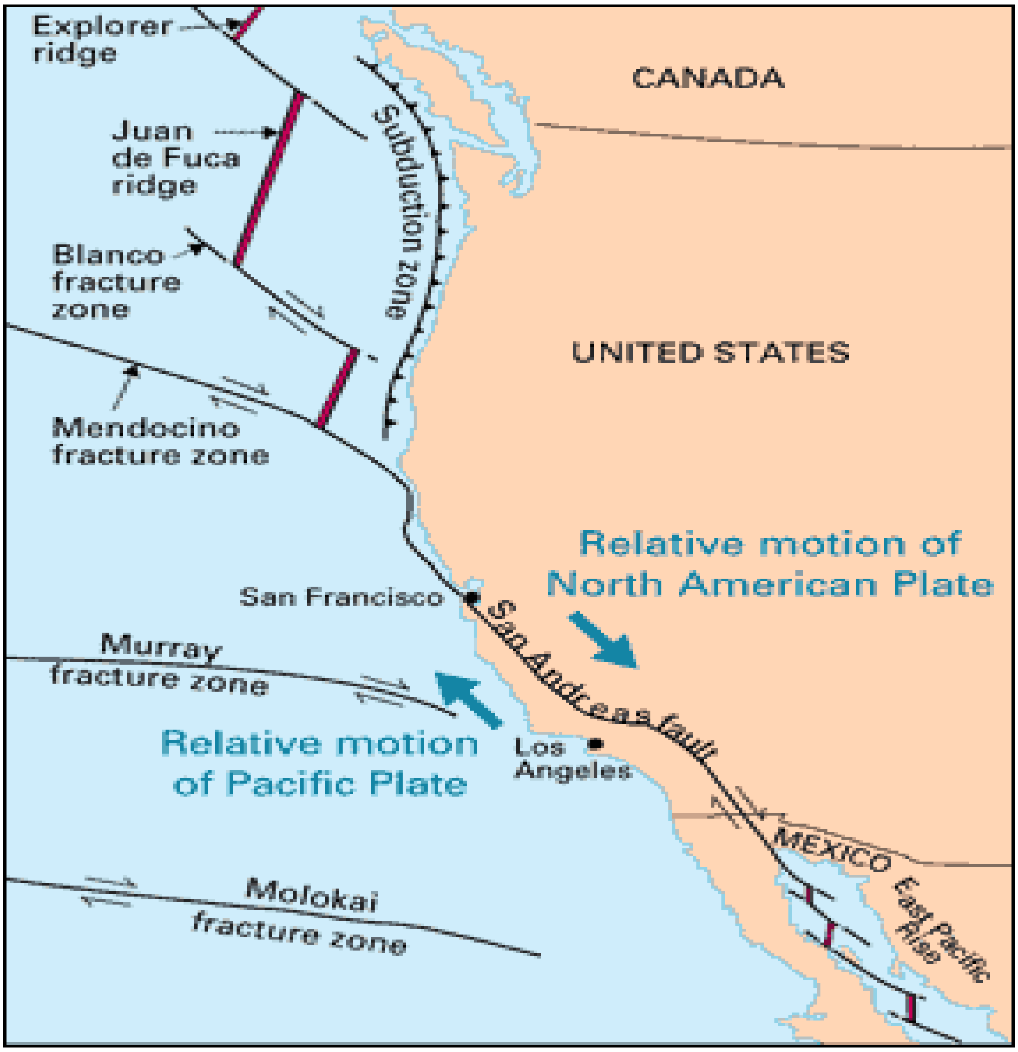
2. **<u>Oceanic-Continental Boundary</u>** - Oceanic plate is dense and subducts under the Lighter continental plate. Produces deep ocean trench at the edge of the continent. About half the oceanic sediment descends with the subducting plate; the other half is piled up against the continent. Subducting plate and sediments partially melt, producing andesitic or granitic magma. Produces volcanic mountain chains on continents called <u>volcanic arcs</u> and batholiths. Part of the oceanic plate can be broken off and thrust up onto the continent during subduction (<u>obduction</u>). Obduction can expose very deep rocks (oceanic crust, sea floor sediment, and mantle material) at the surface. Characterized by shallow to intermediate focus earthquakes with rare deep focus earthquakes.



3. **Continental-Continental Boundary** - Continental crust cannot subduct, so continental rocks are piled up, folded, and fractured into very high complex mountain systems. Characterized by shallow-focus earthquakes, rare intermediate-focus earthquakes. and practically no volcanism.



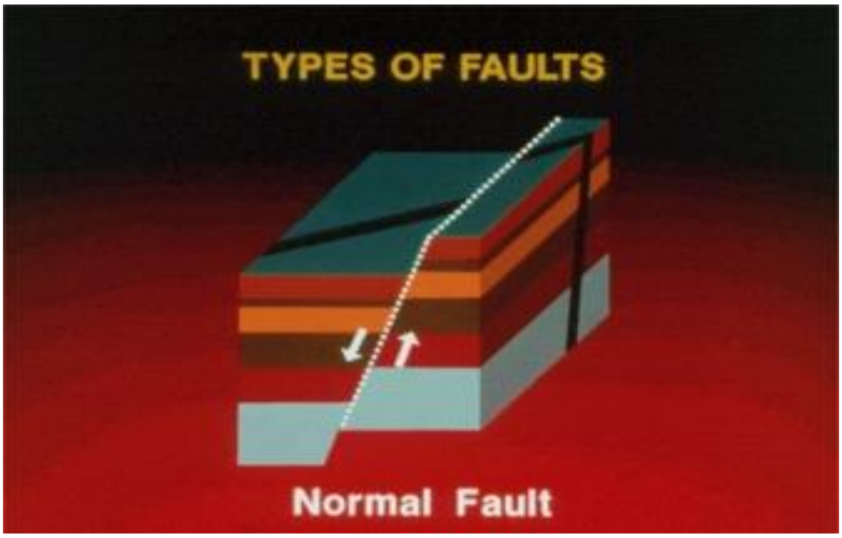
- **<u>Transform</u> -** Plates move laterally past one another. Largely shear stress with lithosphere being neither created nor destroyed. Characterized by faults that parallel the direction of plate movement, shallow-focus earthquakes, intensely shattered rock, and no volcanic activity. Shearing motion can produce both compressional stress and tensional stress where a fault bends. Transform faults occur on land, connect segments of the oceanic ridge, and provide the mechanism by which crust can be carried to subduction zones.



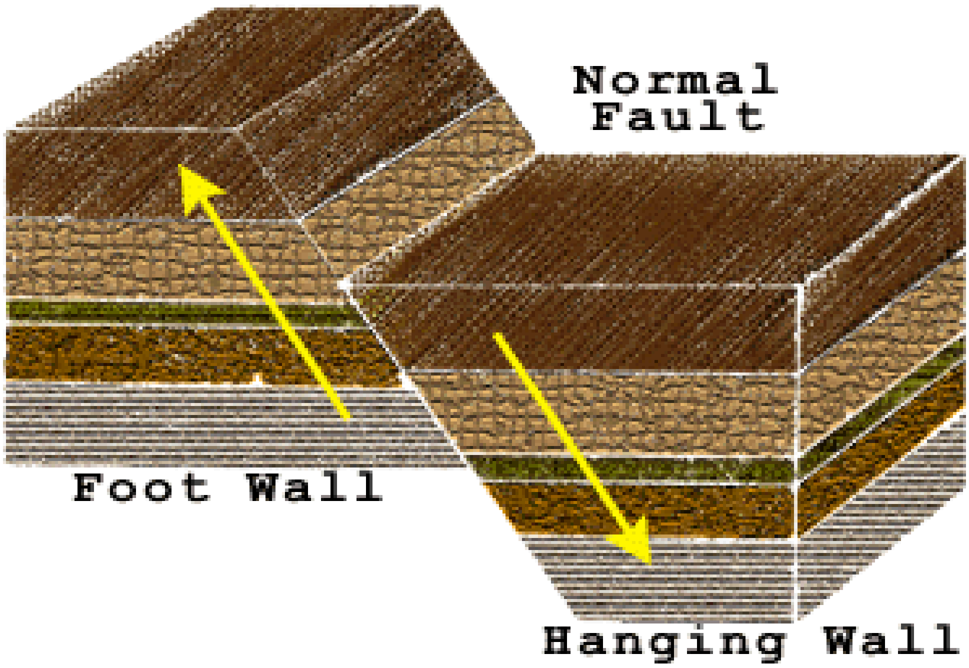
Types of Faults

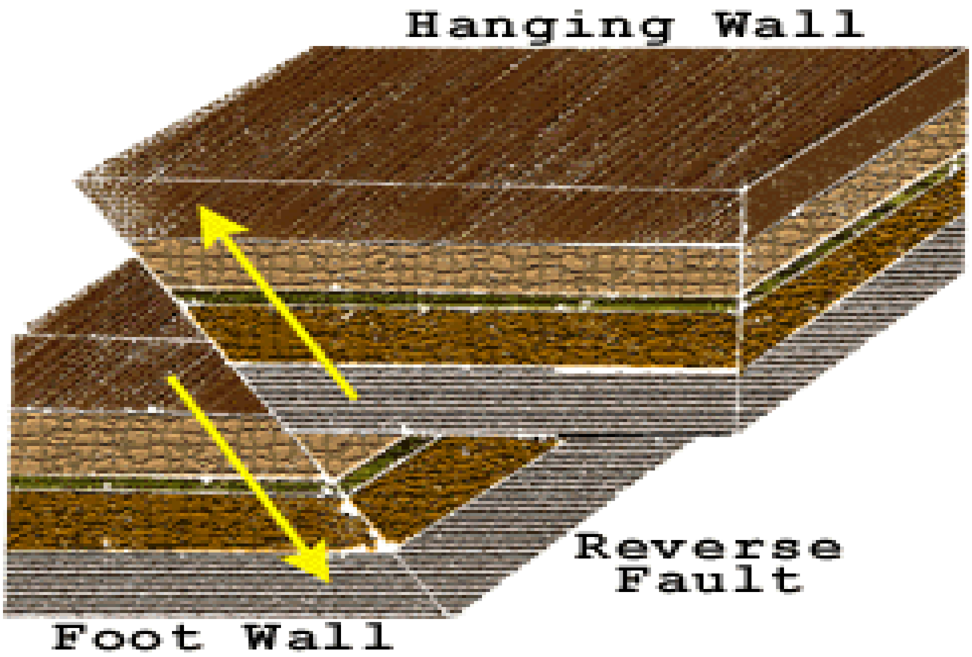
A fault is a fracture or zone of fractures between two blocks of rock. Faults allow the blocks to move relative to each other. This movement may occur rapidly, in the form of an earthquake - or may occur slowly, in the form of creep. Faults may range in length from a few millimeters to thousands of kilometers. Most faults produce repeated displacements over geologic time. During an earthquake, the rock on one side of the fault suddenly slips with respect to the other. The fault surface can be horizontal or vertical or some arbitrary angle in between.

Earth scientists use the angle of the fault with respect to the surface (known as the dip) and the direction of slip along the fault to classify faults. Faults that move along the direction of the dip plane are dip-slip faults and described as either normal or reverse, depending on their motion. Faults that move horizontally are known as strike-slip faults and are classified as either right-lateral or left-lateral. Faults that show both dip-slip and strike-slip motion are known as oblique-slip fault.



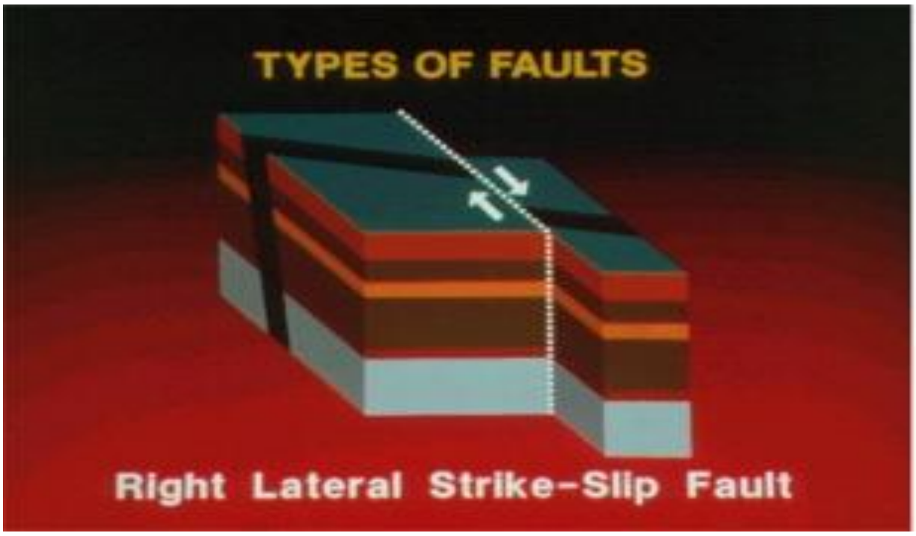
A normal fault is a dip-slip fault in which the block above the fault has moved downward relative to the block below. This type of faulting occurs in response to extension and is often observed in the Western United States Basin and Range Province and along oceanic ridge systems.

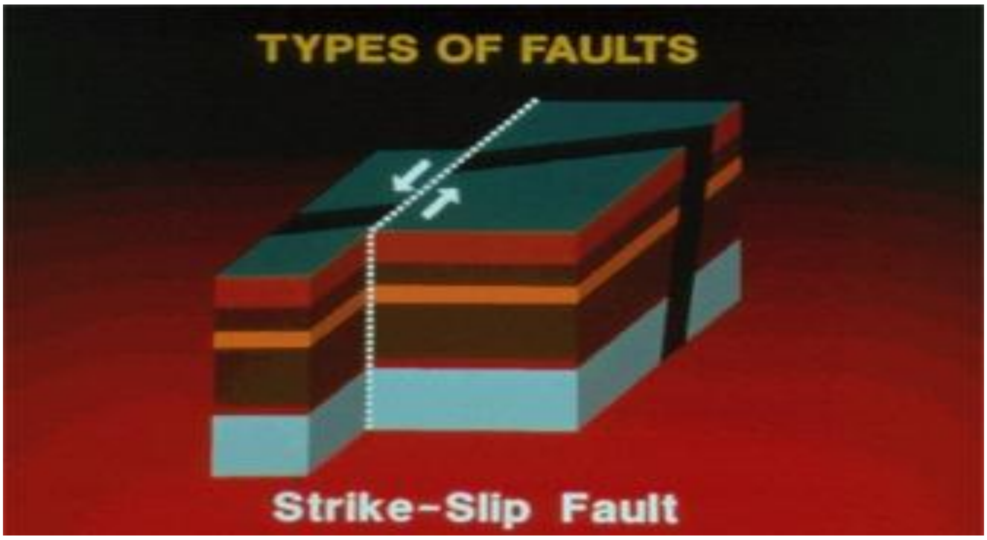




<u>A thrust fault</u> is a dip-slip fault in which the upper block, above the fault plane, moves up and over the lower block. This type of faulting is common in areas of compression, such as regions where one plate is being subducted under another as in Japan and along the Washington coast. When the dip angle is shallow, a reverse fault is often described as a thrust fault.

