Reactor Physics: Basic Definitions and Perspectives

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Summary:
The basic definitions and perspectives for the behaviour of free neutrons as they interact with their surrounding media are introduced. This forms the basis for the detailed study to follow.

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1 Introduction

1.1 Overview

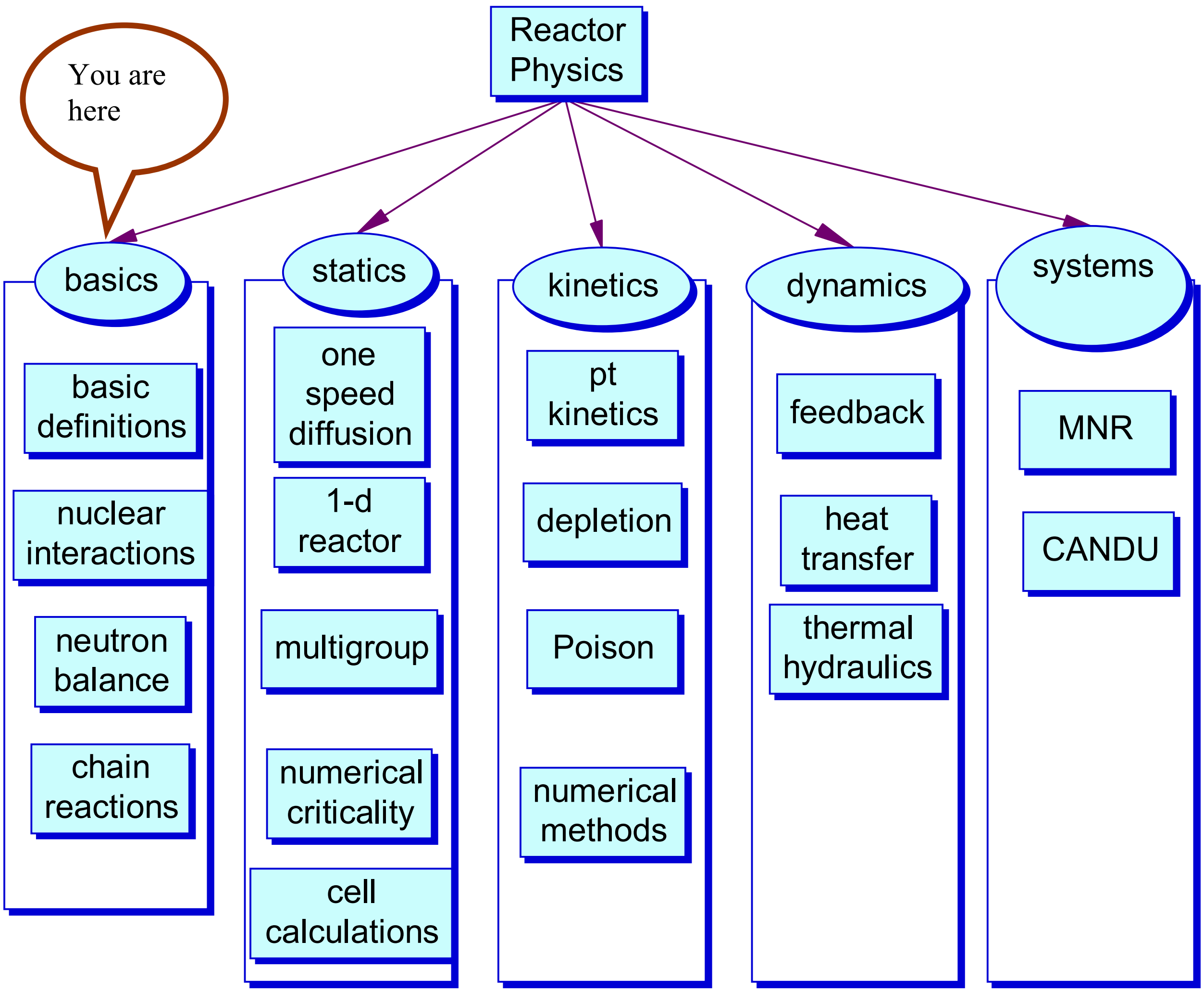


Figure 1 Course Overview

1.2 Learning outcomes

1.2.1 To understand the following physical processes

- fission
- neutron life cycle
- the neutron environment
- neutron energy distribution

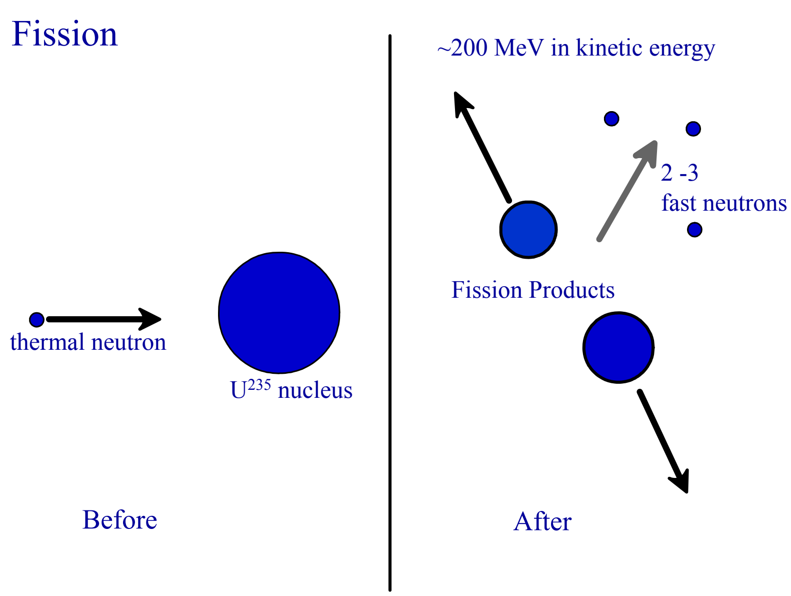
1.2.2 To understand the basics of neutron processes

- decay
- absorption and scattering
- kinematics

1.2.3 To understand the main issues for reactor modelling

2 The Life and Times of the Neutron

2.1 The Fission Event



The neutron, which is uncharged, can interact with a U^{235} nucleus leading to fission.

The result is the creation of fission products (which may be radioactive), radiation (usually γ's and β's) and 2 to 3 neutrons at high energy (1-2 MeV).

The probability of the event is a strong function of the neutron energy, as shown next.

Figure 2 The fission event

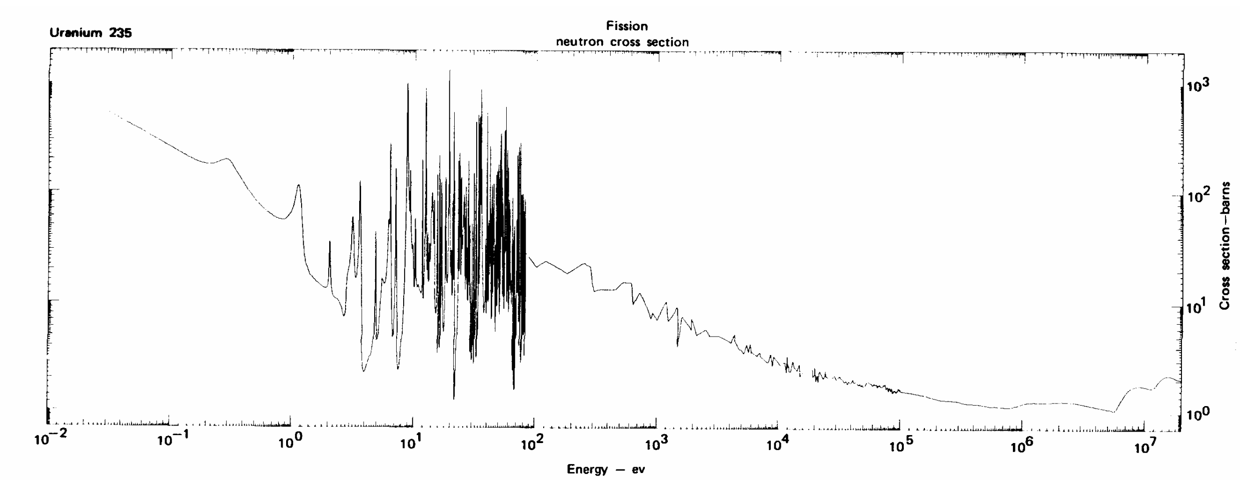


Figure 3 Fission cross section of U-235 [source: DUD1976, figure 2-17]

$\sigma \equiv$ microscopic cross section [cm^2] = effective interaction area
1 barn $\equiv 1 \times 10^{-24}$ cm^2
σ is usually quoted in units of barns since the effective area is so small.