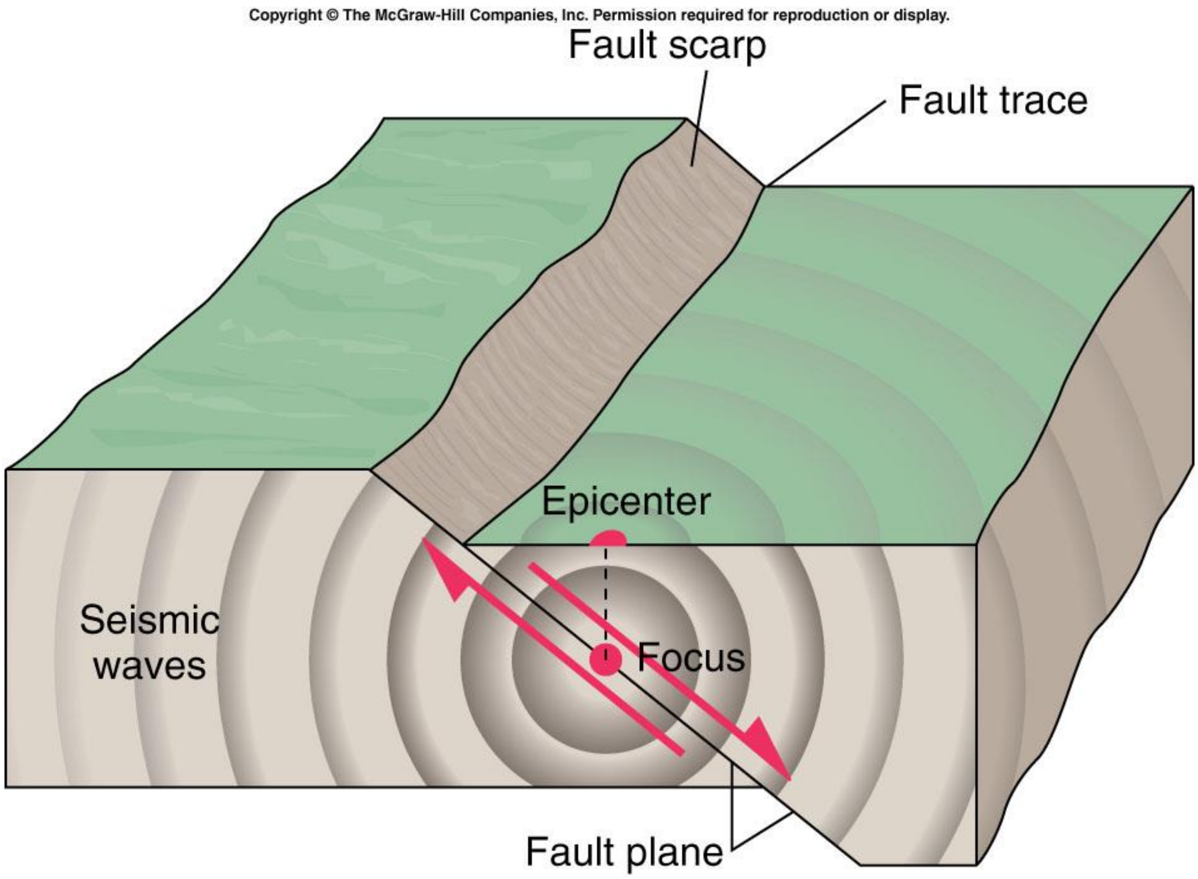
A strike-slip fault is a fault on which the two blocks slide past one another. These faults are identified as either right-lateral or left lateral depending on whether the displacement of the far block is to the right or the left when viewed from either side. The San Andreas Fault in California is an example of a right lateral fault.





CAUSES OF EARTHQUAKES

An earthquake is a sudden shuddering or trembling of the earth produced by shock waves or vibrations passing through it. Earthquakes occur in regions of the earth that are undergoing deformation. Energy is stored in the form of elastic strain as the region is deformed. This process continues until the accumulated strain exceeds the strength of the rock, and then fracture or faulting occurs. The opposite sides of the fault rebound to a new equilibrium position, and the energy is released in the vibrations of seismic waves and in heating and crushing of the rock. Rock fracturing usually start from a point **(<u>focus, hypocenter</u>**) close to one edge of the fault plane and propagates along the plane with a typical velocity of some 3 Km/sec. The vertical projection of the hypocenter onto the earth's surface is called the **<u>epicenter</u>.**



Types of earthquakes

There are many different types of earthquakes: tectonic, volcanic, and explosion. The type of earthquake depends on the region where it occurs and the geological make-up of that region.

1. **tectonic earthquakes.** These occur when rocks in the earth's crust break due to geological forces created by movement of tectonic plates.
2. **volcanic earthquakes,** occur in conjunction with volcanic activity.
3. **Collapse earthquakes** are small earthquakes in underground caverns and mines.
4. **Explosion earthquakes** result from the explosion of nuclear and chemical devices.

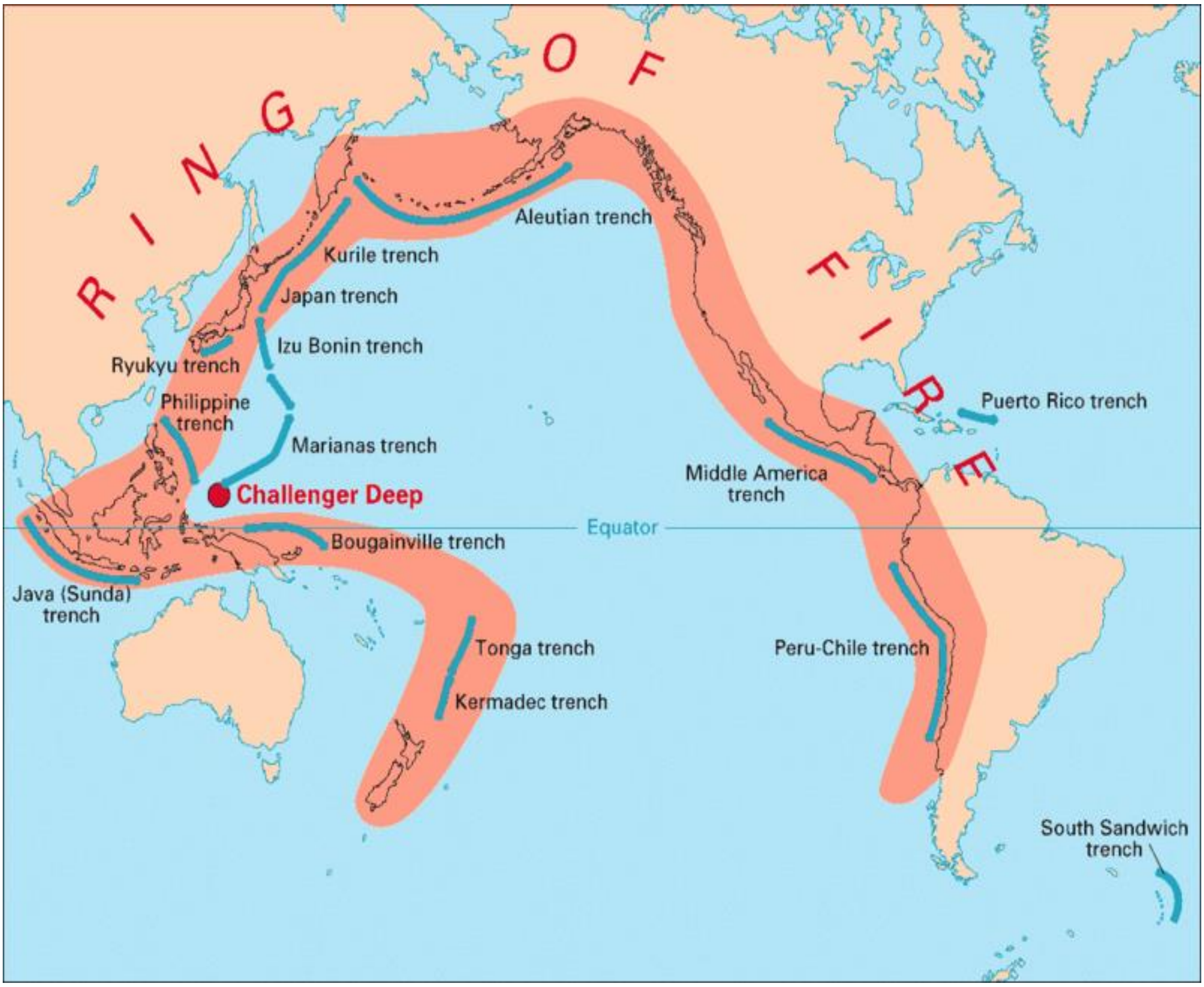
EARTHQUAKES , WHY AND WHERE DO THEY OCCUR ?

It has long been recognized that earthquakes are not evenly distributed over the earth. The eventual correlation of the earthquake pattern with the earth's major surface features was a key to the evolution of the plate tectonics theory. This is the most recent and broadly satisfying explanation theory of the majority of earthquakes. The basic idea is that the earth's outermost part (Lithosphere) consists of several large and fairly stable slabs of solid and relatively rigid rock called plates. Each plate extends to a depth of about 80 Km .

There are two major belts along which most of the world's earthquakes occur (Interplate earthquakes).

1. The circum - pacific belt : A large part (80 %) of the seismic energy released by all earthquakes is released along this belt. This includes the western coasts of South and North America , Japan, Philippines, and a strip through the East Indies and New Zealand.

2. The Alpide (Asiatic - European) belt : A high energy concentration (10 %) can also be seen along this belt. It extends from the Pacific belt in New Guinea through Summatra and Indonesia, the Himalayas, and mountains and faults of the Middle East , the Alps , and into the Atlantic Ocean far as the Azores.



Sporadically, earthquakes also occur at rather large distances from the respective plate margins . These so called **Intraplate earthquakes** , show a diffuse geographical distribution and there origin is still poorly understood. It can be large and because of there unexpectedness and infrequency can cause major disasters.

According to the focal depth, earthquakes are classified into one of the three categories :

1. <u>Shallow-focus</u> earthquakes : have their foci at a depth between 0 and 70 Km. and take place at oceanic ridges and transform faults as well as at subduction zones.

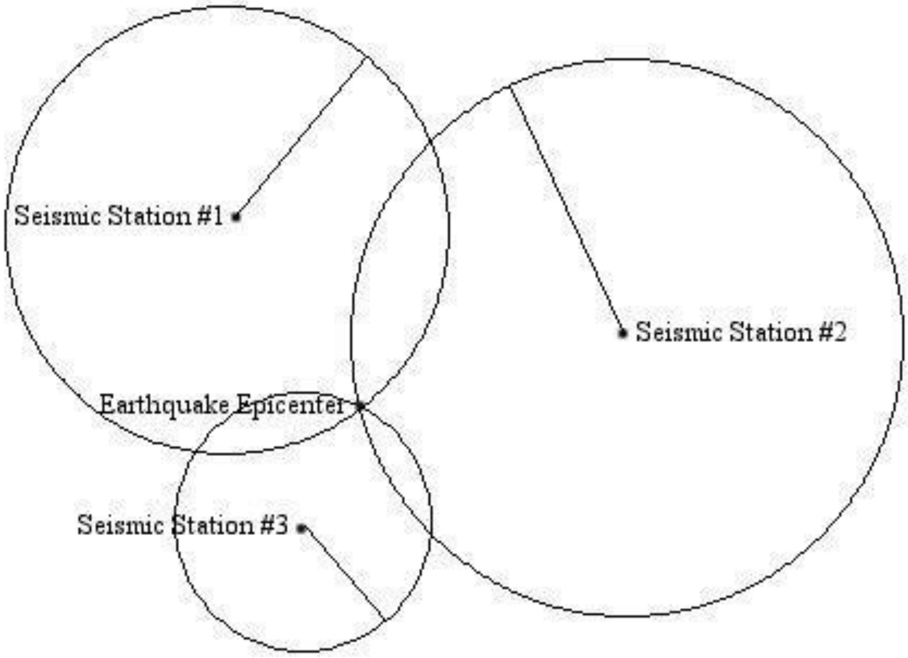
2. <u>Intermediate-focus</u> <u>earthquake</u> : focal depth between 71 and 300 Km.

<u>3. Deep-focus earthquakes</u> : focal depth greater than 300 Km

Most earthquakes originate within the crust. At depth beneath the <u>Moho</u> (Crust-Mantle boundary), the number falls abruptly and dies away to zero at a depth of about 700 Km. Earthquakes along ridges usually occur at a depth of about 10 Km or less and are of moderate size. Transform faults generate large shocks at depth down to about 20 Km. The largest earthquakes occur along subduction zones.

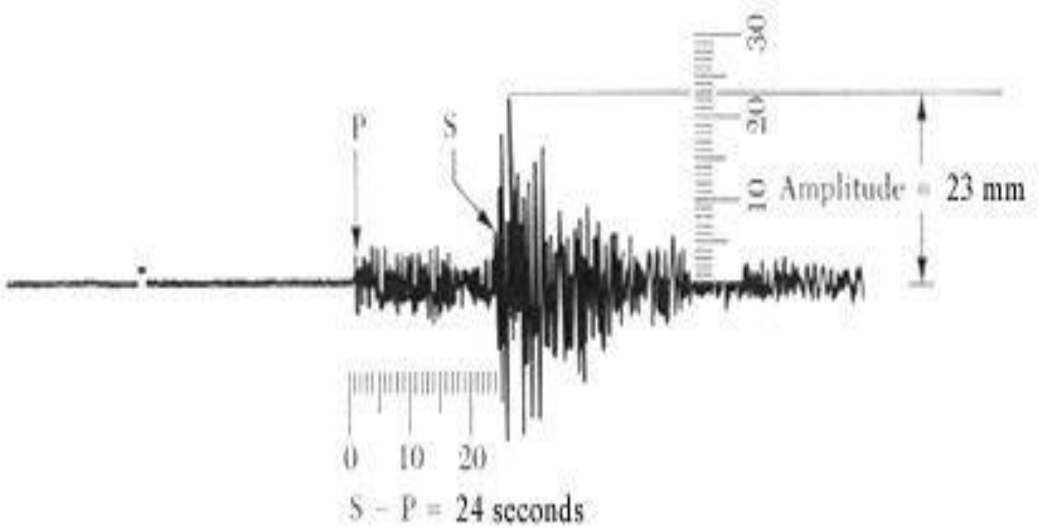
Locating the Epicenter of an Earthquake and Measuring its Magnitude.

P waves and S waves travel at different velocities. The first P wave will arrive at a seismic station before the first S wave. By using the difference in their arrival times you can determine the distance between the epicenter of any Earthquake and a seismic station using the conversion table at the bottom of the page. Once you have determined that distance, you can use it as the radius of a circle and can draw a circle around the seismic station. You need two other seismic stations doing the same thing. Where the three circles intersect is approximately where the epicenter of the Earthquake is.