2.2 Neutron Life Cycle in CANDU

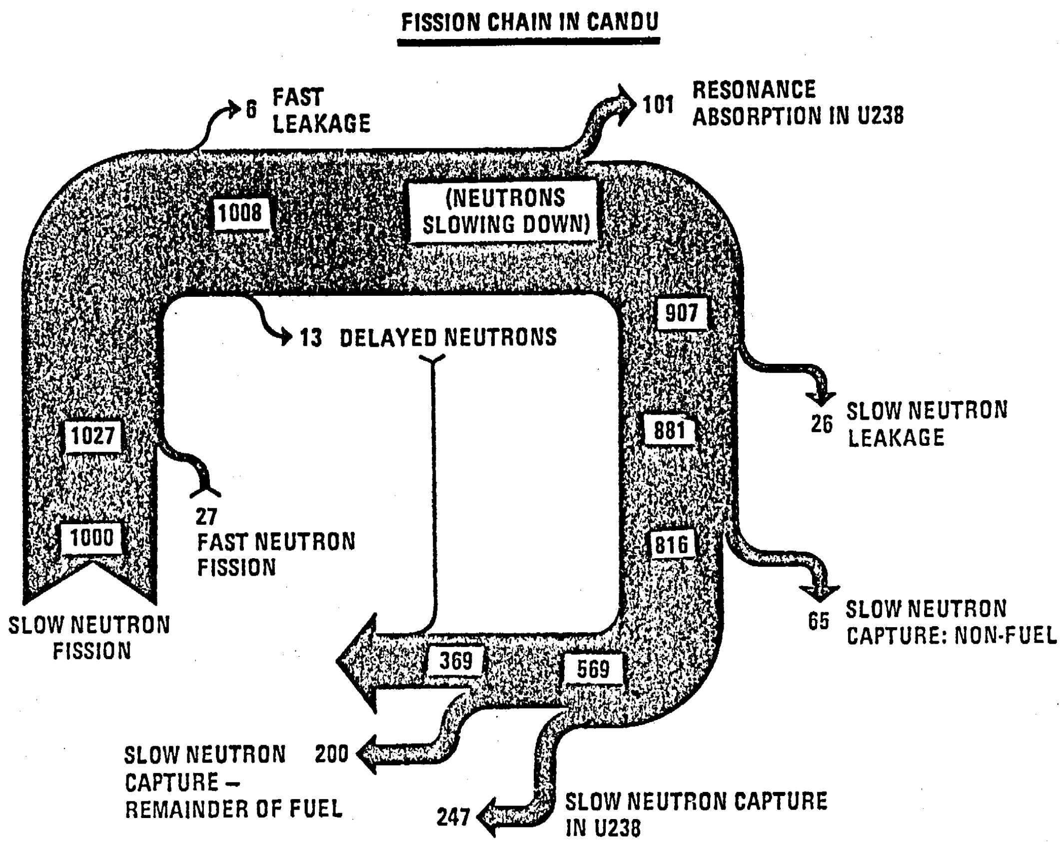


Figure 4 Neutron life cycle [source: unknown]

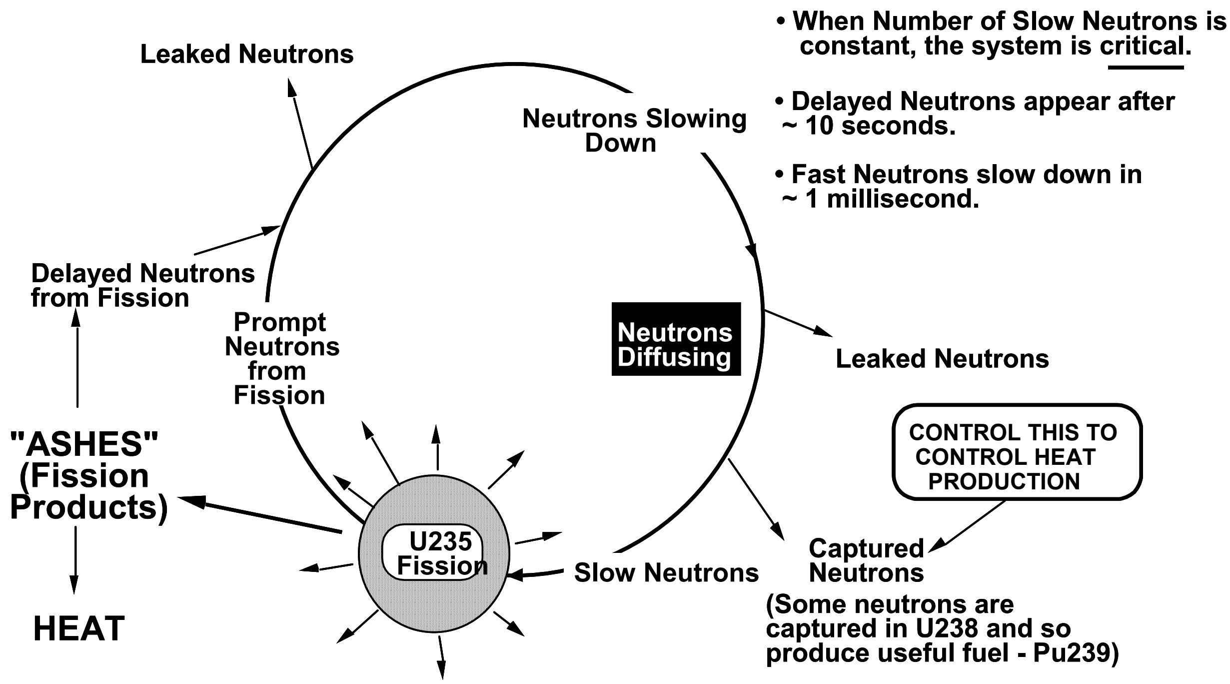


Figure 1 – The Neutron Cycle in a Thermal Reactor

Figure 5 Another view of the neutron life cycle [source: EP712 course notes, chapter 2]

2.3 Density of neutrons required to produce 1 watt/cm^3

Consider a beam of neutrons moving at velocity, v cm/s.

The average distance travelled before a fission interaction is $\bar{x}$ cm (in U^{235})

$\therefore$ Average time per interaction = $\bar{x}$ / v seconds and frequency of interaction = $\dfrac{v}{\bar{x}}$ s^{-1} per neutron.

If the density of neutrons is n neutrons/cm^3, then interaction rate = $\dfrac{nv}{\bar{x}}$ interactions/s-cm^3.

For

$$\frac{1 \text{ watt}}{\text{cm}^3} = \frac{1 \text{ Joule}}{\text{s} - \text{cm}^3} \; ,$$

$$1 \; \frac{\text{J}}{\text{s} - \text{cm}^3} = \frac{\text{energy}}{\text{fission}} \times \frac{\#\text{fissions}}{\text{s} - \text{cm}^3}$$

$$= 200 \times 10^6 \text{ eV} \times 1.602 \times 10^{-19} \; \frac{\text{Joules}}{\text{eV}} \times \frac{n\,v}{\bar{x}}$$

$$\therefore \; n = \frac{\bar{x} \leftarrow \; \sim \; 1 \text{ cm}}{2 \times 10^8 \times 1.6 \times 10^{-19} \times v \leftarrow \sim 2 \times 10^5 \text{ cm/s}}$$

$$\simeq 1.5 \times 10^5 \text{ n/cm}^3$$

Compare this to the typical nuclei density $\sim 10^{22}$/cm^3

<u>Conclusion</u>: Neutrons do not interact with each other.