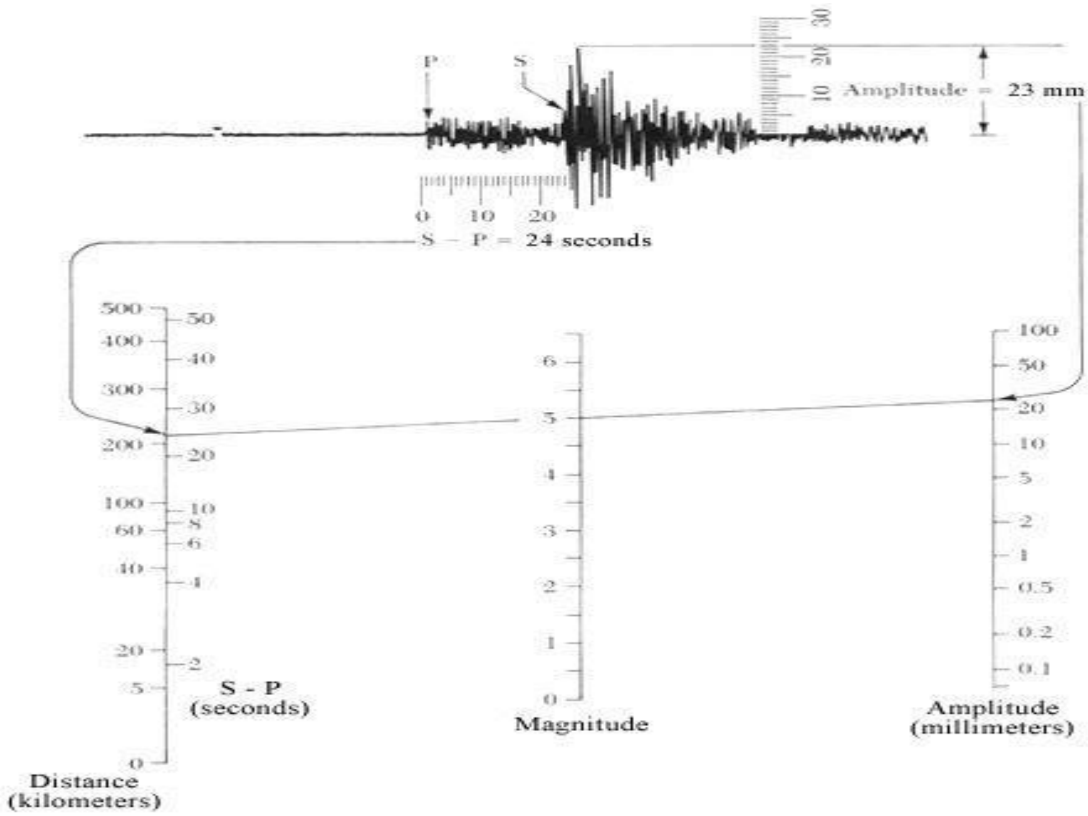


Locating the Epicenter of an Earthquake

To determine the magnitude of an Earthquake you simply measure the greatest amplitude of the first S wave and use the conversion table below to find the Earthquake's magnitude.



Below is the conversion chart used to determine both the distance of the epicenter of an Earthquake and the magnitude of the earthquake.



SCALES OF EARTHQUAKES

Two basically different scales are used to describe the size or strength of an earthquake and its effect :

1. Intensity : earthquake intensity represents the degree of shaking at a particular location on the earth's surface. It indicates the local effect or damage of the earthquake upon people, animals, buildings and objects in the immediate environment. The intensity diminishes generally with increasing distance from the epicenter.

One of the most widely used scales for intensity is the **Modified Mercalli Scale** . The scale has the following values, ranging from I to XII, usually written in Roman numerals :

I. Not felt
II. Felt by persons at rest
III. Felt indoor. Hanging objects swing
IV. Vibration like passing of heavy trucks. Windows rattle.
V. Felt outdoors. Sleepers awakened.
VI. Felt by all Persons walk unsteadily. Glassware broken.
VII. Difficult to stand. Hanging objects quiver.
VIII. Twisting,fall of chimneys,factory stacks,towers.
IX. General panic.Undergroundpipes broken.Frames racked.
X. Most masonry and frame structures destroyed.
XI. Rails bent greatly.Underground pipes out of surface.
XII. Damage nearly total. Objects thrown into the air.

2. Magnitude : The magnitude of an earthquake is an expression of the actual original force or energy of the earthquake at its moment of creation. It is measured directly by instruments, unlike the subjective measurement of intensity.

The **Richter scale** is a standard for expressing magnitude of earthquakes. Magnitude is defined as the logarithm to the base 10 of the amplitude of the largest ground motion traced by a standard type seismograph placed 100 Km from the earthquake's epicenter.

The Richter scale of magnitude runs from 0 through 8.9 , although there is no lower limit or upper limit. Each unit representing a ten-fold increase in amplitude of the measured waves and nearly a 30-fold increase in energy. An earthquake of magnitude 1 can only detected by a seismograph. The weakest earthquakes noticed by people are usually around magnitude 2. Houses and buildings are damaged at magnitude 5. Earthquakes with a magnitude above 6 are capable of producing serious damage. An earthquake with a magnitude of 8 and above is considered a great earthquake.

All the magnitude scales are of the form :

$$\mathbf{M = \log \ (A / T) + q (d , h) + a}$$

Where :

M is the magnitude

A is the maximum amplitude of the wave

T is the period of the wave

q is a function correcting for the decrease of amplitude with distance from the epicenter and focal depth (h)

d is the epicentral distance

a is an empirical constant

When the surface - wave magnitude (Ms) and body-wave magnitude (mb) are calculated for an earthquake, they do not usually have the same value.

$$\mathbf{mb = 2.94 + 0.55 \ Ms}$$

Structural damage is related to ground acceleration, although building respond differently to seismic waves of different periods. Intensity (I) is calibrated in terms of ground acceleration (a) by an approximate relationship

$$\log a = (I / 3) - 2.5$$

The approximate relationship between the magnitude and intensity has been estimated according to the following relationship :

$$M = 2I / 3 + 1.7 \log h - 1.4$$

Magnitude (Richter Scale)	Intensity (Mecalli Scale)	Damage Description
4	**5.5**	Widely felt, plaster cracked
5	**7**	Strong vibration, weak buildings and chimneys damaged
6	**8.5**	Ordinary buildings badly damaged
7	**10**	Well-built buildings destroyed
8	**11.5**	Specially designed buildings damaged
9	**12**	Widespread destruction

An earthquake of Richter magnitude 5.5 turns out to have an energy of about 10 ergs. The relation between energy and magnitude is :

$$\log E = 5.24 + 1.44 \; Ms$$

E : is the total energy measured in **joules**

Ms : is the surface-wave magnitude.

An increase in magnitude Ms of 1 unit increases the amount of seismic energy E released by a factor of about 30. In comparison , the Hiroshima atomic bomb was approximately equivalent in terms of energy to an earthquake of magnitude 5.3. A one megaton nuclear explosion would release about the same amount of energy as an earthquake of magnitude 6.5.

CHAPTER 5

ELECTRICAL METHOD

- Electrical properties of rocks
- Apparent & True resistivity
- Electrode configurations
- Electrical soundings & Profiling
- Applications in groundwater exploration

Problem Set - 4

ELECTRICAL METHODS

ELECTRICAL PROPERTIES OF ROCKS :

- ☒ Resistivity (or conductivity), which governs the amount of current that passes when a potential difference is created.
- ☒ Electrochemical activity or polarizability, the response of certain minerals to electrolytes in the ground, the bases for SP and IP.
- ☒ Dielectric constant or permittivity. A measure of the capacity of a material to store charge when an electric field is applied . It measure the polarizability of a material in an electric field = $1 + 4\pi X$
  X : electrical susceptibility .

Electrical methods utilize direct current or Low frequency alternating current to investigate electrical properties of the subsurface.
Electromagnetic methods use alternating electromagnetic field of high frequencies.
Two properties are of primary concern in the Application of electrical methods.

(1) The ability of Rocks to conduct an electrical current.
(2) The polarization which occurs when an electrical current is passed through them (IP).

Resistivity

For a uniform wire or cube, resistance is proportional to length and inversely proportional to cross-sectional area. Resistivity is related to resistance but it not identical to it. The resistance R depends an length, Area and properties of the material which we term resistivity (ohm.m) .

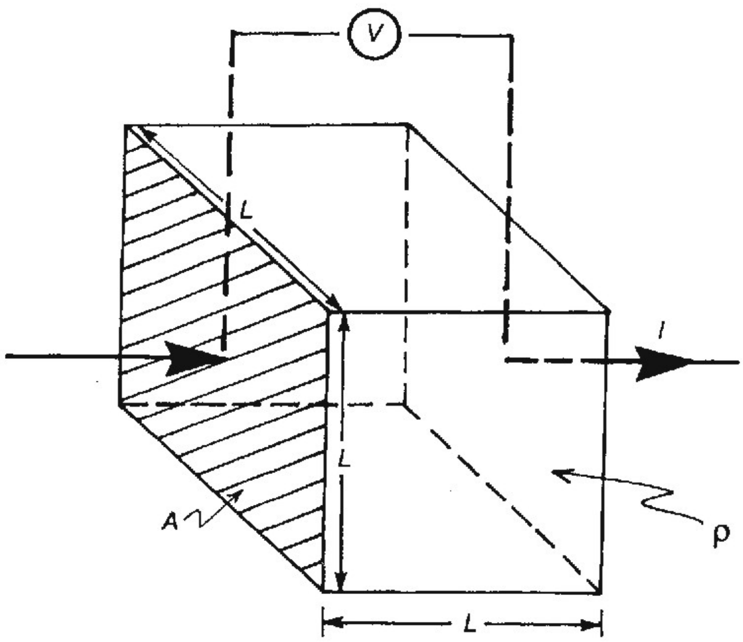
Constant of proportionality is called Resistivity :

$$R = \rho \frac{L}{A}$$

<u>Resistivity is the fundamental physical property of the metal in the wire</u>

$$\rho = \frac{VA}{IL}$$

Resistivity is measured in ohm-m



<u>Conductivity</u> is defined as $1/\rho$ and is measured in Siemens per meter (S/m), equivalent to $ohm^{-1}m^{-1}$.

Classification of Materials according to Resistivities Values

a) Materials which lack pore spaces will show high resistivity such as :
   - massive limestone
   - most igneous and metamorphic (granite, basalt)

   - Materials whose pore space lacks water will show high resistivity such as : - dry sand and gravel , Ice .

b) Materials whose connate water is clean (free from salinity) will show high resistivity such as :
   - clean sand or gravel , even if water saturated.

c) most other materials will show medium or low resistivity, especially if clay is present such as :
   - clay soil
   - weathered rock.

The presence of clay minerals tends to decrease the Resistivity because

1) The clay minerals can combine with water .

2) The clay minerals can absorb cations in an exchangeable state on the surface.

3) The clay minerals tend to ionize and contribute to the supply of free ions.

Factors which control the Resistivity

(1) Geologic Age
(2) Salinity.
(3) Free-ion content of the connate water.
(4) Interconnection of the pore spaces (Permeability).
(5) Temperature.
(6) Porosity.
(7) Pressure
(8) Depth

Archie's Law

Empirical relationship defining bulk resistivity of a saturated porous rock. In sedimentary rocks, resistivity of pore fluid is probably single most important factor controlling resistivity of whole rock.

Archie (1942) developed empirical formula for effective resistivity of rock:

$$\rho_0 = a\rho_w\phi^{-m}$$

ρ_0 = ***bulk rock resistivity***
ρ_w = *pore-water resistivity*
a = empirical constant ($0.6 < a < 1$)
m = *cementation factor* (1.3 poor, unconsolidated) < m < 2.2 (good, cemented or crystalline)
φ = *fractional porosity* (vol liq. / vol rock)

Formation Factor*:*

$$F = \frac{\rho_0}{\rho_w} = a\phi^{-m}$$

Effects of Partial Saturation:

$$\rho_t = S_w^{-n} a\rho_w\phi^{-m}$$

Sw is the volumetric saturation.
n is the *saturation coefficient* ($1.5 < n < 2.5$).

- Archie's Law ignores the effect of pore geometry, but is a reasonable approximation in many sedimentary rocks

Current Flow in A Homogeneous Earth

1. **Point current Source :**

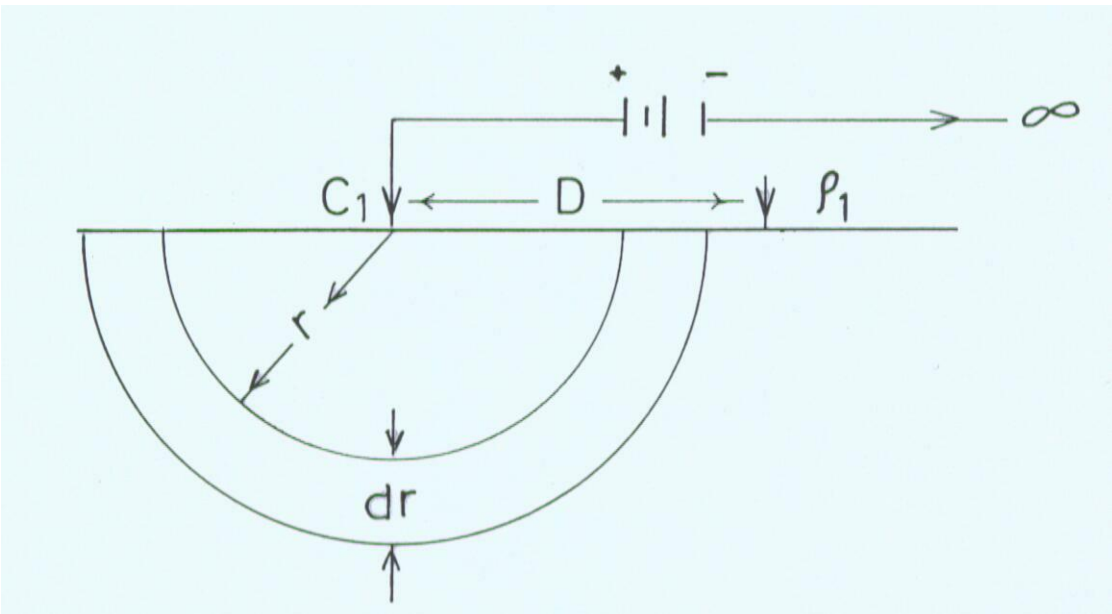
   If we define a very thin shell of thickness dr we can define the potential different dv

   $$dv = I (R) = I (\rho L / A) = I (\rho \; dr / 2\pi r^2)$$

   To determine V a t a point . We integrate the above eq. over its distance D to to infinity :

   $$V = I \rho / 2\pi D$$

   C: current density per unit of cross sectional area :

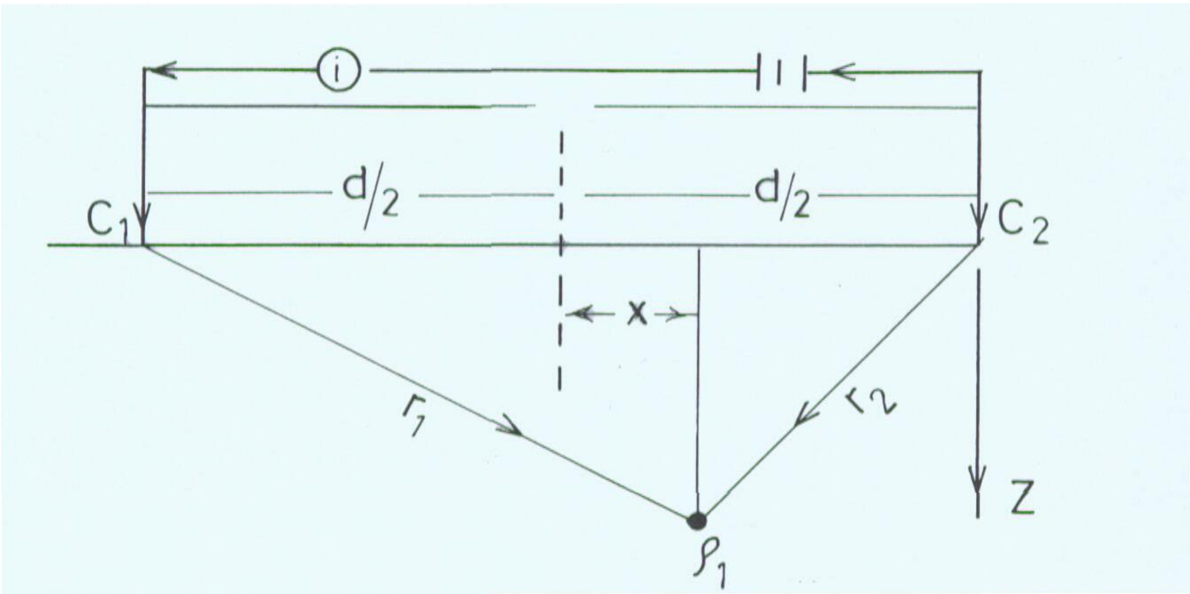


2. Two current electrodes

To determine the current flow in a homogeneous, isotropic earth when we have two current electrodes. The current must flow from the positive (source) to the resistive (sink).