This is an important conclusion.

2.4 Neutron Energy

Thermal distribution:

$$n(v) = 4\pi\left(\frac{m}{2\pi\, kT}\right)^{3/2} n_0\ v^2\ e^{-mv^2/2kT}$$

$$\Downarrow \qquad\qquad \left(E = \frac{1}{2}\, mv^2\right)$$

$$\left.\begin{aligned} n(E) &= \frac{2\pi n_0}{(\pi\, kT)^{3/2}}\ E^{1/2}\ e^{-E/kT} \\ \phi(E)\ &\equiv\ n(E)\, v = vn_0\ M(E) \\ &= \frac{2\pi n_0}{(\pi kT)^{3/2}}\left(\frac{2}{m}\right)^{1/2} E\, e^{-E/kT} \end{aligned}\right\}\ \text{Maxwellian Distribution}$$

Now, $n_0 = \int_o^\infty n(E)dE = \int_o^\infty n(v)dv$

<u>Note</u>: n(E) = # of neutrons in interval dE [# / eV]

n(v) = # of neutrons in interval dv [# / (m/s)]

Thus $n\left(\frac{1}{2}\, mv^2\right) \neq n(v)$ since interval size is different

But n(E) d(E) = n(v)dv so that $\int_0^\infty n(E)dE = \int_0^\infty n(v)dv$

<u>Most probable vel</u>:

$$\frac{dn(v)}{dv} = 0 \Rightarrow v_p = \sqrt{\frac{2kT}{m}}$$

$$= 2200 \text{ m/s}$$

<u>Most probable energy</u>:

$$\frac{dn(E)}{d\,E} = 0 \Rightarrow E_p = \frac{1}{2}\ kT$$

$$\bar{E} = \frac{3}{2}\ kT$$

$$\Rightarrow\ E(v_p) = kT = 0.025\text{eV at } 20^\circ\text{ C}$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}}$$

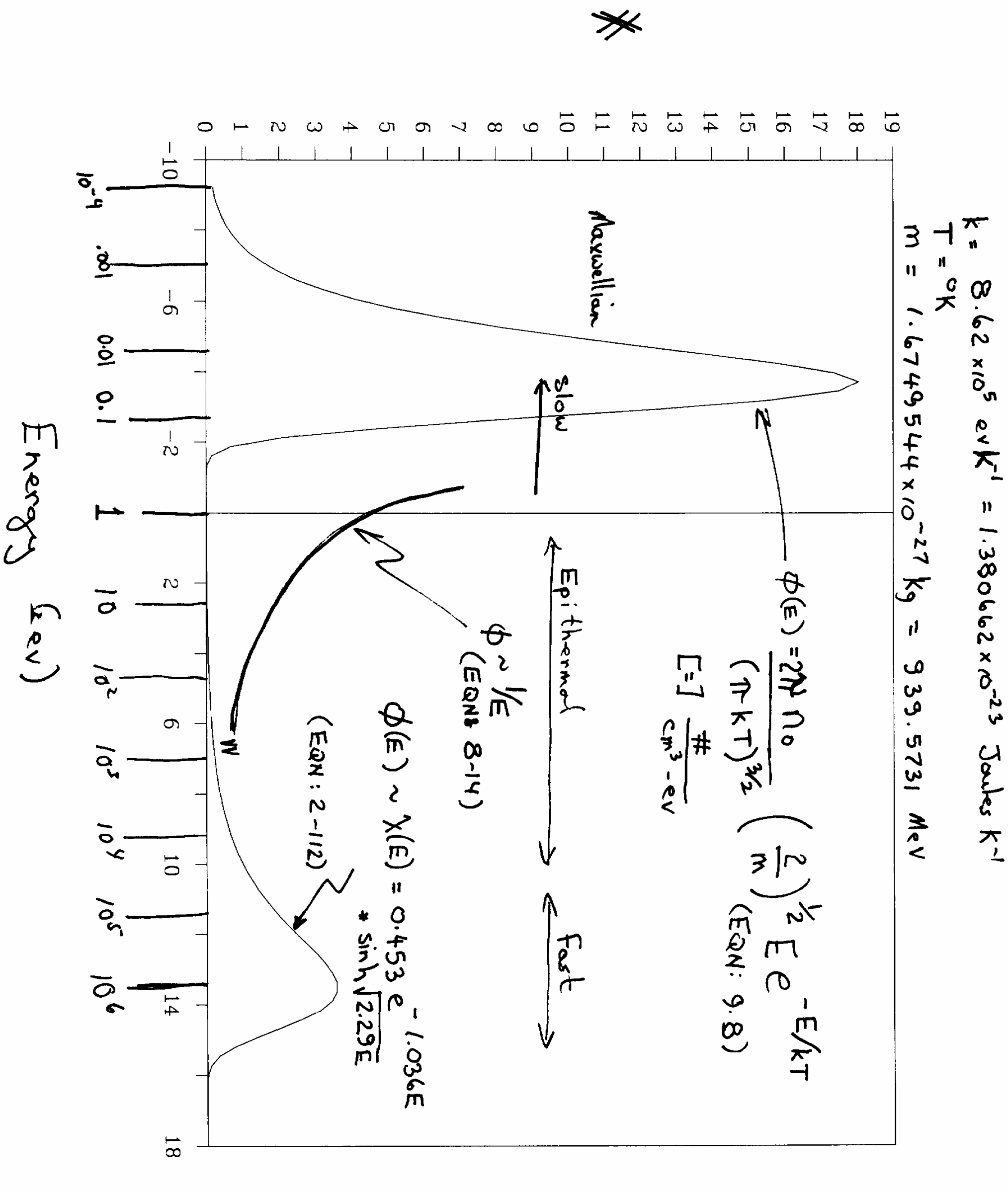


Figure 6 Neutron Energy Distribution

2.5 Units

$$v_p = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2\,x\,1.3806x10^{-23}\ \text{Joules}/\text{K}\ x\ 293.13\text{K}}{1.67x10^{-27}\,\text{kg}}}$$

$$= 2201\ \text{m}/\text{s}$$

$$E\ [\equiv]\ \text{gm}\ \frac{\text{cm}^2}{\text{sec}^2} = \text{erg} \quad \text{or} \quad \text{kg}\frac{\text{m}^2}{\text{sec}^2} = \text{Joules}$$

<u>Recall</u>: $F = ma \Rightarrow \text{dyne} = \text{gm cm/sec}^2$

$\therefore\ E = F \bullet x = \text{dyne - cm} = \text{erg} = 10^{-7}\ \text{J}.$

3 1/E Spectrum

3.1 Derivation of 1/E spectrum (equation 8-14 of D & H)

Assume the neutron is slowing down in H in the absence of absorption. Further assume that there is no upscatter.

$$\underbrace{[\Sigma_s(E) + \Sigma_a(E)]\phi(E)}_{\text{\# of neutrons leaving energy E}} = \underbrace{\int_E^\infty \Sigma_s(E' \to E)\,\phi(E')\,dE' + S(E)}_{\text{\# scattering down to energy E}}$$

Since $\Sigma_a(E) = 0$, we have

$$\underbrace{\Sigma_s(E)\,\phi(E)}_{\equiv F(E)} = \int_E^\infty \underbrace{\frac{\Sigma_s(E')}{E'}}_{\substack{\text{equal probability}\\ \text{of scatter (isotropic)}}} \phi(E')dE' + S(E)$$

$$\Downarrow$$

$$\therefore \qquad F(E) = \int_E^{E_0} \frac{F(E')}{E'}\,dE' + S_0\,\delta(E - E_0)$$

$$\Downarrow$$

$$\frac{d\,F(E)}{d\,E} = -\frac{1}{E}\,F(E)$$

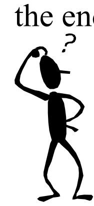
$$\Downarrow$$

$$F(E) = \frac{S_0}{E} + S_0\,\delta(E - E_0)$$

$$\therefore\ \phi(E) = \frac{S_0}{\Sigma_s(E)\,E} + \frac{S_0}{\Sigma_s(E)}\,\delta(E-E_0)$$

$$\Sigma_s \sim \text{const} \Rightarrow \phi \sim \frac{1}{E}$$

At this point you should be able to answer Questions 1, 2, 3 and 4 at the end of this chapter.