The effect of the source at C1 (+) and the sink at C2 (-)

$$\mathbf{Vp_1 = i\,\rho / 2\pi\, r_1 + (- i\rho / 2\pi\, r_2)}$$



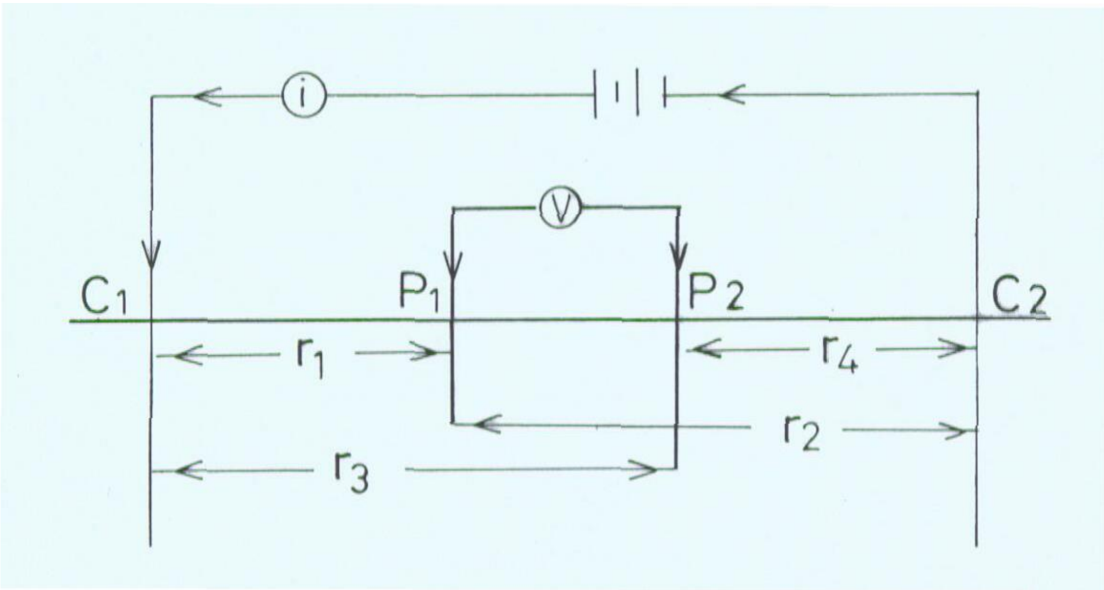
$$\mathbf{Vp_1 = i\rho / 2\pi \{ 1/ [(d/2 + x)^2 + Z^2]^{0.5} - 1 / [(d/2 - x)^2 + Z^2]^{0.5} \}}$$

3. Two potential Electrodes

$$Vp_1 = i\rho / 2\pi r_1 \quad - i\rho / 2\pi r_2$$

$$Vp_2 = i\rho / 2\pi r_3 \quad - i\rho / 2\pi r_4$$

$$\Delta V = Vp_1 - Vp_2 = i\rho / 2\pi \left(1/r_1 - 1/r_2 - 1/r_3 + 1/r_4 \right)$$



ELECTRODE CONFIGURATIONS

The value of the apparent resistivity depends on the geometry of the electrode array used (K factor)

1- Wenner Arrangement
Named after wenner (1916) .
The four electrodes A , M , N , B are equally spaced along a straight line. The distance between adjacent electrode is called "a" spacing . So AM=MN=NB=
⅓ AB = a.

$$P_{a=}\ 2\pi\ a \quad V\ /\ I$$

The wenner array is widely used in the western Hemisphere. This array is sensitive to horizontal variations.

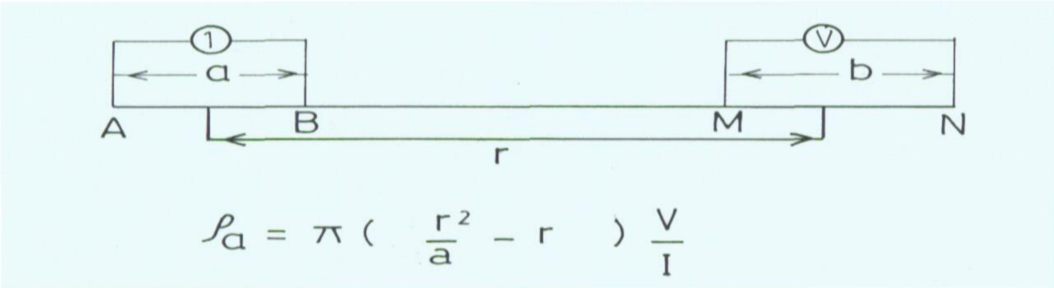
2) Schlumberger Arrangement .
This array is the most widely used in the electrical prospecting . Four electrodes are placed along a straight line in the same order AMNB , but with AB ≥ 5 MN

$$\rho a = \pi \times \frac{V}{I} \times \left[\frac{\left(\frac{AB}{2}\right)^2 - \left(\frac{MN}{2}\right)^2}{MN} \right]$$

This array is less sensitive to lateral variations and faster to use as only the current electrodes are moved.

3. Dipole – Dipole Array .
The use of the dipole-dipole arrays has become common since the 1950's , Particularly in Russia. In a dipole-dipole, the distance between the current electrode A and B (current dipole) and the distance between the potential electrodes M and N (measuring

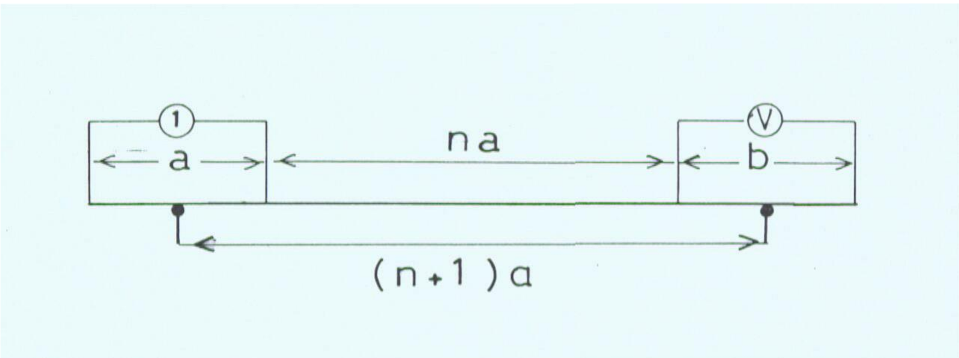
dipole) are significantly smaller than the distance r , between the centers of the two dipoles.



$$\rho_a = \pi \left[\left(r^2 / a \right) - r \right] v/i$$

Or . if the separations a and b are equal and the distance between the centers is **(n+1) a** then

$$\rho_a = n\,(n+1)\,(n+2)\,.\,\pi\; a.\, v/i$$



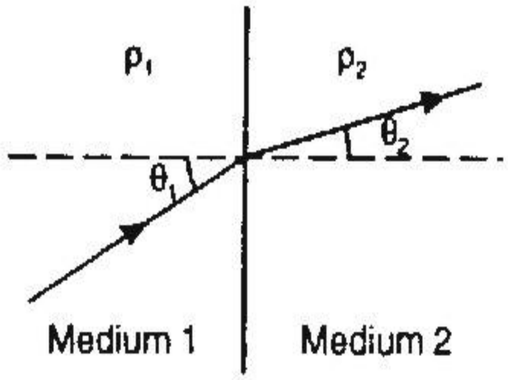
This array is used for deep penetration $\approx$ 1 km.

REFRACTION OF ELECTRICAL RESISTIVITY

A. Distortion of Current flow

At the boundary between two media of different resistivities the potential remains continuous and the current lines are refracted according to the law of tangents.

$\tan \theta_1 / \tan \theta_2 = P_2 / P_1$

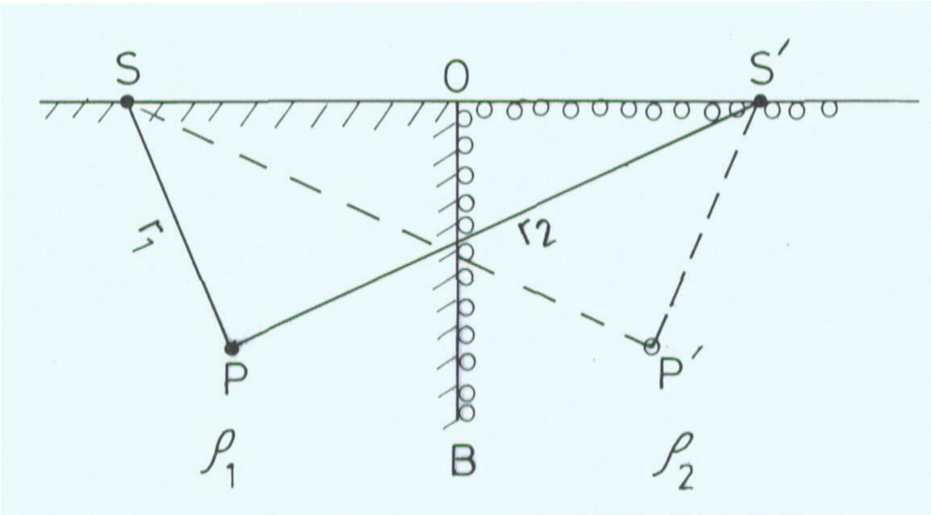


$P_1 \tan \theta_1 = P_2 \tan \theta_2$

If $\rho 2 < p1$, The current lines will be refracted away from the Normal. The line of flow are moved downward because the lower resistivity below the interface results in an easier path for the current within the deeper zone.

B. Distortion of Potential

Consider a source of current I at the point S in the first layers P1 of Semi infinite extent. The potential at any point P would be that from S plus the amount reflected by the layer P2 as if the reflected amount were coming from the image $S^{/}$



$$V_1 (P) = i\, \rho_1 / 2\pi \left[(1 / r_1) + (K / r_2) \right]$$

$$K = \textbf{Reflection coefficient} = \rho_2 - \rho_1 \ / \ \rho_2 + \rho_1$$

In the case where P lies in the second medium $\rho 2$, Then transmitting light coming from S. Since only 1 – K is transmitted through the boundary.
The Potential in the second medium is

$$V_2(P) = I\, \rho_2 / 2\pi \left[(1 / r_1) - (K / r_1) \right]$$

Continuity of the potential requires that the boundary where r1 = r2 , V1(p) must be equal to V2 (P).
At the interface r1 = r2 , V1= V2

$$\frac{\rho_1}{\rho_2} = \frac{1-k}{1+k} \qquad\qquad k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

k is electrical reflection coefficient and used in interpretation

The value of the dimming factor , K always lies between ± 1

If the second layer is a pure insulator
(ρ_2 = ω) then K = + 1

If the second layer is a perfect conductor

(ρ_2 = O) then K = - 1

When $\rho_1 = \rho_2$ then No electrical boundary Exists and K = O .

SURVEY DESIGN

Two categories of field techniques exist for conventional resistivity analysis of the subsurface. These techniques are vertical electric sounding (VES), and Horizontal Electrical Profiling (HEP).

1- Vertical Electrical Sounding (VES) .

The object of VES is to deduce the variation of resistivity with depth below a given point on the ground surface and to correlate it with the available geological information in order to infer the depths and resistivities of the layers present.

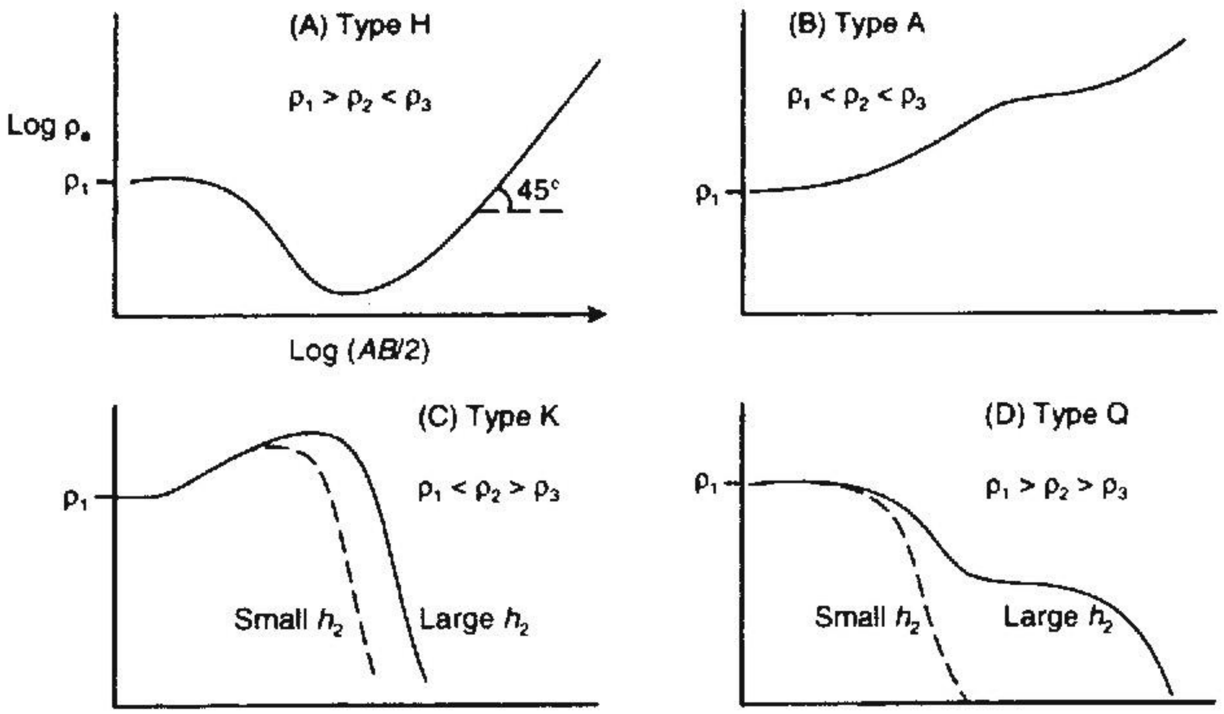
2- Horizontal Electrical profiling (HEP) .

The object of HEP is to detect lateral variations in the resistivity of the ground, such as lithological changes, near- surface faults……
.

Multiple Horizontal Interfaces

For Three layers resistivities in two interface case , four possible curve types exist.

1- Q – type $\rho_1 > \rho_2 > \rho_3$
2- H – Type $\rho_1 > \rho_2 < \rho_3$
3- K – Type $\rho_1 < \rho_2 > \rho_3$
4- A – Type $\rho_1 < \rho_2 < \rho_3$



Applications of Resistivity Techniques

1. Bedrock Depth Determination

Both VES and CST are useful in determining bedrock depth. Bedrock usually more resistive than overburden. HEP profiling with Wenner array at 10 m spacing and 10 m station interval used to map bedrock highs.

2. Location of Permafrost

Permafrost represents significant difficulty to construction projects due to excavation problems and thawing after construction.

- Ice has high resistivity of 1-120 ohm-m

3. Landfill Mapping

Resistivity increasingly used to investigate landfills:

- Leachates often conductive due to dissolved salts
- Landfills can be resistive or conductive, depends on contents

Limitations of Resistivity Interpretation

1- **Principle of Equivalence**.