4 Decay

$$\frac{-\,d\,N(t)}{dt} = \lambda\, N\,(t)$$

$$\Downarrow$$

$$N(t) = N_0\ e^{-\lambda t}$$

$$\therefore\ -\frac{dN(t)}{dt} = \text{RATE} = \lambda\, N_0\ e^{-\lambda t}$$

decaying in dt at t = - dN (t)

$$= \lambda\, N_0\ e^{-\lambda t}\ dt$$

fraction of initial decaying in dt at t = $\lambda\, e^{-\lambda t}\ dt = \underbrace{p(t)}_{\text{probability}}\ dt$

mean lifetime, $\bar{t} = \int_0^\infty p(t)\ t dt = \lambda \int_0^\infty t e^{-\lambda t} dt = \frac{1}{\lambda}$

$$\therefore\quad \bar{t} = \frac{1}{\lambda}$$

<u>Half Life</u>, $T_{1/2}$

$$N(T_{1/2}) = \frac{N_0}{2} = N_0\ e^{-\lambda T_{1/2}}$$

$$\Rightarrow\ T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

4.1 Math aside

If we have two functions f(x) + g(x):

$$d(fg) = f'g + g'f \Rightarrow \int d(fg) = fg$$

$$= \int f'g\,dx + \int g'f\,dx$$

$$\therefore \quad \int_0^\infty \underbrace{t}_{f}\underbrace{e^{-\lambda t}}_{g'}\,dt = -\left.\frac{te^{-\lambda t}}{\lambda}\right|_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{(-\lambda)}dt$$

$$= 0 + \frac{1}{\lambda^2}$$

4.2 Example (D&H #2.3)

Decay chain for an initially pure radioactive sample.

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad \Rightarrow N_1(t) = N_1(0)\, e^{-\lambda_1 t}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad \Rightarrow N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

$$\left.\begin{aligned} \frac{dN_3}{dt} &= \lambda_2 N_2 - \lambda_3 N_3 \\ &\;\vdots \\ \frac{dN_N}{dt} &= \lambda_{N-1} N_{N-1} - \lambda_N N_N \end{aligned}\right\} \rightarrow \text{messy solutions}$$

To solve for N_2:

Rewrite to get:

$$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1$$

$$\Rightarrow d N_2 + \lambda_2 N_2\, dt = \lambda_1 N_1\, dt$$

Multiply by $e^{\lambda_2 t}$:

$$\Rightarrow e^{\lambda_2 t} dN_2 + \lambda_2 e^{\lambda_2 t} N_2 dt = \lambda_1 N_1 e^{\lambda_2 t} dt$$

$$\Rightarrow d(e^{\lambda_2 t} N_2) = \lambda_1 N_1 (0) e^{(\lambda_2 - \lambda_1)t} dt$$

$$\therefore e^{\lambda_2 t} N_2 = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} e^{(\lambda_2 - \lambda_1)t} + C$$

Now at $t = 0$, $N_2(0) = 0 \Rightarrow C = \dfrac{-\lambda_1 N_1(0)}{\lambda_2 - \lambda_1}$

$$\therefore N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} [e^{(\lambda_2 - \lambda_1)t} - 1] e^{-\lambda_2 t} = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}]$$

For fast decay of 1 and slow decay of 2 $(\lambda_1 >> \lambda_2)$

$$N_2(t) \sim \frac{\lambda_1 N_1(0)}{\underbrace{\cancel{\lambda_2}}_{0} - \lambda_1} [\underbrace{\cancel{e^{-\lambda_1 t}}}_{0} - e^{-\lambda_2 t}]$$

$$\sim N_1(0)\, e^{-\lambda_2 t}$$

i.e. decay dominated by decay of 2.

For slow decay of 1 and fast decay of 2 $(\lambda_2 >> \lambda_1)$

$$N_2(t) \sim \frac{\lambda_1 N_1(0)}{\lambda_2} e^{-\lambda_1 t} = N_1(t) \frac{\lambda_1}{\lambda_2}$$

i.e. $\lambda_2 N_2(t) = \lambda_1 N_1(t)$

This is called "$\underbrace{\text{secular}}$ equilibrium".

(lasting a long time, indifferent, not religious)

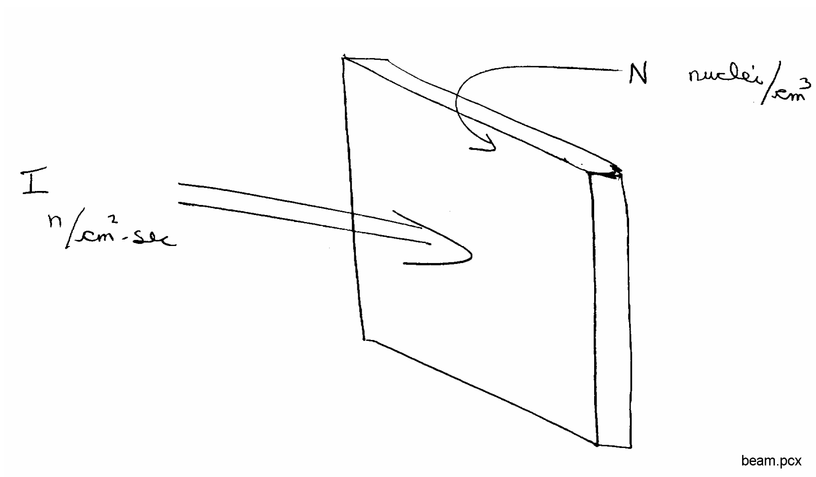
5 Cross Section

5.1 Microscopic cross section, σ [cm^2]

Consider a beam of neutrons incident on a target. The rate of interaction (neutron-nuclei) is

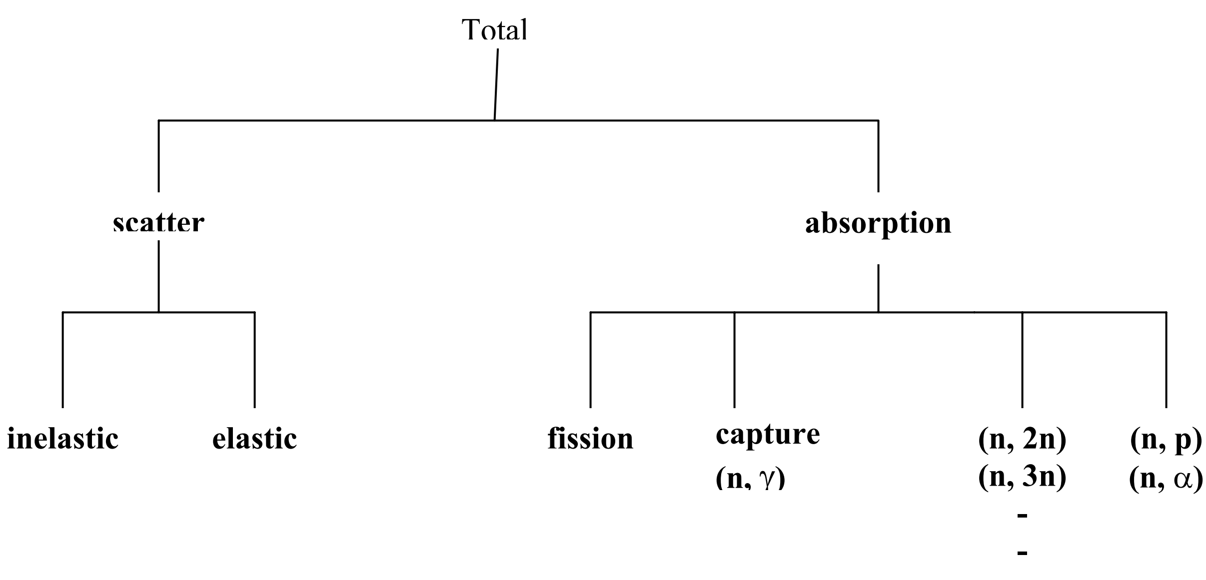
$$\text{Rate of interaction} = \underbrace{\sigma}_{\mathrm{cm}^2}\ \underbrace{I}_{\frac{\#}{\mathrm{cm}^2-\mathrm{s}}}\ \underbrace{N}_{\frac{\#}{\mathrm{cm}^3}}\ [\equiv]\frac{\#}{\mathrm{cm}^3-\mathrm{s}}$$