If we consider three-lager curves of K ($\rho_1 < \rho_2 > \rho_3$) or Q type ($\rho_1 > \rho_2 > \rho_3$) we find the possible range of values for the product $T_2 = \rho_2\ h_2$ Turns out to be much smaller. This is called T-equivalence. H = thickness, T : Transverse resistance it implies that we can determine T_2 more reliably than ρ_2 and h_2 separately. If we can estimate either ρ_2 or h_2 independently we can narrow the ambiguity. Equivalence: several models produce the same results. Ambiguity in physics of 1D interpretation such that different layered models basically yield the same response.

Different Scenarios: Conductive layers between two resistors, where lateral conductance (σh) is the same. Resistive layer between two conductors with same transverse resistance (ρh).

2- Principle of Suppression.

This states that a thin layer may sometimes not be detectable on the field graph within the errors of field measurements. The thin layer will then be averaged into on overlying or underlying layer in the interpretation. Thin layers of small resistivity contrast with respect to background will be missed. Thin layers of greater resistivity contrast will be detectable, but equivalence limits resolution of boundary depths, etc. The detectibility of a layer of given resistivity depends on its relative thickness which is defined as the ratio of Thickness/Depth.

Comparison of Wenner and Schlumberger

(1)	In Sch.	MN	≤	1/5	AB
	Wenner	MN	=	1/3	AB

(2) In Sch. Sounding, MN are moved only occasionally.
In Wenner Soundings, MN and AB are moved after each measurement.

(3) The manpower and time required for making Schlumberger soundings are less than that required for Wenner soundings.

(4) Stray currents that are measured with long spreads effect measurements with Wenner more easily than Sch.

(5) The effect of lateral variations in resistivity are recognized and corrected more easily on Schlumberger than Wenner.

(6) Sch. Sounding is discontinuous resulting from enlarging MN.

Disadvantages of Wenner Array

1. All electrodes must be moved for each reading
2. Required more field time
3. More sensitive to local and near surface lateral variations
4. Interpretations are limited to simple, horizontally layered structures

Advantages of Schlumberger Array

1. Less sensitive to lateral variations in resistivity
2. Slightly faster in field operation
3. Small corrections to the field data

Disadvantages of Schlumberger Array

1. Interpretations are limited to simple, horizontally layered structures
2. For large current electrodes spacing, very sensitive voltmeters are required.

Advantages of Resistivity Methods

1. Flexible
2. Relatively rapid. Field time increases with depth
3. Minimal field expenses other than personnel
4. Equipment is light and portable
5. Qualitative interpretation is straightforward
6. Respond to different material properties than do seismic and other methods, specifically to the water content and water salinity

Disadvantages of Resistivity Methods

1- Interpretations are ambiguous, consequently, independent geophysical and geological controls are necessary to discriminate between valid alternative interpretation of the resistivity data (Principles of Suppression & Equivalence)

2- Interpretation is limited to simple structural configurations.

3- Topography and the effects of near surface resistivity variations can mask the effects of deeper variations.

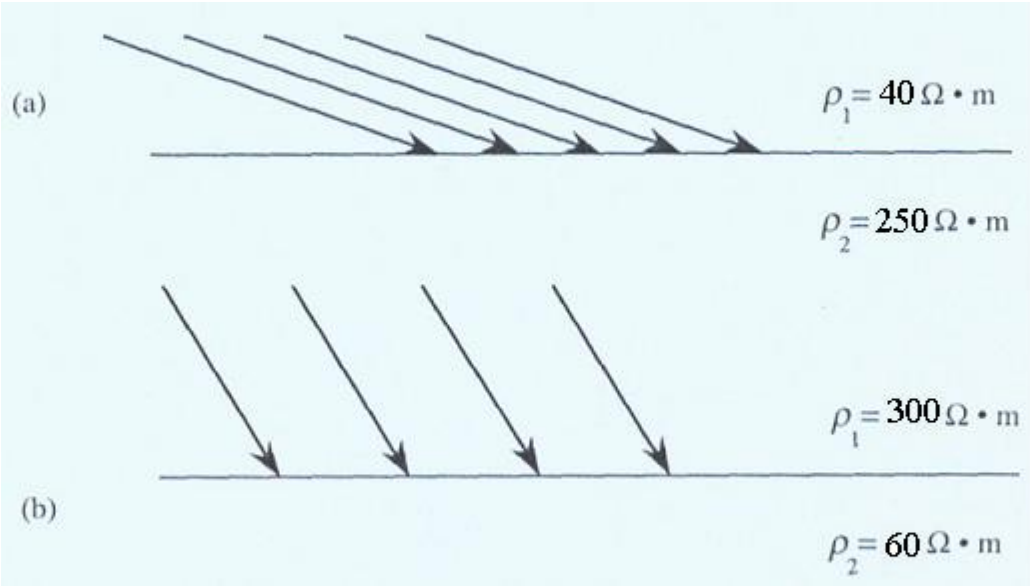
4- The depth of penetration of the method is limited by the maximum electrical power that can be introduced into the ground and by the practical difficulties of laying out long length of cable. The practical depth limit of most surveys is about 1 Km.

5. Accuracy of depth determination is substantially lower than with seismic methods or with drilling.

Problem Sets

1. Copper has ρ =1.7 X 10^{-8} ohm.m. What is the resistance of 20 m of copper with a cross-sectional radius of 0.005 m .

2. Construct the current-flow lines beneath the interface in (a) and (b).



3. Assume a homogeneous medium of resistivity 120 ohm-m. Using the wenner electrode system with a 60-m spacing, assume a current of 0.628 ampere. What is the measured potential difference? What will be the potential difference if we place the sink (negative-current electrode) at **infinity?**

4. Why are the electrical methods of exploration particularly suited to hydrogeological investigations? Describe other geophysical methods which could be used in this context, stating the reasons why they are applicable.

CHAPTER 6

GRAVITY METHOD

- Fundamental principles
- Measurements
- Data reduction
- Interpretation & Applications

Solved Problems

Problem Set - 5

GRAVITY METHOD

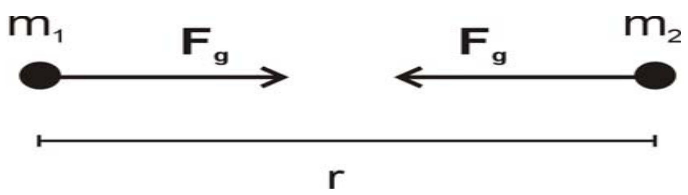
The gravity method is a nondestructive geophysical technique that measures differences in the earth's gravitational field at specific locations. It has found numerous applications in engineering and environmental studies including locating voids and karst features, buried stream valleys, water table levels and the determination of soil layer thickness. The success of the gravity method depends on the different earth materials having different bulk densities (mass) that produce variations in the measured gravitational field. These variations can then be interpreted by a variety of analytical and computers methods to determine the depth, geometry and density the causes the gravity field variations.

Gravity data in engineering and environmental applications should be collected in a grid or along a profile with stations spacing 5 meters or less. In addition, gravity station elevations must be determined to within 0.2 meters. Using the highly precise locations and elevations plus all other quantifiable disturbing effects, the data are processed to remove all these predictable effects. The most commonly used processed data are known as Bouguer gravity anomalies, measured in mGal.

The gravity method can be a relatively easy geophysical technique to perform and interpret. It requires only simple but precise data processing, and for detailed studies the determination of a station's elevation is the most difficult and time-consuming aspect. The technique has good depth penetration when compared to ground penetrating radar, high frequency electromagnetics and DC-resistivity techniques and is not affected by the high conductivity values of near-surface clay rich soils. Additionally, lateral boundaries of subsurface features can be easily obtained especially through the measurement of the derivatives of the gravitational field.

The main drawback is the ambiguity of the interpretation of the anomalies. This means that a given gravity anomaly can be caused by numerous source bodies. An accurate determination of the source usually requires outside geophysical or geological information.

Geophysical interpretations from gravity surveys are based on the mutual attraction experienced between two masses* as first expressed by **Isaac Newton**. Newton's law of gravitation states that the mutual attractive force between two point masses**, m1 and m2, is proportional to one over the square of the distance between them. The constant of proportionality is usually specified as *G*, the gravitational constant. Thus, the law of gravitation



$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

where *F* is the force of attraction, *G* is the gravitational constant ($\mathbf{G = 6.67 \times 10^{-11}\ m^3\,kg^{-1}\,s^{-2}}$) and *r* is the distance between the two masses, m1 and m2.

Mass is formally defined as the proportionality constant relating the force applied to a body and the acceleration the body undergoes as given by Newton's second law, usually written as ***F=ma*****.** Therefore, mass is given as ***m=F/a*** and has the units of force over acceleration.

Gravitational Acceleration

When making measurements of the earth's gravity, we usually don't measure the gravitational force, *F*. Rather, we measure the gravitational acceleration, *g*. The gravitational acceleration is the time rate of change of a body's speed under the influence of the gravitational force. That is, if you drop a rock off a cliff, it not only falls, but its speed increases as it falls. In addition to defining the law of mutual attraction between masses, Newton also defined the relationship between a force and an acceleration.

Newton's second law states that force is proportional to acceleration. The constant of proportionality is the mass of the object. Combining Newton's second law with his law of mutual attraction, the gravitational acceleration on the mass m2 can be shown to be equal to the mass of attracting object, m1, over the squared distance between the center of the two masses, *r*.

$$\boxed{F = m_2 g} \quad \boxed{g = \frac{G m_1}{r^2}}$$

If an object such as a ball is dropped, it falls under the influence of gravity in such a way that its speed increases constantly with time. That is, the object accelerates as it falls with constant acceleration. At sea level, the rate of acceleration is about 9.8 meters per second squared.

In gravity surveying, we will measure variations in the acceleration due to the earth's gravity. Variations in this acceleration can be caused by variations in subsurface geology. Acceleration variations due to geology, however, tend to be *much* smaller than 9.8 meters per second squared. Thus, a meter per second squared is an inconvenient system of units to use when discussing gravity surveys.

The units typically used in describing the gravitational acceleration variations observed in exploration gravity surveys are specified in **milliGals**. A Gal is defined as a **cm / sec.^{2}** (**1 mGal=10^{-3} Gal**) and microgal (**1μGal = 10^{-6} Gals**).

Thus, the Earth's gravitational acceleration is approximately 980 Gals. The Gal is named after **Galileo Galilei**. The milliGal (mgal) is 0.001 Gal. In milliGals, the Earth's gravitational acceleration is approximately 980,000.

Gravity Measurements

The instrument used to measure gravity is called a **gravimeter**.

Absolute gravity

This technique makes measurements of the **total gravity field** at a site. There are a number of types of instruments, including free-fall devices, the reversible pendulum, and superconducting gravimeters. The equipment is very expensive and bulky. Lengthy observation times (24+ hrs) are required to obtain accurate readings (0.001- 0.01 mgal).

Relative gravity

In general, for interpreting gravity data, only the relative gravitational acceleration is required. Therefore, we usually don't need to know the absolute gravity at every station, just how gravity changes between stations. The relative gravity readings can be converted into absolute gravity if one of the survey sites is chosen to be a place where the absolute gravity was measured previously.

a) Portable pendulum:

- based on the idea that the period of a pendulum (time taken for the pendulum to swing back and forth) is given by:

$$T = 2\Pi\sqrt{\frac{L}{g}}$$

where L is the pendulum length and g is the gravity.

- measure the period (T1) at one location and then move the pendulum to another location and measure the period (T2). The change in period (T2- T1) is proportional to the change in gravity between the two locations.
  - accuracy of 0.25 mgal

Factors Affecting Gravitational Acceleration

Factors can be subdivided into two categories: those that give rise to temporal variations and those that give rise to spatial variations in the gravitational acceleration.

A. Temporal Based Variations - These are changes in the observed acceleration that are time dependent. In other words, these factors cause variations in acceleration that would be observed even if we didn't move our gravimeter.

Instrument Drift - Changes in the observed acceleration caused by changes in the response of the gravimeter over time.

Tidal Affects - Changes in the observed acceleration caused by the gravitational attraction of the sun and moon.

B. Spatial Based Variations - These are changes in the observed acceleration that are space dependent. That is, these change the gravitational acceleration from place to place, just like the geologic affects, but they are not related to geology.

Latitude Variations - Changes in the observed acceleration caused by the ellipsoidal shape and the rotation of the earth.

Elevation Variations - Changes in the observed acceleration caused by differences in the elevations of the observation points.

Bouguer Effects - Changes in the observed acceleration caused by the extra mass underlying observation points at higher elevations.

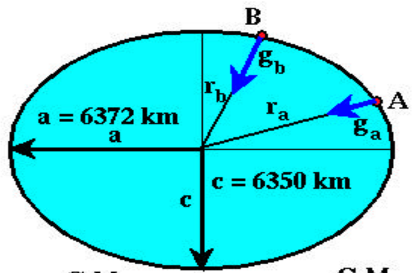
Topographic Effects - Changes in the observed acceleration related to topography near the observation point.

Latitude Variations:

Two features of the earth's large-scale structure and dynamics affect our gravity observations: its **shape and its rotation**.

Although the difference in earth radii measured at the poles and at the equator is only 22 km (this value represents a change in earth radius of only 0.3%), this, in conjunction with the earth's rotation, can produce a measurable change in the **gravitational acceleration** with latitude. Because this produces a spatially varying change in the gravitational acceleration, it is possible to confuse this change with a change produced by local geologic structure. Fortunately, it is a relatively simple matter to correct our gravitational observations for the change in acceleration produced by the earth's elliptical shape and rotation.

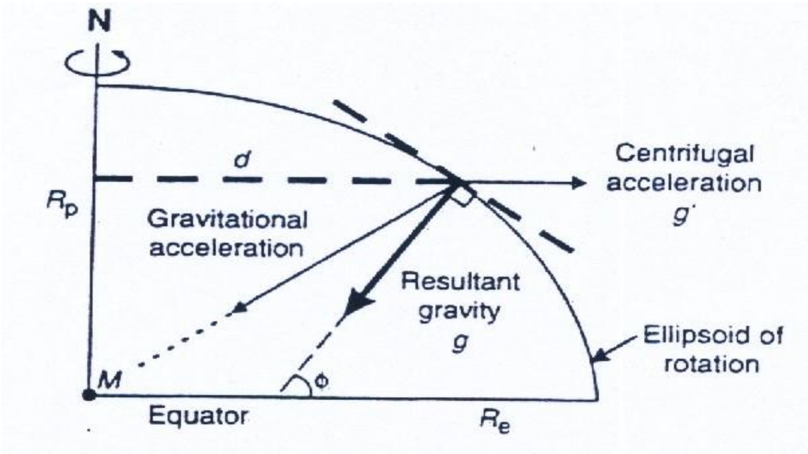
To first order*, the elliptical shape of the earth causes the gravitational acceleration to vary with latitude because the distance between the gravimeter and the earth's center varies with latitude. The magnitude of the gravitational acceleration changes as one over the distance from the center of mass of the earth to the gravimeter squared. Thus, qualitatively, we would expect the gravitational acceleration to be smaller at the equator than at the poles, because the surface of the earth is farther from the earth's center at the equator than it is at the poles.



$$g_a \approx \frac{GM}{r_a^2} \qquad g_b \approx \frac{GM}{r_b^2}$$

But, $r_a > r_b$, therefore,

$$g_b > g_a$$



$$g_\phi = g_0(1 + \alpha \sin^2 \phi - \beta \sin^2 2\phi)$$

The mathematical formula used to predict the components of the gravitational acceleration produced by the earth's shape and rotation is called the *Geodetic Reference Formula of 1967*. The predicted gravity is called the ***normal gravity*** **(g_n).**

How large is this correction to our observed gravitational acceleration? And, because we need to know the latitudes of our observation points to make this correction, how accurately do we need to know locations? At a latitude of 45 degrees, the gravitational acceleration varies approximately 0.813 mgals per kilometer. Thus, to achieve an accuracy of 0.01 mgals, we need to know the north-south location of our gravity stations to about 12 meters.

$g_n = 978.03185\ (1.0 + 0.005278895 \sin^2 \phi - 0.000023462 \sin^4 \phi)\ (cm/s^2)$

g_n = Normal Gravity: Gravitational acceleration expected for a rotating ellipsoidal earth without any geologic complications and no surface features

ϕ = Latitude

At any latitude **$g_n = 0.812 \sin 2\varphi$ mgal/m**

Observed Gravity (gobs) - Gravity readings observed at each gravity station after corrections have been applied for instrument drift and tides.

Latitude Correction (gn) - Correction subtracted from ***g****obs* that accounts for the earth's elliptical shape and rotation. The gravity value that would be observed if the earth were a perfect (no geologic or topographic complexities), rotating ellipsoid is referred to as the ***normal gravity***.

Free Air *Corrected Gravity (**g**fa)* - The Free-Air correction accounts for gravity variations caused by elevation differences in the observation locations. The form of the Free-Air gravity anomaly, ***g****fa*, is given by;

gfa = gobs - gn + 0.3086h *(mgal)*

where *h* is the elevation at which the gravity station is above the elevation datum chosen for the survey (this is usually sea level).

Bouguer Slab *Corrected Gravity (**g**b)* - The Bouguer correction is a first-order correction to account for the excess mass underlying observation points located at elevations higher than the elevation datum. Conversely, it accounts for a mass deficiency at observations points located below the elevation datum. The form of the Bouguer gravity anomaly, ***g****b*, is given by;

gb = gobs - gn + 0.3086h - 0.04193rh (mgal)

where *r* is the average density of the rocks underlying the survey area.

Terrain *Corrected Bouguer Gravity (**g**t)* - The Terrain correction accounts for variations in the observed gravitational acceleration caused by variations in topography near each observation point. The terrain correction is positive regardless of whether the local topography consists of a mountain or a valley. The form of the Terrain corrected, Bouguer gravity anomaly, ***g****t*, is given by;

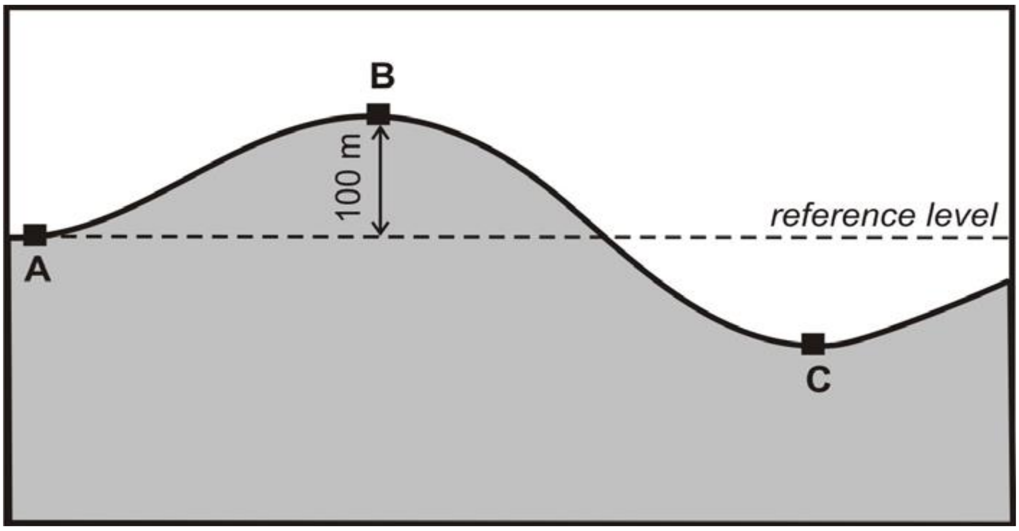
gt = gobs - gn + 0.3086h - 0.04193r + TC ***(mgal)***

where *TC* is the value of the computed Terrain correction. Assuming these corrections have accurately accounted for the variations in gravitational acceleration they were intended to account for, any remaining variations in the gravitational acceleration associated with the Terrain Corrected Bouguer Gravity, ***gt***, can now be assumed to be caused by geologic structure.

FREE AIR AND BOUGUER ANOMALIES

A. Free Air anomaly

The figure below shows three gravity sites in an area with a hill and a valley.



As you go from site A to site B, you climb a hill that is 100 m high. The corresponding change in gravity is: **Δg = 0.3086 x 100 = 30.86 mgal**. The gravity at the top of the hill will be 30.86 mgal **less** than the gravity at the bottom because you have moved **further** from the centre of the Earth.

This variation in gravity must be removed from your data. The correction for elevation is called the **Free Air Correction.** To apply this, it is necessary to define a reference level for the survey. Any reference level can be chosen:

- for surveys in coastal areas, sea level is often chosen as the reference
- for a survey far away from the coast, the average elevation of the survey area could be used.
- in the figure, the elevation of Site A was chosen. The Free Air Correction is written as: **CFA** = 0.3086 Δh, where Δh is the **difference in elevation** between the site and the reference level

• If the site is **above** the reference level (e.g., Site B), CFA is **added** to the observed gravity value.

• If the site is **below** the reference level (e.g., Site C), CFA is **subtracted** from the observed gravity value. The resulting gravity value is called the **free air anomaly**. $\Delta g_F = g_{obs} - g_\varphi + C_F$

B. Bouguer Anomaly

Consider Sites A and B. The Free Air correction will correct the gravity observed at B for the 100 m difference in elevation. However, there is still a difference in the amount of mass below each station. The gravity at Site B will be affected by the gravitational pull of the 100 m thickness of the material between it and the reference level. This "excess" gravity has to be removed in order to compare the gravity at A and B.

To first order, the difference in gravity between Site A and Site B can be approximated by an infinite slab of uniform density and thickness. The gravitational attraction of this layer is: $g = 2\pi G\rho\Delta h$, where Δh is the thickness and ρ is the density of the material . The correction for the difference in mass due to a difference elevation (Δh) is called the

Bouguer correction (CB): C = $2\pi G\rho\Delta h$ = B 0.00004193 ρ Δh (CB in mgal). The Bouguer correction at Site B would be: 0.1119 Δh = 0.1119×100 m = 11.2 mgal.

This value must be **subtracted** – we want to take away the effect of the hill. Conversely, after the gravity data at site C have been corrected for elevation (CFA), they will be "too low", because they were made at a lower elevation and thus there was less mass below the station. In this case, the Bouguer correction will **add** the "missing" mass to the original gravity measurement. The resulting gravity value is the **Bouguer anomaly,**

$$\Delta g_B = g_{obs} - g_\varphi + C_F - C_B + C_T$$

this is the gravity anomaly due to local geology.

Generally, measurement **above** reference level **Add** Free Air **Subtract** Bouguer correction correction. Measurement **below** reference level **Subtract** Free Air **Add** Bouguer correction.

Applications of the Gravity Method

- Determining Shape of the earth (Geodesy)
- Detection of subsurface voids including caves, mine shafts
- Determining the amount of subsidence in surface collapse features over time
- Determination of soil and glacier sediment thickness (bedrock topography)
- Location of buried sediment valleys
- Determination of groundwater volume and changes in water table levels over time in alluvial basins
- Mapping the volume, lateral and vertical extent of landfills
- Mapping steeply dipping contacts including faults

SOLVED PROBLEMS IN GRAVITY

Problem 1:

Given :

Observed gravity at base 980.30045 Gals
Observed gravity at station relative to base + 5.65 mGal
Theoretical gravity at sea level at latitude of station 980.30212 Gals
Elevation of station 100 m above sea level
Density of rock above sea level 2.0 g/cc
Terrain effect 0.15 mGals

Compute : Bouguer gravity anomaly
Free air anomaly

$\Delta g_B = g_{obs} - g_\varphi + C_F - C_B + C_T$

$= (980300.54 + 5.65) - (980302.12 + (0.3086 \times 100) - (0.0419 \times 100 \times 2) + 0.15$

$g = 980306.19 - 980302.12 + 30.86 - 8.38 + 0.15$

$\Delta g_B = $ **26.55 mGal**

$\Delta g_F = g_{obs} - g_\varphi + C_F$

= 34.9 mGal

<u>Problem 2 :</u>

Given :

Observed gravity at base 980.30045 Gals

Observed gravity at station relative to base + 5.65 mGal

Theoretical gravity at sea level at latitude of station 980.30212 Gals

Elevation of station 100 m below sea level

Density of rock above sea level 2.0 g/cc

Terrain effect 0.15 mGals

Compute :

1 - Free air anomaly

2 - Bouguer gravity anomaly

$\Delta \mathbf{g_F = g_{obs} - g_\varphi + C_F}$

$= 980306.19 - 980302.12 + (0.3086 \times -100)$

$\mathbf{= -26.79\ mGal}$

$\Delta \mathbf{g_B = g_{obs} - g_\varphi + C_F - C_B + C_T}$

$= 980306.19 - 980302.12 + (0.3086 \times -100) - (0.0419 \times 2.0 \times -100) + 0.15$

$\mathbf{= -18.26\ mGal}$

<u>Problem 3 :</u>

Given :

Observed gravity relative to base + 30 mGal
Elevation of station 150 m above base
Station is 1000 m north of base
Latitude effect is 0.00025 mGal/m
Density is 1.8 g/cc
Terrain effect is 0.05 mGal

Compute : Free-air and Bouguer Anomalies

$\Delta g_F = g_{obs} - g_\varphi + C_F$

$= 30 - (0.00025 \times 1000) + 0.3086 \times 150)$

$= 30 - 0.25 + 46.29$

$\mathbf{= 76.04\ mGal}$

$\Delta g_B = g_{obs} - g_\varphi + C_F - C_B + C_T$

$= 76.04 - (0.0419 \times 1.8 \times 150) + 0.05$

$= 76.04 - 11.313 + 0.05$

$\mathbf{= 64.777\ mGal}$

Problem 4 :

Given:

Observed gravity relative to base - 12.5 mGal

Elevation of station is 100 m below the base

Station is 2000 m south of base

Latitude effect is 0.00025 mGal/m

Density is 1.8 g/cc

Terrain effect is 0.1 mGal

Compute:

1 - Free air anomaly

2 - Bouguer gravity anomaly

$\boldsymbol{\Delta g_F = g_{obs} - g_\varphi + C_F}$

$= - 12.5 – (0.00025 \times - 2000) + (0.3086 \times - 100)$

$= - 12.5 + 0.5 - 30.86$ **= - 42.86 mGal**

$\boldsymbol{\Delta g_B = g_{obs} - g_\varphi + C_F - C_B + C_T}$

$= - 42.86 - (0.0419 \times 1.8 \times - 100) + 0.1$

$= - 42.86 +7.542+ 0.1$

= - 35.218 mGal

Problem Sets

Q.1 What is the average gravitational acceleration at the surface of the Earth? Mass of the Earth (ME) = 5.974×10^{24} kg . Average radius of the Earth (r) = 6371 km

Q. 2 What is the expected value of gravity at latitude 53.52589^0 N ?

Q.3 Surface gravity at a measuring site is 9.803244 ms^{-2}, the site has latitude 43.1° N and elevation 54 m. Obtain the free air gravity anomalies.

CHAPTER 7

MAGNETIC METHOD

- Basic concepts
- Description of the magnetic field
- Source of magnetic anomalies
- Measurements
- Interpretation & Applications

Problem Set – 6

MAGNETIC METHODS

Magnetic methods are one of the most commonly used geophysical tools. This stems from the fact that magnetic observations are obtained relatively easily and cheaply and few corrections must be applied to the observations. Despite these obvious advantages, like the gravitational methods, interpretations of magnetic observations suffer from a lack of uniqueness.

Magnetic Monopoles

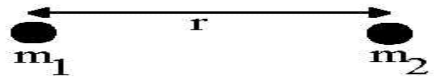
Charles Coulomb, in 1785, showed that the force of attraction or repulsion between electrically charged bodies and between magnetic poles also obey an inverse square law like that derived for gravity by Newton. The mathematical expression for the magnetic force experienced between

two magnetic monopoles is given by

$$\boxed{F_m = \frac{1}{\mu}\frac{p_1 p_2}{r^2}}$$

where μ is a constant of proportionality known as the *magnetic permeability*, *p1* and *p2* are the strengths of the two magnetic monopoles, and *r* is the distance between the two poles. the magnetic permeability, μ, is a property of the material in which the two monopoles, *p1* and *p2*, are located. If they are in a vacuum, μ is called the magnetic permeability of free space. *p1* and *p2* can be either positive or negative in sign. If *p1* and *p2* have the same sign, the force between the two monopoles is repulsive. If *p1* and *p2* have opposite signs, the force between the two monopoles is attractive.

From Coulomb's expression, we know that force must be given in Newtons,*N*, Permeability,mu, is defined to be a unit less constant. The units of pole strength are defined such that if the force, *F*, is 1 N and the two magnetic poles are separated by 1 m, each of the poles has a strength of 1 Amp - m (Ampere - meters). In this case, the poles are referred to as *unit poles*.



The *magnetic field strength*, *H*, is defined as the force per unit pole strength exerted by a magnetic monopole, *p1*. *H* is nothing more than Coulomb's expression divided by *p2*.

$$\boxed{H = \frac{F_m}{p_2} = \frac{p_1}{\mu r^2}}$$

Given the units associated with force, *N*, and magnetic monopoles, *Amp -m*, the units associated with magnetic field strength are Newtons per Ampere-meter, *N / (Amp - m)*. A *N / (Amp - m)* is referred to as a *tesla (T)*, named after the renowned inventor **Nikola Tesla**

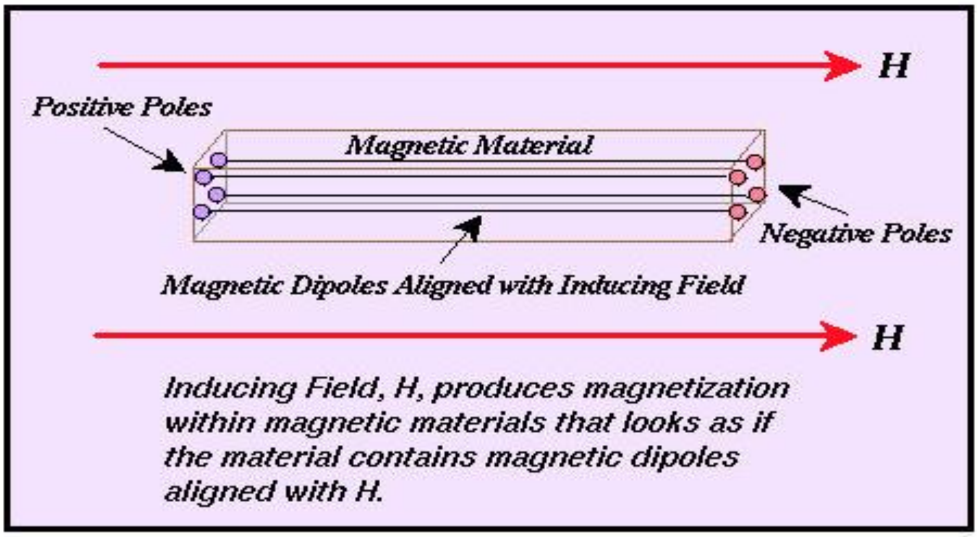
When describing the magnetic field strength of the earth, it is more common to use units of nanoteslas (nT), where one nanotesla is 1 billionth of a tesla. The average strength of the Earth's magnetic field is about 50,000 nT. A nanotesla is also commonly referred to as a *gamma*.