Recall that 1 barn = 10^{-24} cm^2



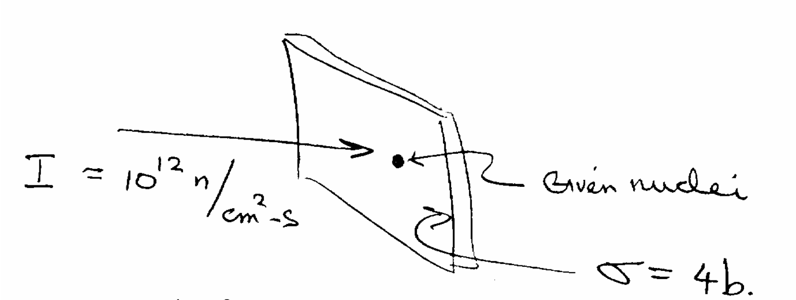
The total cross section, $\sigma_{total} = \sigma_{scatter} + \sigma_{absorption}$

ie. $\sigma_T = \sigma_s + \sigma_a$



5.2 Example (D & H 2.7)

Question: How long, on average for a given nuclei to suffer a neutron interaction?



beam2.pcx

$$\frac{\text{Rate}}{N} = \sigma I$$

$$= 4 \times 10^{-24} \times 10^{12} \text{ interactions/sec}$$

$$= 4 \times 10^{-12} \text{ interactions/sec for 1 nuclei}$$

$$\therefore \text{ seconds/interactions for 1 nuclei} = \frac{1}{4 \times 10^{-12}} \text{ s}$$

$$= 2.5 \times 10^{11} \text{ seconds}$$

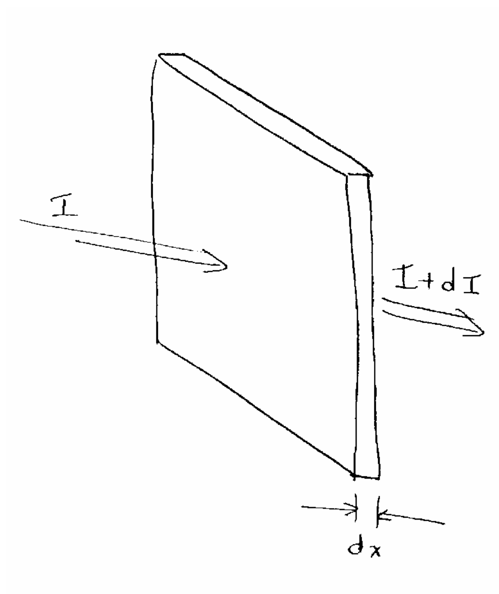
5.3 Macroscopic Cross Section, Σ [cm^{-1}]

$$\text{Rate} = \sigma I N \equiv \Sigma I$$

$$= -\frac{dI}{dx}$$

$$\Sigma \equiv \sigma N \left[\frac{cm^2 \cdot \#}{cm^3}\right]$$

$$= cm^{-1}$$



$$\frac{\left(-\frac{dI}{I}\right)}{dx} = \Sigma$$

= fractional change of I in distance dx

= probability of reaction per unit length

$$\Downarrow$$

$$I(x) = I_o e^{-\Sigma x}$$

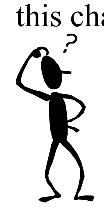
$$\frac{I(x)}{I_o} = \text{probability of going x with no interaction}$$

$$= e^{-\Sigma x}$$

Probability of interaction at x in dx is: (p(x)dx)

$$-\frac{dI}{I_o} = \frac{I(x)}{I_o} \cdot \Sigma dx = \underbrace{\Sigma e^{-\Sigma x}}_{p(x)} dx$$

At this point you should be able to answer Questions 5 at the end of this chapter.



5.4 Mean Free Path

$$\bar{x} = \int_0^\infty p(x)\, x dx = \int_0^\infty \Sigma\, e^{-\Sigma x}\, x dx$$
$$= \frac{1}{\Sigma} = \text{ mean free path}$$

$$\text{cf: } \bar{t} = \int_0^\infty \lambda e^{-\lambda t}\, t dt = \frac{1}{\lambda}$$

Mean time between collisions: $\dfrac{\bar{x}}{\text{velocity}} = \dfrac{1}{v\Sigma}$

$$\text{Collision frequency} = \frac{1}{\text{time}} = v\Sigma$$

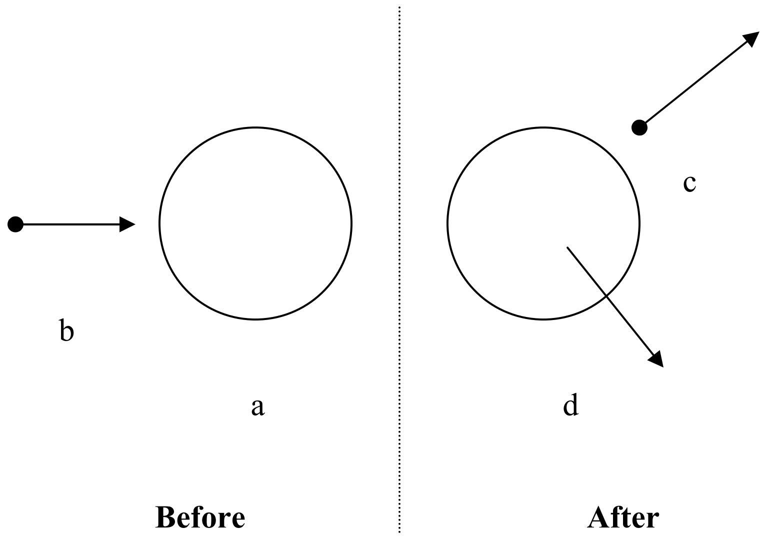
5.5 Calculation of Nuclei Density

$$\Sigma_t = \Sigma_a + \Sigma_s \quad \leftarrow \quad \Sigma_s = \sum_i N_i \sigma_s^i$$

$$\Sigma_a = N_x\, \sigma_a^x + N_y\, \sigma_a^y + \ldots$$
$$= \sum_i N_i \sigma_a^i$$

$$N_i = \frac{A \left(\dfrac{\#}{\text{gm - mole}}\right) \bullet \rho \left(\dfrac{\text{gm}}{\text{cm}^3}\right)}{A\ \dfrac{\text{gm}}{\text{gm - mole}}}$$, where A = Avogadro's number, $6.0221367\text{x}10^{23}$

6 Nuclear Reactions



<u>Reactions:</u>

$$a+b \longrightarrow c+d$$
$$a\ (b,c)\ d$$

<u>Example:</u>

$$n_0^1 + U_{92}^{235} \to U_{92}^{236} + \gamma$$

$$U_{92}^{235}\ (n, \gamma)\ U_{92}^{236}$$

Radioactive Capture: (n, γ)

Fission: $n + X \to X_1 + X_2 + \underbrace{\nu}_{0\to 3} n + \text{energy}$

Scattering: (n,n) elastic
(n, n') inelastic

<u>Source of neutrons</u>:

1. Fission
   A. Initiated by cosmic radiation
   B. Spontaneous
   C. Neutron absorption

2. (α, n)

$$Be_4^9 + \underbrace{He_2^4}_{\text{from radium, Pu, polonium}} \to C_6^{12} + n_0^1$$

3. (γ, n) (photoneutrons)

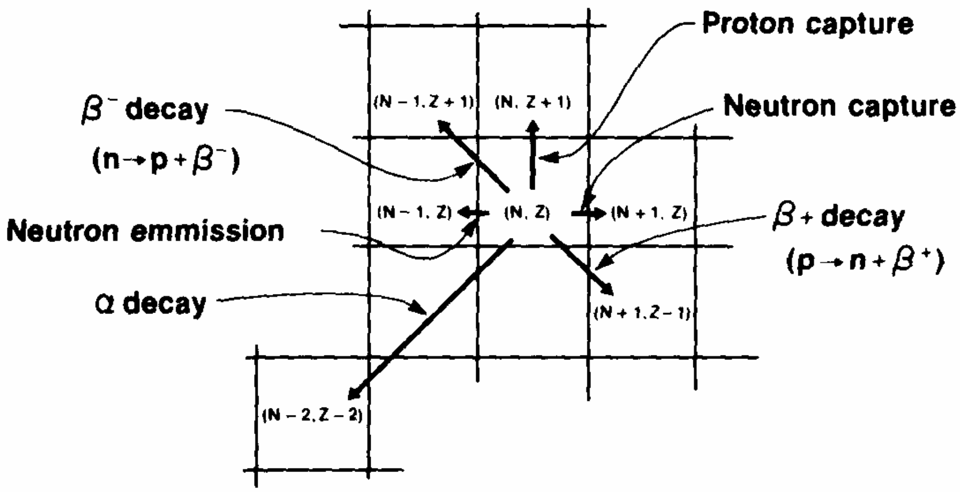
$$H_1^2 + \gamma \to H_1^1 + n_0^1 \qquad Be_4^9 + \gamma \to Be_4^8 + n_0^1$$

X_Z^A

X = some nucleus
Z = Atomic number = number of protons
N = number of neutrons
A = Mass number = N+Z
= total number of nucleons

PART C:

Principle nuclear transformations



CREDITS:
Design: A.A. Harms, Graphics: C.M. Leng,
***Data and design features based on* (1) S. Segre, (2) S.E. Liverhart, (3) *Table of Isotopes (1967),* (4) General Electric Chart of the Nuclides *(undated),* (5) Battelle Northwest Chart of the *Nuclides* (1969), (6) Karlsruhe Nukidkarte (1974).**

Figure 7 Nuclear Transformations [Source: A. A. Harms, McMaster University]

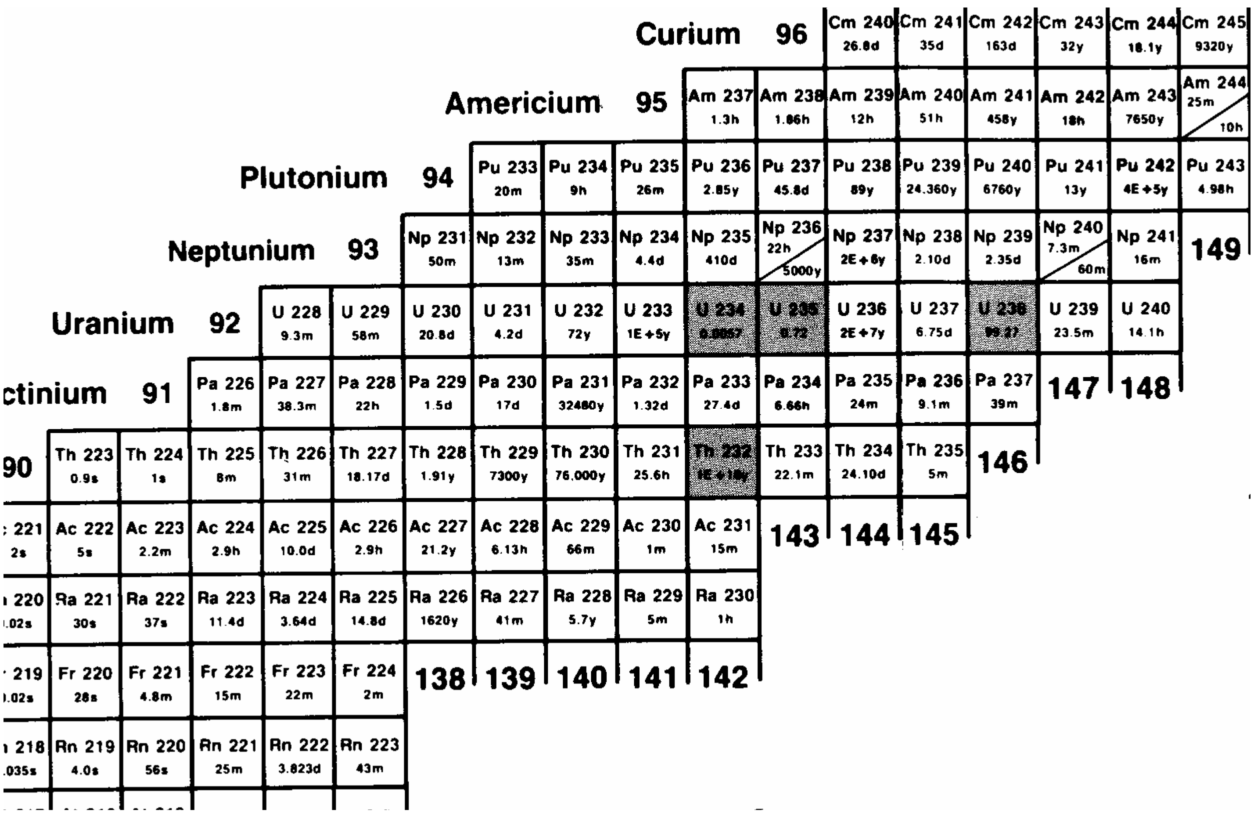


Figure 8 Segment of the Chart of the Nuclides [Source: A. A. Harms, McMaster University]