Magnetic Induction

When a magnetic material, say iron, is placed within a magnetic field, *H*, the magnetic material will produce its own magnetization. This phenomena is called *induced magnetization*.

In practice, the induced magnetic field will look like it is being created by a series of magnetic dipoles located within the magnetic material and oriented parallel to the direction of the inducing field, *H*.

The strength of the magnetic field induced by the magnetic material due to the inducing field is called the *intensity of magnetization*, *I*.



The intensity of magnetization, *I*, is related to the strength of the inducing magnetic field, *H*, through a constant of proportionality, known as the magnetic susceptibility. $\boxed{I = kH}$

The magnetic susceptibility (K) is a unitless constant that is determined by the physical properties of the magnetic material. It can take on either positive or negative values. Positive values imply that the induced magnetic field, *I*, is in the same direction as the inducing field, *H*.

Negative values imply that the induced magnetic field is in the opposite direction as the inducing field.

Magnetic Susceptibility K is dependent on :

1- The state of magnetization.
2- Intensity of saturation magnetization.
3- Grain size.
4- Internal stress.
5- Shape
6- Mode of dispersion.

The intensity of magnetization $I = M / V$
M = magnetic moment = $m L$
V = Volume
m = pole strength
L = length

Intensity of magnetization ∞ H and has the same direction.

$$I = \frac{M}{volume} = \frac{ml}{volume} = \frac{m}{area}$$

Mechanisms for Induced Magnetization

The nature of magnetization material is in general complex, governed by atomic properties, and well beyond the scope of this series of notes. Suffice it to say, there are three types of magnetic materials: paramagnetic, diamagnetic, and ferromagnetic.

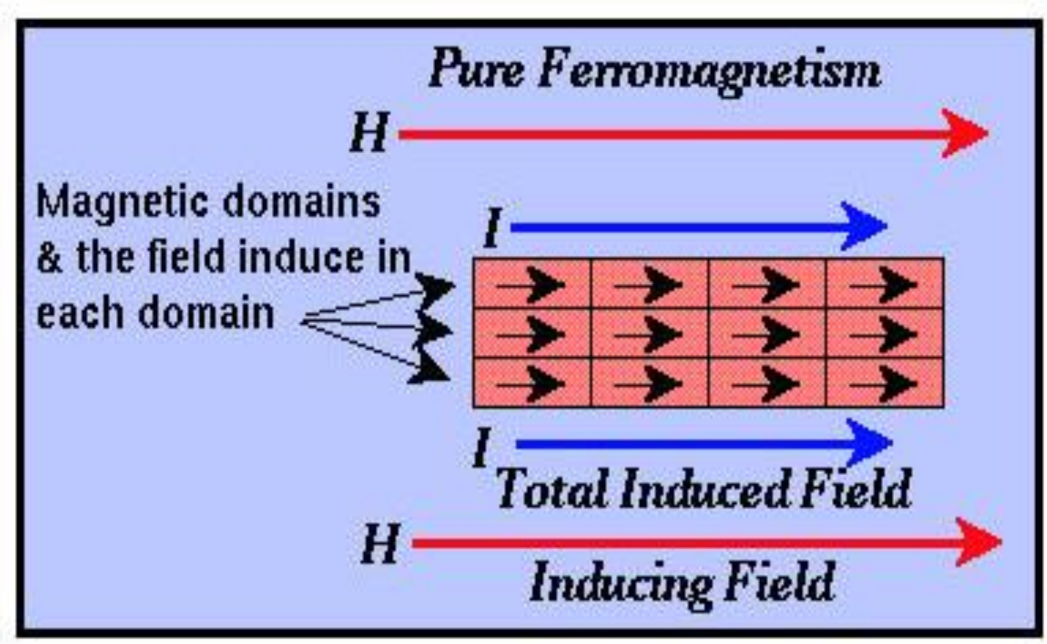
Diamagnetism - Discovered by Michael Faraday in 1846. This form of magnetism is a fundamental property of all materials and is caused by the alignment of magnetic moments associated with orbital electrons in the presence of an external magnetic field. For those elements with no unpaired electrons in their outer electron shells, this is the only form of magnetism observed. The susceptibilities of diamagnetic materials are relatively small and negative. Quartz and salt are two common diamagnetic earth materials.

Paramagnetism - This is a form of magnetism associated with elements that have an odd number of electrons in their outer electron shells. Paramagnetism is associated with the alignment of electron spin directions in the presence of an external magnetic field. It can only be observed at relatively low temperatures. The temperature above which paramagnetism is no longer observed is called the *Curie Temperature*. The susceptibilities of paramagnetic substances are small and positive.

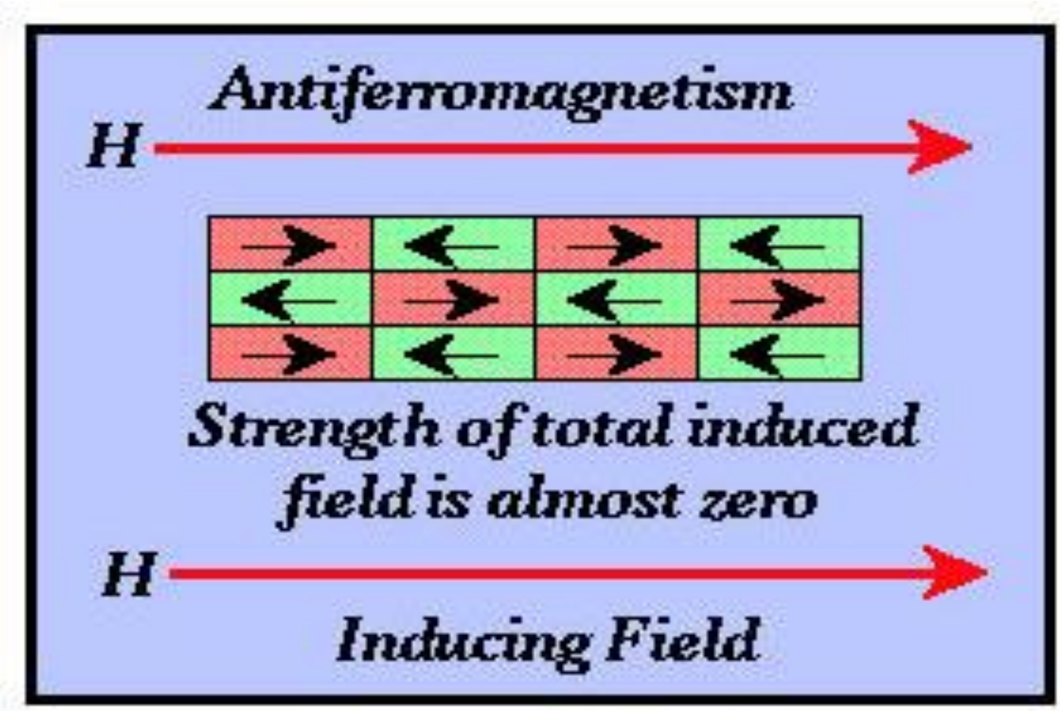
Ferromagnetism - This is a special case of paramagnetism in which there is an almost perfect alignment of electron spin directions within large portions of the material referred to as *domains*.

Like paramagnetism, ferromagnetism is observed only at temperatures below the Curie temperature. There are three varieties of ferromagnetism.

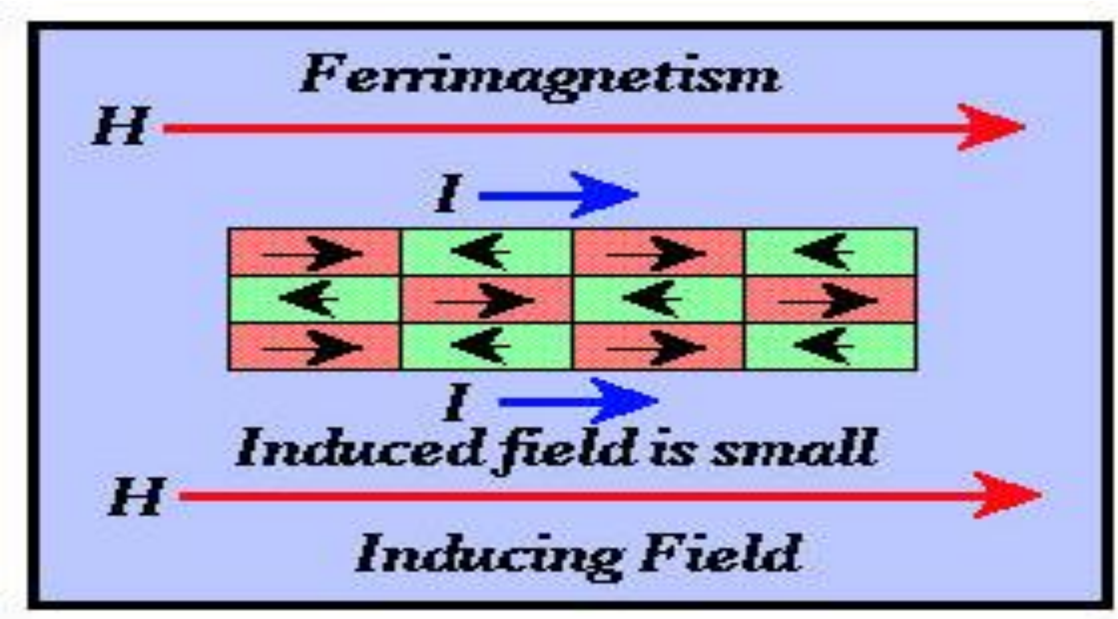
Pure Ferromagnetism - The directions of electron spin alignment within each domain are almost all parallel to the direction of the external inducing field. Pure ferromagnetic substances have large (approaching 1) positive susceptibilities. Ferrromagnetic minerals do not exist, but iron, cobalt, and nickel are examples of common ferromagnetic elements.



Antiferromagnetism - The directions of electron alignment within adjacent domains are opposite and the relative abundance of domains with each spin direction is approximately equal. The observed magnetic intensity for the material is almost zero. Thus, the susceptibilities of antiferromagnetic materials are almost zero. Hematite is an antiferromagnetic material



Ferromagnetism - Like antiferromagnetic materials, adjacent domains produce magnetic intensities in opposite directions. The intensities associated with domains polarized in a direction opposite that of the external field, however, are weaker. The observed magnetic intensity for the entire material is in the direction of the inducing field but is much weaker than that observed for pure ferromagnetic materials. Thus, the susceptibilities for ferromagnetic materials are small and positive. The most important magnetic minerals are ferromagnetic and include magnetite, titanomagnetite, ilmenite, and pyrrhotite.



Remanent Magnetization in Rocks

- Remanent field (remains even after external field removed)
  - thermoremanent
  - detrital remanent
  - chemical remanent

Total magnetization (J) = Remanent (Jr) + induced (Ji)
intensity of Jr is large in igneous and thermally metamorphosed Rocks.
Koenigsberger ratio. Q = Remanent (Jr) / induced (Ji)
Q > 1, Jr of sediments is smaller than Ji.

Magnetic Field Elements

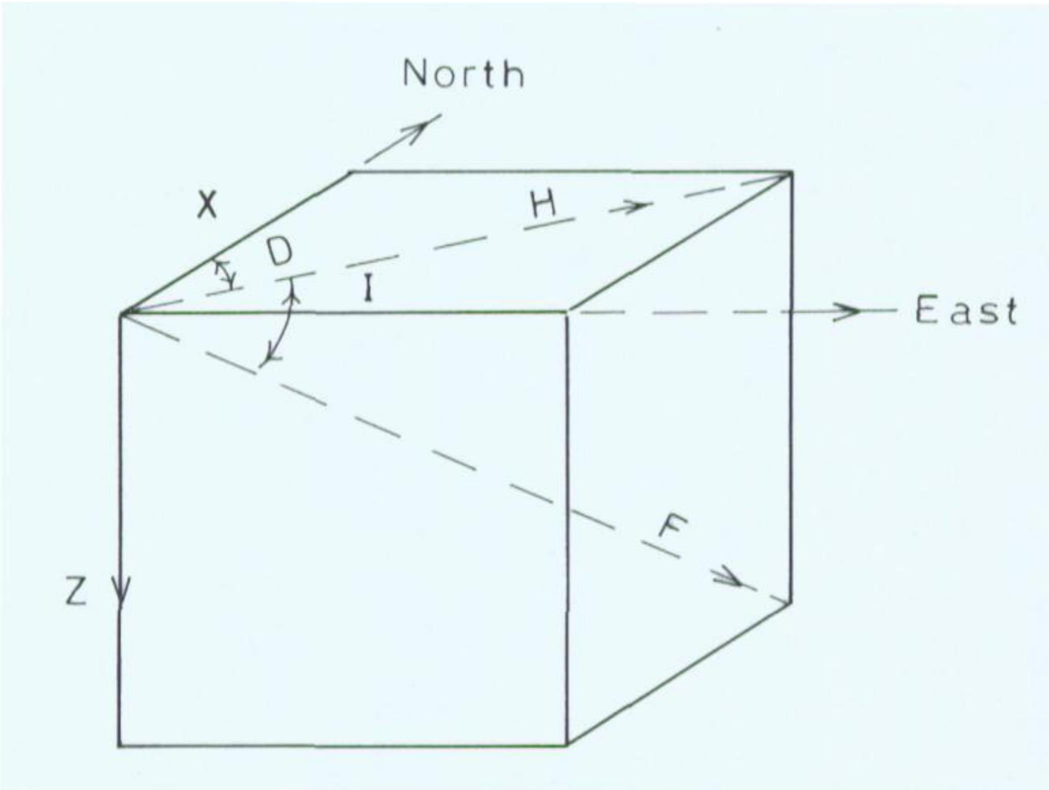
At any point on the Earth's surface, the magnetic field, *F**, has some strength and points in some direction. The following terms are used to describe the direction of the magnetic field.

Declination - The angle between north and the horizontal projection of *F*. This value is measured positive through east and varies from 0 to 360 degrees.

Inclination - The angle between the surface of the earth and *F*. Positive inclinations indicate *F* is pointed downward, negative inclinations indicate *F* is pointed upward. Inclination varies from -90 to 90 degrees.

Magnetic Equator - The location around the surface of the Earth where the Earth's magnetic field has an inclination of zero (the magnetic field vector *F* is horizontal). This location *does not* correspond to the Earth's rotational equator.

Magnetic Poles - The locations on the surface of the Earth where the Earth's magnetic field has an inclination of either plus or minus 90 degrees (the magnetic field vector *F* is vertical). These locations *do not* correspond to the Earth's north and south poles.



The total field F is resolved into its horizontal components H (x , y) and it vertical components Z. The angle which F makes with its horizontal components H is the inclination (I), and the angle between H and X (points North) is the declination (D).

$$F^2 \quad = X^2 + y^2 + Z^2$$
$$F^2 \quad H^2 + Z^2$$
$$H \quad = F \cos I$$
$$Z \quad F \sin I$$
$$X \quad = H \cos D$$
$$Z / H = \tan I$$

$$\boxed{\tan I = 2 \tan \text{Ø} \rightarrow \text{Latitude}}$$

F at North Pole = 60,000 nT
F at South Pole = 70,000 nT
F at equator = 30.000 nT
$F_{pole} = 2\ F_{Equator}$

The Earth's Magnetic Field

Ninety percent of the Earth's magnetic field looks like a magnetic field that would be generated from a dipolar magnetic source located at the center of the Earth and aligned with the Earth's rotational axis. The strength of the magnetic field at the poles is about 60,000 nT. The remaining 10% of the magnetic field can not be explained in terms of simple dipolar sources.

If the Earth's field were simply dipolar with the axis of the dipole oriented along the Earth's rotational axis, all declinations would be 0 degrees (the field would always point toward the north).

The magnetic field can be broken into three separate components.

Main Field - This is the largest component of the magnetic field and is believed to be caused by electrical currents in the Earth's fluid outer core. For exploration work, this field acts as the inducing magnetic field.

External Magnetic Field - This is a relatively small portion of the observed magnetic field that is generated from magnetic sources external to the earth. This field is believed to be produced by interactions of the Earth's ionosphere with the solar wind. Hence, temporal variations associated with the external magnetic field are correlated to solar activity.

Temporal Variations of the Earth's Magnetic Field

The magnetic field varies with time. When describing temporal variations of the magnetic field, it is useful to classify these variations into one of three types depending on their rate of occurrence and source. Three temporal variations:

Secular Variations - These are long-term (changes in the field that occur over years) variations in the main magnetic field that are presumably caused by fluid motion in the Earth's Outer Core. Because these variations occur slowly with respect to the time of completion of a typical exploration magnetic survey, these variations will not complicate data reduction efforts.

Diurnal Variations - These are variations in the magnetic field that occur over the course of a day and are related to variations in the Earth's external magnetic field. This variation can be on the order of 20 to 30 nT per day and should be accounted for when conducting exploration magnetic surveys.

Magnetic Storms - Occasionally, magnetic activity in the ionosphere will abruptly increase. The occurrence of such storms correlates with enhanced sunspot activity. The magnetic field observed during such times is highly irregular and unpredictable, having amplitudes as large as 1000 nT.

Exploration magnetic surveys should not be conducted during magnetic storms. This is because the variations in the field that they can produce are large, rapid, and spatially varying. Therefore, it is difficult to correct for them in acquired data.

Measuring the Earth's Magnetic Field

Magnetometers are highly accurate instruments, allowing the local magnetic field to be measured to accuracies of 0.002%. The proton precession, caesium vapour and gradiometer magnetometer systems are used for commercial applications. The systems operate on broadly similar principles utilizing proton rich fluids surrounded by an electric coil. A momentary current is applied through the coil, which produces a corresponding magnetic field that temporarily polarizes the protons. When the current is removed, the protons realign or process into the orientation of the Earth's magnetic field.

Gradiometers measure the magnetic field gradient rather than total field strength, which allows the removal of background noise. Magnetic gradient anomalies generally give a better definition of shallow buried features such as buried tanks and drums, but are less useful for investigating large geological features. Unlike EM surveys, the depth penetration of magnetic surveys is not impeded by high electrical ground conductivities associated with saline groundwater or high levels of contamination.

Flux-gate magnetometer

- relative instrument
- can be used to find vector components, direction of field
- portable instruments usually set up to read H_Z (vertical component)

Proton-precession magnetometer

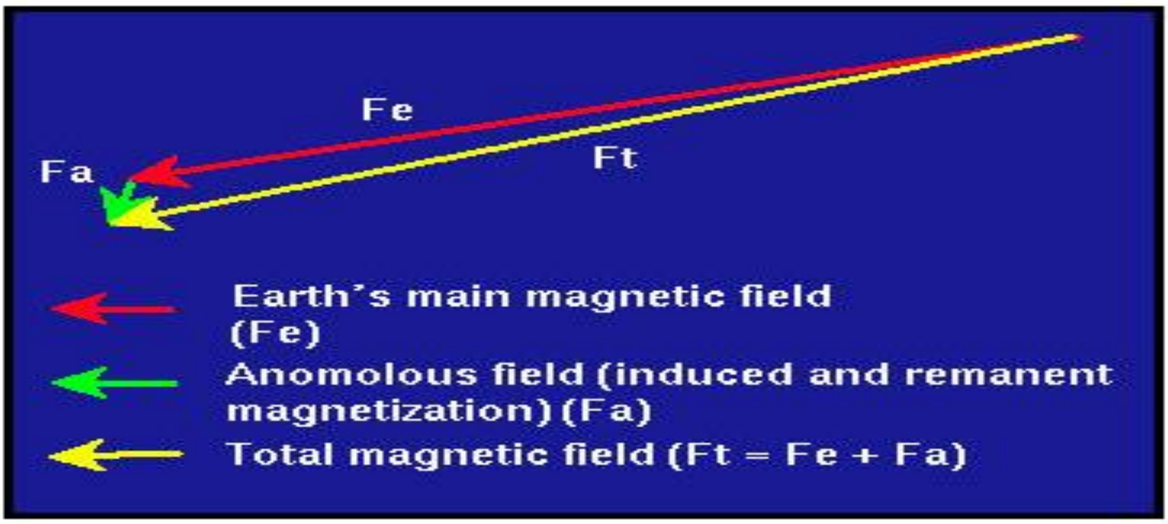
- simple, inexpensive, accurate, portable instrument
- measures absolute, total value of field
- 1 nT precision
- susceptible to strong magnetic gradients

It shows no appreciable instrument drift with time. One of the important advantages of the proton precession magnetometer is its ease of use and reliability. Sensor orientation need only be set to a high angle with respect to the Earth's magnetic field. No precise leveling or orientation is needed. The magnetic field we record with proton precession magnetometer has two components:

The main magnetic field, or that part of the Earth's magnetic field generated by deep (outer core) sources. The direction and size of this component of the magnetic field at some point on the Earth's surface is represented by the vector labeled *Fe* in the figure.

The anomalous magnetic field, or that part of the Earth's magnetic field caused by magnetic induction of crustal rocks or remanent magnetization of crustal rocks. The direction and size of this component of the magnetic field is represented by the vector labeled *Fa* in the figure. The total magnetic field we record, labeled *Ft* in the figure, is nothing more than the sum of *Fe* and *Fa*.

Typically, *Fe* is much larger than *Fa*, as is shown in the figure (50,000 nT versus 100 nT). If *Fe* is much larger than *Fa*, then *Ft* will point almost in the same direction as *Fe* regardless of the direction of *Fa*.That is because the anomalous field, *Fa*, is so much smaller than the main field, *Fe*, that the total field, *Ft*, will be almost parallel to the main field.



Similarities Between Gravity and Magnetics

- Geophysical exploration techniques that employ both gravity and magnetics are passive (measure a naturally occurring field of the earth.
- Collectively, the gravity and magnetic methods are often referred to as *potential methods*.
- Identical physical and mathematical representations can be used to understand magnetic and gravitational forces. For example, the fundamental element used to define the gravitational force is the point mass and the fundamental magnetic element is called a magnetic monopole.
- The acquisition, reduction, and interpretation of gravity and magnetic observations are very similar.
- Both gravity and magnetic vary in time and space and used as reconnaissance tools in exploration.

Differences Between Gravity and Magnetics

- The fundamental parameter that controls gravity variations is rock density and the fundamental parameter controlling the magnetic field variations is magnetic susceptibility.
- Unlike the gravitational force, which is always attractive, the magnetic force can be either attractive or repulsive.
- Unlike the gravitational case, single magnetic point sources (monopoles) can never be found alone in the magnetic case. Rather, monopoles always occur in pairs. A pair of magnetic monopoles, referred to as a *dipole*, always consists of one positive monopole and one negative monopole.
- A properly reduced gravitational field is always generated by subsurface variations in rock density. A properly reduced magnetic field, however, can have as its origin at least two possible sources. It can be produced via an *induced magnetization*, or it can be produced via a *remnant magnetization*.
- Unlike the gravitational field, which does not change significantly with time**, the magnetic field is highly time dependent.
- gravity requires 0.1 ppm accuracy, magnetic > 10 ppm
- gravimeter is relative instrument; magnetometer is absolute

- densities vary from 1 to 4; susceptibility over several orders of magnitude
- gravity anomalies smooth, regional; magnetic anomalies sharp, local
- tides are only external gravity effect, can be corrected. Effect of magnetic storms cannot be removed.
- gravity corrections: drift, latitude, free air, Bouguer, terrain, etc.; magnetic corrections: ± drift, IGRF
- gravity surveys slow, expensive; magnetic costs about 1/10 of g

Applications of Rock magnetism in paleomagnetism :

a- Reversals of the earth's field. (most recent reversal about 20.000 y. ago., 50/50 N/R.

b- Sea floor spreading.

c- Secular variation and paleo intensity of the earth field.

d- Polar wander and continental drift.

e- Paleo climatology.

f- Magnetic dating of rocks by :

- secular variation 10^3 y.
- polarity zones $10^4 – 10^6$ y.
- average paleomagnetic pole positions $10^7 – 10^9$ y.
- Q ratio.

g. Tectonic movements involving rotation. Ex. Japan. By NRM.

GENARAL APPLICATIONS

- Finding buried steel tanks and waste drums
- Detecting iron and steel obstructions
- Accurately mapping archaeological features
- Locating unmarked mineshafts
- Mapping basic igneous intrusives & faults
- Evaluating the size and shape of ore bodies

Problem Sets

PROBLEM 1

IF THE MAGNETIC SUSCEPTIBILITY OF A SPHERICAL PLUTON IS 0.0003 AND THE EARTH'S MAGNETIC FIELD (B) IS 0.0006 TESLA. THE RADIUS OF THE PLUTON IS 1 KM AND THE MAGNETIC PERMEABILITY IS $4\ \pi\ X\ 10^{-7}$.

COMPUTE :

1. **THE MAGNETIC FIELD STRENGTH (H)**
2. **THE INTENSITY OF MAGNETIZATION (I)**
3. **THE MAGNETIC MOMENT OF THE PLUTON (M).**

مصطلحات الإستكشاف السيزمي

المصطلح	الترجمة العربية
Elasticity	المرونة
Stress System	نظام الإجهاد
Poisson's ratio	معامل بوايسون
Tangential Stress	الإجهاد المماسي
Transverse Stress	الإجهاد المستعرض
Transverse Strain	التشوه (الانفعال) المستعرض
Normal Stress	الإجهاد العمودي
Rigidity Modulus	معامل الصلابة
Shear Modulus	معامل القص
Hooke's Law	قانون هوك
Elastic Limit	حد المرونة
Plastic Point	نقطة اللدونة
Anelastic Materials	المواد اللامرنة
Shear resistance	مقاومة القص
Young's modulus	معامل يونج
Compressibility	الانضغاطية
Dilatation	تمدد حجمي
Wave Propagation	الانتشار الموجي
Body Waves	الموجات الباطنية
Surface Waves	الموجات السطحية
Longitudinal Waves	الموجات الطولية
Primary Waves	الموجات الأولية
Compressional Waves	الموجات التضاغطية
Shear Waves	موجات القص
Transverse Waves	موجات مستعرضة
Secondary Waves	موجات ثانوية
Birch's Law	قانون بيرش
Seismic Velocities	سرع سيزميه
Rayleigh Waves (LR)	موجات رايلي
Love Waves (LQ)	موجات لوف
Dispersion	تشتت
Amplitude	سعة الموجة
Wavelength	طول الموجة
Frequency	تردد

Seismic Refraction	الانكسار السيزمي
Seismic Reflection	الانعكاس السيزمي
Critical Distance	المسافة الحرجة
Thickness	سماكة
Depth	عمق
Seismic Source	مصدر سيزمي
Transmitter	مرسل
Receiver	مستقبل
Geophones	سماعات أرضية
Fermat's Principle	مبدأ فيرمات
Huygen's Principle	مبدأ هايجن
Reflection Coefficient (R_c)	معامل الانعكاس
Transmission Coefficient (T_c)	معامل الاختراق
Acoustic Impedance	العائق الصوتي
Wavefront	مقدمة الموجة
Raypath	مسار الموجة
Snell's Law	قانون سنيل
Critical Refraction	الانكسار الحرج
Low- Velocity – Layer	طبقة منخفضة السرعة
Hidden Layer	طبقة مختبئة
Blind Layer	طبق عمياء
Thin Layer	طبقة رقيقة
Diffraction	الحيود
Delay Time	زمن التأخير
Dipping Layers	طبقات مائلة
Green Equation	معادلة جرين
Dynamic Correction	التصحيح الديناميكي
Multiple Reflection	انعكاس متعدد
Time- Average Equation	معادلة معدل الزمن
Faust Equation	معادلة فوست
Apparent Velocity	سرعة ظاهرية
Average Velocity (VA)	معدل سرعة
Interval Velocity (VI)	سرعة بينية
Root Mean Square Velocity	سرعة تربيع متوسط الجذر
Dix Equation	معادلة ديكس
Data Processing	معالجة المعلومات
Cross Over Distance	مسافة العبور
Seismic Attenuation	تعتيم سيزمي

	مصطلحات الإستكشاف الكهربائي
Self Potential	جهد ذاتي
Galvanometer	مقياس الجهد الكهربائي
Apparent Resistivity	مقاومية ظاهرية
Equipotential line	خط متساوي الجهد
Principle of Equivalence	مبدأ التعادل
Principle of Suppression	مبدأ الغطس
Conductivity	التوصيلية
Vertical Electrical Sounding	الجس الكهربائي العمودي
Horizontal Electrical Profiling	المقطع الكهربائي الأفقي
Dipole	ثنائي القطب
Schlumberger Arrangement	ترتيب شلمبرجير
Induced Polarization	الإستقطابية المستحثة
Passive Methods	الطرق الخامدة
Electrical Conduction	التوصيل الكهربائي
Archie's Law	قانون أرشي
Electrical Reflection Coefficient	معامل الإنعكاسية الكهربائية
Ground Penetrating Radar (GPR)	رادار الإختراق الأرضي
Airborne Electromagnetic	المسح الكهرومغناطيسي الجوي
Very Low Frequency (VLF)	التردد المنخفض جدا

مصطلحات الإستكشاف الجاذبي والمغناطيسي

Absolute gravity	**جاذبية مطلقة**
Magnetic Field Strength	**قوة المجال المغناطيسي**
Magnetic Induction	**الحث المغناطيسي**
Centrifugal Force	**قوه طرد مركزية**
Intensity of Magnetization	**شدة التمغنط**
Magnetic Declination	**انحراف مغناطيسي**
Diamagnetic	**ضعيف النفاذية المغناطيسية**
Diurnal Correction	**تصحيح يومي**
Elevation Correction	**تصحيح الارتفاع**
Equator	**خط الاستواء**
Magnetic Permeability	**النفاذية المغناطيسية**
Ferro magnetic	**مغناطيس حديدي**
Gravitational Acceleration	**التسارع الجاذبي**
Gravity Anomaly	**شاذة الجاذبية**
Magnetic Inclination	**الميل المغناطيسي**
Isostatic Correction	**تصحيح ايزوستاتي**
Lunar Variations	**تغيرات قمرية**
Magnetic Moment	**العزم المغناطيسي**
Magnetic Storms	**عواصف مغناطيسية**
Magnetometer	**جهاز قياس المغناطيسية**
Magnetic Susceptibility	**قابلية مغناطيسية (التأثرية المغناطيسية)**
Observed Gravity	**جاذبية مقاسة**
Paleomagnetism	**مغناطيسية قديمة**
Residual Magnetism	**مغناطيسية متخلفة**
Remanent Magnetism	**مغناطيسية متبقية**
Secular Variations	**تغيرات متناهية البطء**
Geoid	الجيوئد (سطح متساوي الجهد)
Gravimeter	**جهاز قياس الجاذبية**
Bouguer Anomaly	**شاذة بوجير**
Latitude Correction	**تصحيح خط العرض**