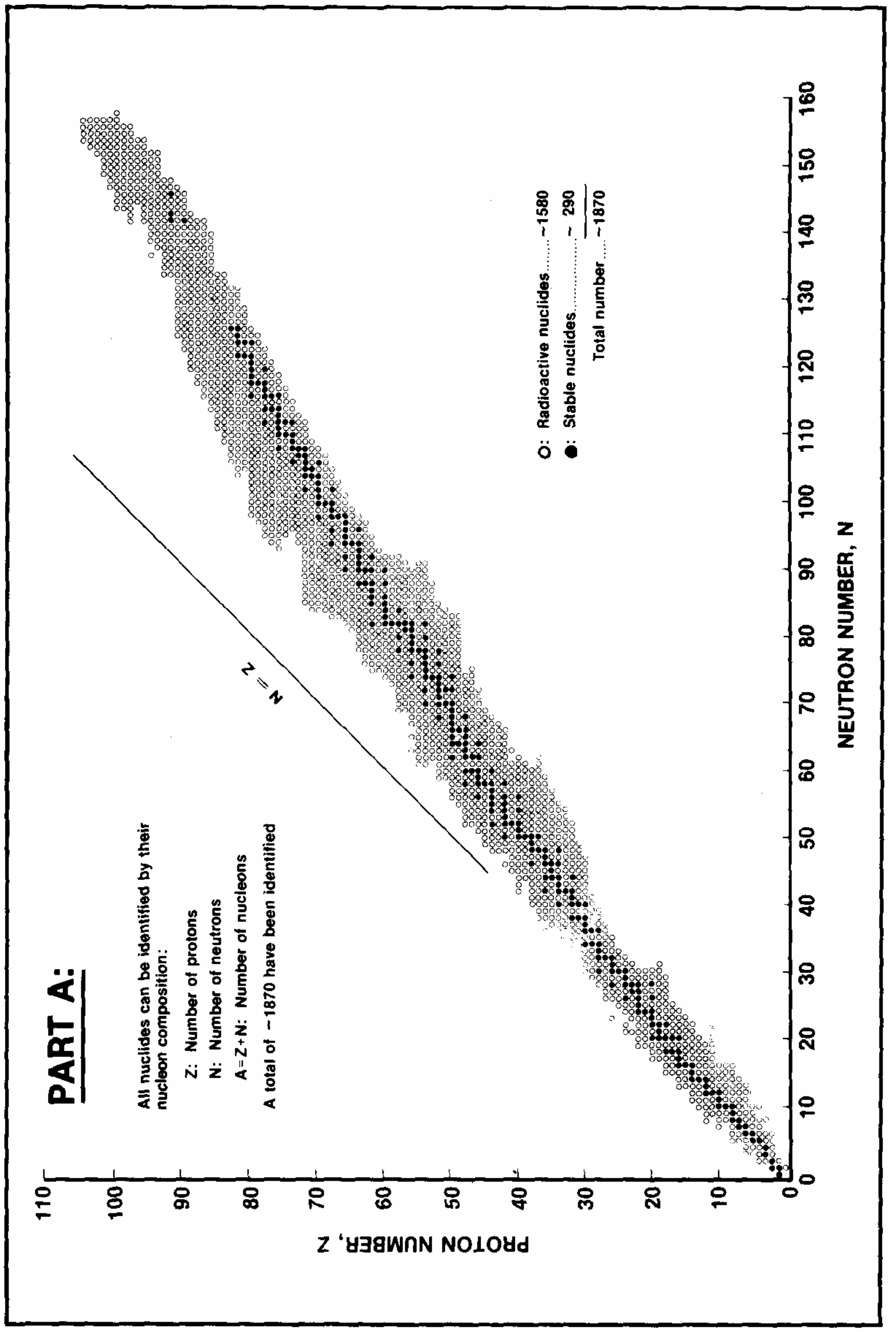


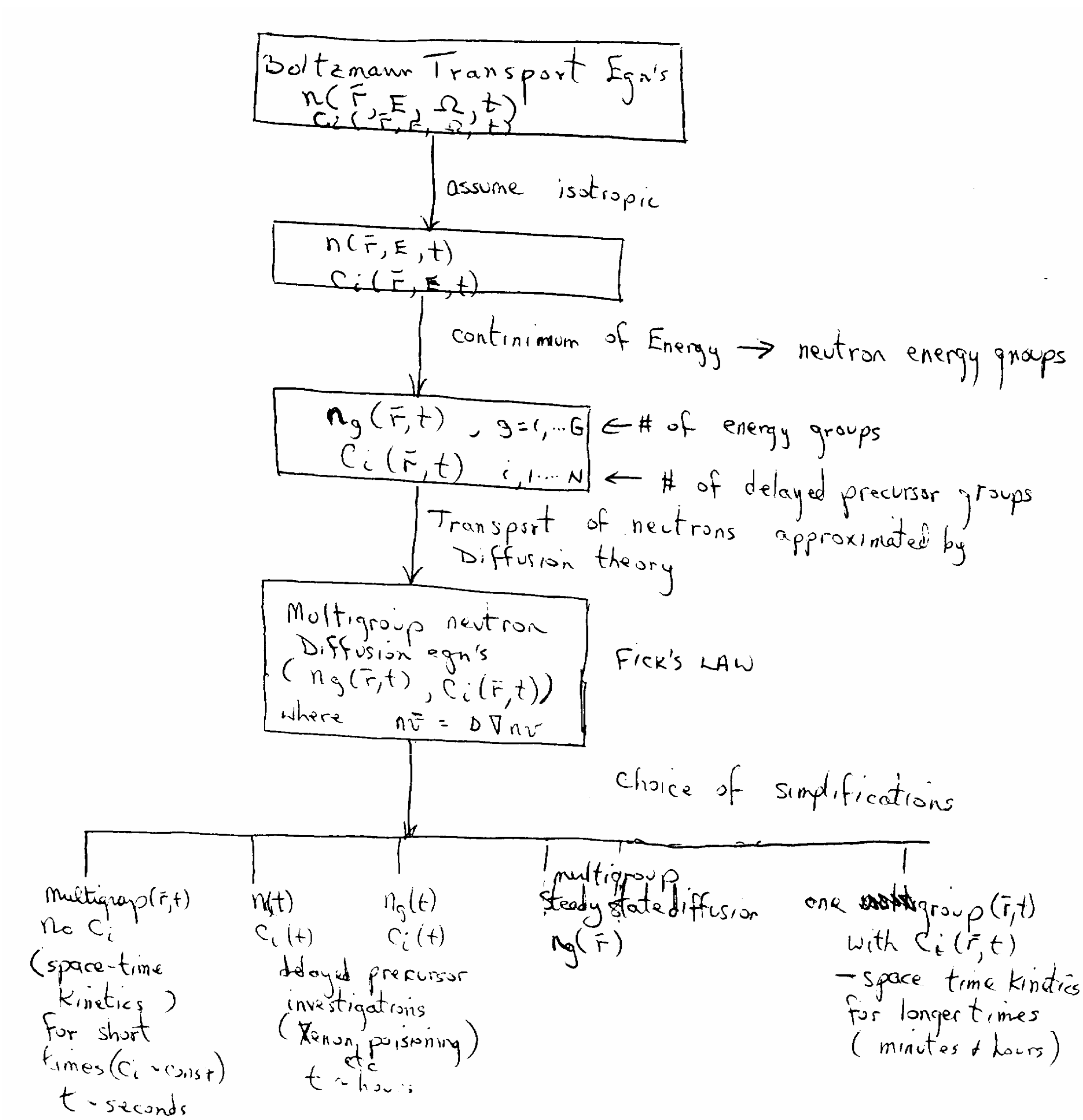
Figure 9 Number of neutrons and protons in stable nuclei [Source: A. A. Harms, McMaster University]

7 Summary

7.1 Summary of key concepts

- neutrons do not interact with each other
- life cycle
- neutron energy spectrum
- decay
- chart of the nuclides
- $\sigma, \sum$
- mechanics of collision (addendum)

7.2 Summary of approximations



8 A Look Ahead

8.1 The neutron balance

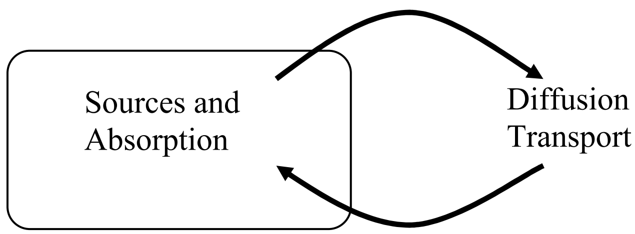


Figure 10 Neutron processes

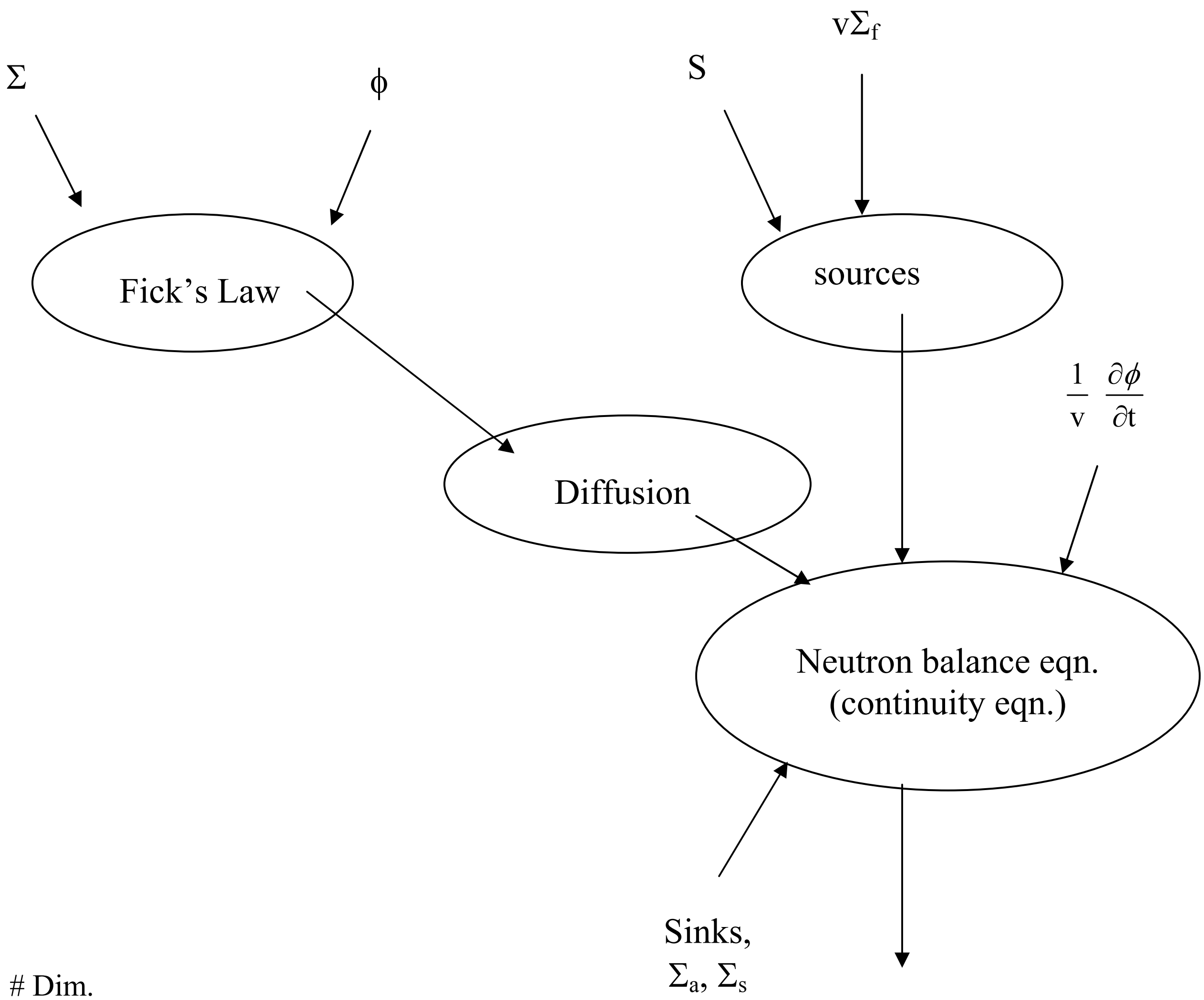
Neutron balance:

$$\frac{\partial n\,(\mathbf{r},t)}{\partial t} \equiv \frac{1}{v}\frac{\partial \phi(\mathbf{r},t)}{\partial t} = S(\mathbf{r},t) - \Sigma_a \phi(\mathbf{r},t) \underbrace{- \nabla \cdot \mathbf{J}(\mathbf{r},t)} \qquad \mathbf{(2)}$$

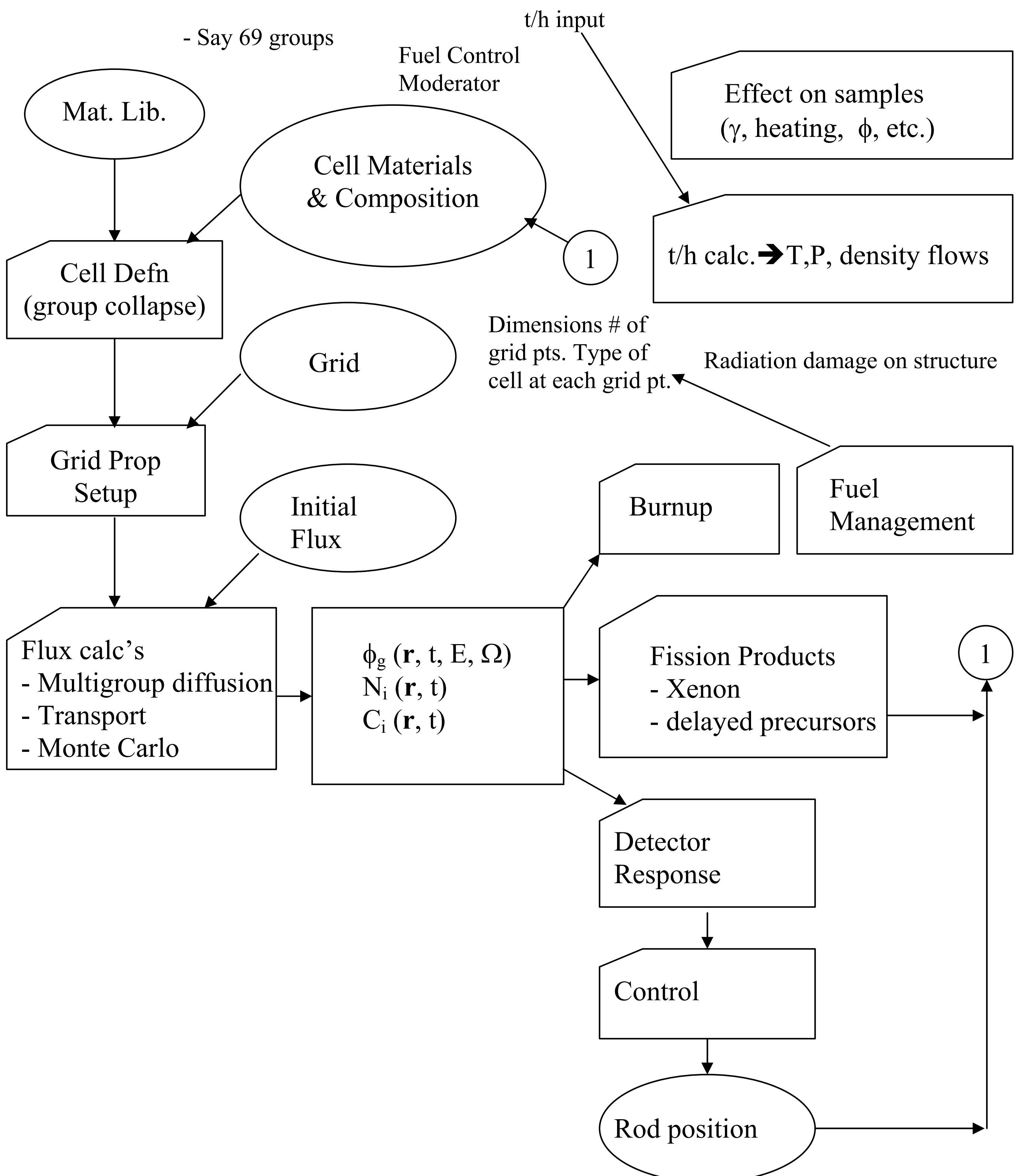
$= +\nabla \bullet D \nabla \phi(\mathbf{r},t)$ from Fick's Law : $\mathbf{J} \equiv -D\nabla\phi$

Reactor physics is all about the calculation of the neutron density, n, or flux, ϕ.

8.2 The Central Role of Flux



\# Dim.
SS or tran.
Delayed precursors
\# of groups
variance in material properties.
- -depletion
- -inhomo
- -control
- -power dependence (Temperature effects)


- Say 69 groups
t/h input
Fuel Control
Moderator
Mat. Lib.
Cell Materials
& Composition
Effect on samples
(γ, heating, ϕ, etc.)
t/h calc. ➔ T,P, density flows
1
Cell Defn
(group collapse)
Grid
Dimensions # of
grid pts. Type of
cell at each grid pt.
Radiation damage on structure
Grid Prop
Setup
Initial
Flux
Burnup
Fuel
Management
Flux calc's
- Multigroup diffusion
- Transport
- Monte Carlo
ϕ_g (**r**, t, E, Ω)
N_i (**r**, t)
C_i (**r**, t)
Fission Products
- Xenon
- delayed precursors
1
Detector
Response
Control
Rod position

9 Some Questions

9.1 Question on characteristics

Given this brief look at neutrons and their life cycle, what are some of the issues/characteristics that you would expect to arise in the design of a nuclear power plant?

9.2 Reactor Modelling Issues

Imagine a reactor consisting of a central fuel region surrounded by a moderator. There is a variable absorber for control. What are some of the issues to consider in setting up a model of the reactor?

9.3 Question of n(E)

Illustrate on a graph of n(E) vs. ln(E) the life cycle of a neutron in a fission reactor.

9.4 Question of non-Maxwellian

Illustrate how the thermal neutron spectrum differs from a Maxwellian and explain why.

9.5 Question on Cross section

Consider:

$$I(x) = I_o e^{-\Sigma x}$$

What are some of the assumptions in or limitations of this equation?

What are some of the things implied by this equation?

About this document:

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Revision 1.0, September 14, 2004, initial creation from hand written notes.

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Notes: